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Two-colour coherent control of multiphoton ionization: a comparison between long-range and short-range potential model atoms*

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Using the numerical solution of the time-dependent Schrödinger equation of a one-dimensional model atom in a two-colour laser field, we have investigated the effects of the potential models on coherent control of atomic multiphoton ionization. It is found that the photoelectron spectra are obviously different for the long-range (Coulomb-like) and short-range (with no excited bound states) potential model atoms, which are produced by two-colour coherent control of atomic multiphoton ionization in a few laser cycles. Our results indicate that two-colour coherent control of atomic multiphoton ionization can be observed in simulations, depending on the choice of the model potentials.

Keywords: coherent control, multiphoton ionization, model potential
PACC: 3280, 4250

1. Introduction

Over the past decade, the coherent control of intense laser–atom interactions by application of two strong lasers of different frequencies, such as controlling multiphoton ionization (MPI)[1−4] and high-order harmonic generation (HHG),[5,6] has attracted many experimental and theoretical investigations. This is for controlling the interaction dynamics through laser parameters such as relative intensity or relative phase difference between the laser field and its harmonics. It was first suggested by Shapiro *et al*[7] that by applying two laser fields, they were able to show that in the weak field limit through quantum mechanical interference the yield of final states of photodissociation could be adjusted. More recent experiments have been carried out by Schumacher *et al*,[8] in which the phase can be determined and controlled accurately. With the initial experimental impetus, the study on two-colour coherent control of intense laser–atom interactions has proceeded rapidly, especially theoretically. Works have been carried out to investigate the effects of intensity control or phase control on scattering,[9] autoionization,[10] MPI and HHG etc. However, for the two-colour coherent control of MPI of atoms, most of previous works were carried out by choosing a long-range model potential. But the short-range potential is also a valid model for the description of a negatively charged ion in an intense laser field.[11−15] To what extent it can also be employed to model an atom in an intense laser field or, in other words, just how important the long-range Coulomb potential is in this situation, has been the object of some debate.[16−18] From this point of view, it is important to compare the two-colour coherent control of atomic MPI of the long-range potential model atom with that of short-range one. However, to our best knowledge, a comparison between the long-range and short-range potential model atoms has not been dealt with yet.

In this paper, we will choose two types of atomic potentials commonly used in the studies on two-colour coherent control of atomic MPI. One is the long-range potential, with a Coulomb tail, which simulates the dynamics of ionization in a single-active-electron atom system, and the other is the short-range one with an exponential decay, which simulates the dynamics of ionization in negatively charged ions. The main difference is that our two types of atomic potentials have a uniform ground-state energy, which is convenient for comparing the effects of different model potentials on two-colour coherent control of atomic MPI. For the external laser fields, we choose a combination of a laser

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field and its seventh-order harmonics field. Indeed, most previous studies of two-colour coherent control atomic MPI considered the low-order harmonics, typically second (2ω) or third (3ω) harmonics, with intensities comparable to that of the fundamental.

This paper is organized as follows. In Section 2 we will introduce our theoretical model, which is a one-dimensional atom in an intense laser field combined with its seventh-order harmonics. We describe the numerical procedure for solving the time-dependent Schrödinger equation (TDSE) and the methods used to calculate the MPI spectra of the atom. In Section 3 we present a set of simulations performed for investigating the two-colour coherent control of MPI for the long-range and the short-range potential model atoms, and make the comparison between them. The conclusions are given in Section 4.

2. Theoretical method

For a one-dimensional model atom under the influence of the time-dependent external laser field the dynamical behaviour of laser–atom interaction can be described by the TDSE. In the dipole approximation, it is given by (atomic units are used throughout this paper)

$$\mathrm{i}\frac{\partial\psi(x,t)}{\partial t} = [H_0 + xE(t)]\psi(x,t), \tag{1}$$

where H_0 is the one-dimensional atomic Hamiltonian, which is given by

$$H_0 = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x). \tag{2}$$

Here $V(x)$ is the atomic potential. In order to model the influence of the potential on the coherent control of atomic MPI in the external laser fields, we have chosen two different model potentials: one is long-range potential,[19] which is given by

$$V(x) = -\frac{1}{\sqrt{a+x^2}}, \tag{3}$$

where the choice of parameters a determines the depth and the range of the potential. It is then possible to select the number and the energies of the bound states supported by this potential. Here, with $a = 2.0$, the ground-state energy is $E_0 = -0.5$ a.u. It behaves asymptotically as a one-dimensional Coulomb potential and supports infinite numbers of bound states. This potential has been shown to be useful for reproducing the main features of laser–atom interactions in the strong-field regime for a linearly polarized laser. And the other is short-range potential[20] with an exponentially decreasing tail, supporting only finite numbers of bound states. They can mimic the dynamics of negative-ion species. It is given by

$$V(x) = -b\exp\Big(-\frac{x^2}{c}\Big), \tag{4}$$

where the choice of parameters b and c determines the ground-state energy and the number of bound states. Here, for $b = 1.986$ and $c = 1.202$, there is only one bound state with energy $E_0 = -0.5$ a.u., i.e., the same as for the long-range potential discussed above.

For external laser fields, we choose a combination of a laser field and its seventh-order harmonics field,

$$E(t) = F_{\mathrm{L}}\sin(\omega_{\mathrm{L}}t) + F_{\mathrm{H}}\sin(\omega_{\mathrm{H}}t + \phi), \tag{5}$$

where F_{L} and F_{H} are the field amplitudes for the fundamental and seventh-order harmonics,respectively; ω_{L} and ω_{H}(with ω_{H}=7ω_{L}) are the corresponding frequencies; ϕ is the relative phase between the two fields.

In order to obtain the photoelectron energy spectra, we first find energy eigenvalues E_n and eigenvectors $\varphi_n(x)$ of the one-dimensional atomic Hamiltonian H_0. The eigenvectors $\varphi_n(x)$ satisfy the equation

$$H_0\varphi_n(x) = E_n\varphi_n(x). \tag{6}$$

In our calculations, the eigenvectors $\varphi_n(x)$ are constructed from the bound states without the external laser fields and the linear combination of the truncated Fourier series.[21] It can be expanded as

$$\varphi_n(x) = \sum_{i=1}^{N_1} A_{ni}(x)\xi_i(x) + \sum_{i=1}^{N_2}\Big[B_{ni}\cos\Big(\frac{n\pi x}{x_{\max}}\Big) + C_{ni}\sin\Big(\frac{n\pi x}{x_{\max}}\Big)\Big], \tag{7}$$

where A_{ni}, B_{ni} and C_{ni} are expanded coefficients, $x_{\max}$ is the maximal range of space, $\xi_i(x)$ are the bound state wavefunctions for H_0, which are constructed from the B-spline functions,[22,23] and can be obtained by solving Eq.(6).

We deal with the problem in this way because the B-spline functions are used to construct the bound states mostly, while the sine and cosine functions are used to generate pseudo-continuum states to represent the continuum spectra. Substituting Eq.(7) into Eq.(6) and diagonalizing it, we can not only obtain energy eigenvalues and eigenvectors of the bound states, but also the positive-energy eigenvalues and their corresponding eigenvectors. Next, we solve the time-dependent Schrödinger equation (1) by

the split-operator method.[24] Thus we obtain numerically the time evolution of the wavefunction $\psi(x,t)$. To find the energy spectra of the ionized photoelectrons, let the evolved wavefunction be overlapped with the continuum-energy eigenstates. The continuum is modelled by a discrete set of pseudostates. In this case, we define the photoelectron spectra as[25]

$$P\Big(\frac{E_{n-1}+E_n+E_{n+1}+E_{n+2}}{4}\Big) = \frac{|\langle\varphi_n(x)|\psi(x,T)\rangle|^2}{E_{n+1}-E_{n-1}} + \frac{|\langle\varphi_{n+1}(x)|\psi(x,T)\rangle|^2}{E_{n+2}-E_n}, \quad (8)$$

where $\psi(x,T)$ is the wavefunction at the end time of the laser pulse. In this equation we take into account the large differences in population between adjacent states of opposite parity in the continuum.

3. Results and discussion

In this section, we will rely on a high-precision numerical solution of TDSE to investigate how the form of the model potential affects the coherent control of atomic MPI. We first compare the photoelectron spectra of the long-range potential model atoms (Eq.(3)) with those of the short-range potential (Eq.(4)) ones, which are produced by two-colour coherent intensity control of atomic MPI in 8.25 laser cycles of fundamental frequency. The results are illustrated in Fig.1. Here, we consider the radiation pulses containing the fundamental frequency $\omega_{\rm L} = 0.1$ a.u. and its 7th harmonics with $\omega_{\rm H} = 0.7$ a.u., lying well in the ultraviolet range. The harmonics field intensity $F_{\rm H} = 0.0675$ a.u. is kept fixed, while the fundamental field intensity is varied among $F_{\rm L} = 0$ (Fig.1(a)), $F_{\rm L} = 0.0055$ a.u. (Fig.1(b)), $F_{\rm L} = 0.021$ a.u. (Fig.1(c)), and $F_{\rm L} = 0.044$ a.u. (Fig.1(d)).

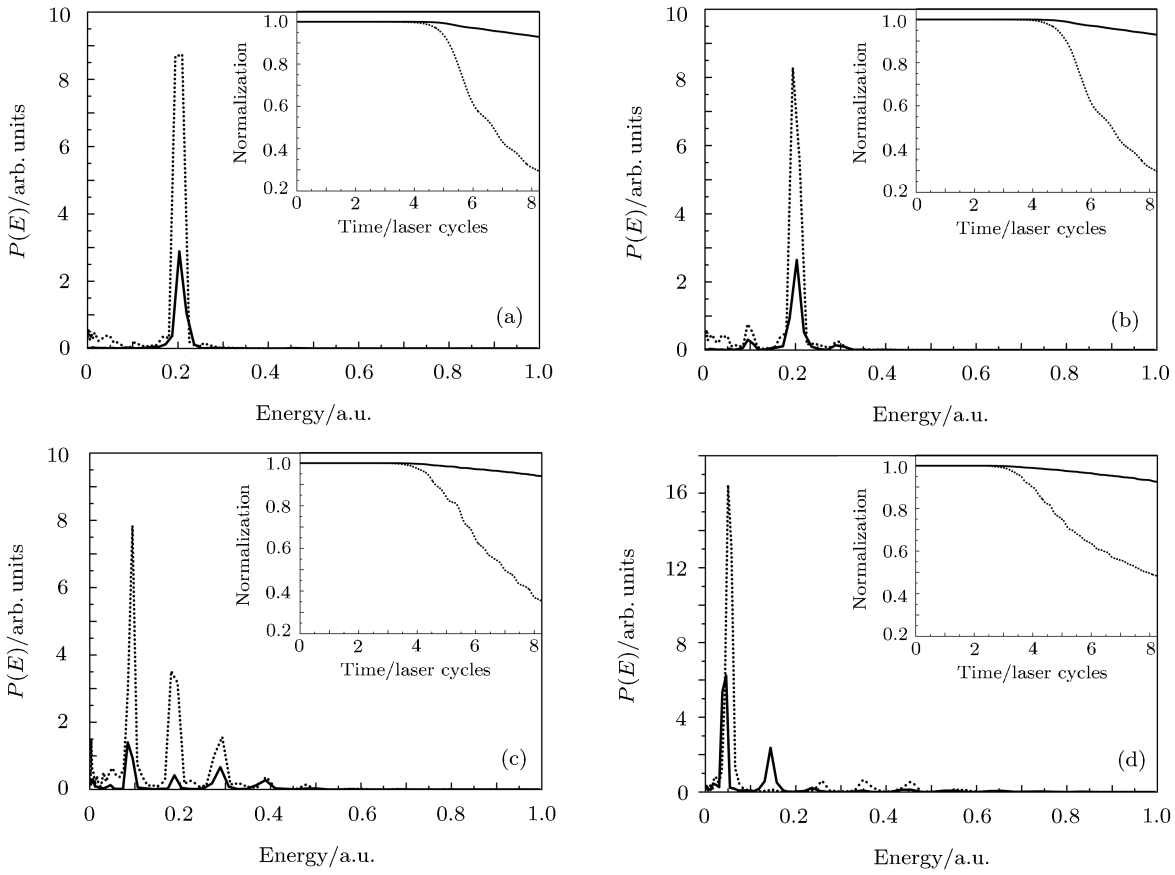


Fig.1. Photoelectron spectra obtained by using two-colour coherent intensity control of atomic MPI for the long-range potential, Eq.(3) (solid line), and short-range potential, Eq.(4), (dotted line). The radiation pulses contain the fundamental frequency $\omega_{\rm L} = 0.1$ a.u. and its 7th harmonics, and the duration of 8.25 laser cycles of fundamental frequency. Two-colour field amplitude $F_{\rm H} = 0.0675$ a.u. is set at a fixed intensity, the relative phase between the two laser fields $\phi = 0$. (a) $F_{\rm L} = 0$; (b) $F_{\rm L} = 0.0055$ a.u.; (c) $F_{\rm L} = 0.021$ a.u.; (d) $F_{\rm L} = 0.044$ a.u. The inset shows the corresponding ionization yields.

In Fig.1, we show the photoelectron spectra obtained at some moderate relative intensities of two-colour laser fields for the long-range and the short-range potential model atoms. From Fig.1(a), for $F_{\rm L} = 0$, we can see that only one photoelectron peak is visible. In fact, according to the energy-conservation relation[26]

$$\varepsilon_{\pm n} \approx \omega_{\rm H} \pm n\omega_{\rm L} - |E_0| - U_{\rm p}, \tag{9}$$

where E_0 is the ionization energy for the ground state, and

$$U_{\rm p} = F_{\rm L}^2/4\omega_{\rm L}^2$$

is the ponderomotive energy. Equation (9) represents a good approximation of the kinetic energy of a photoelectron that absorbs n photons (here the absorption of one high-frequency photon $\omega_{\rm H} = 7\omega_{\rm L}$) from the ground state. Therefore, the maximum of the photoelectron peak is almost exactly located at $\varepsilon_0 = \omega_{\rm H} - |E_0| = 0.2$ a.u. In Fig.1(b), for $F_{\rm L} = 0.0055$ a.u., we can see that two satellite peaks are clearly visible and one on each side of the main peak which centres at 0.2 a.u., same as in Fig.1(a). They corresponds to the two-photon transitions, implying the absorption of a total energy $\omega_{\rm H} \pm \omega_{\rm L}$ by the atoms. In Fig.1(c), for $F_{\rm L}$=0.021 a.u., we can see that the number of satellite peaks increases obviously, and the interval between two peaks of the photoelectron spectra is $\omega_{\rm L}$. This is because a large number of laser photons exchange with the atom system. Figure 1(d), which corresponds to $F_{\rm L} = 0.044$ a.u., shows the multiphoton above-threshold ionization (ATI) spectra obtained at higher laser field strengths. We see that there is an important growth of low-energy peaks as well as a significant extension of the spectra toward higher energies.

From the above analysis, we find that the MPI of atoms can be controlled by the relative intensities of the two-colour laser fields. Obviously, we can find an interesting thing that the different model potentials also can affect the magnitudes of the photoelectron spectra obtained by means of two-colour coherent intensity control of atomic MPI. We see that the photoelectron peaks of the short-range potential model atoms are always higher than those of long-range potential one. Indeed, this is because ionization yield is different for different model potentials of atoms. For the short-range potential, the ionization yield is always higher than that for the long-range potential. This is illustrated in the insets of the figures. Therefore, the photoelectron spectra of the short-range potential model atoms are intensified. On the other hand, the whole photoelectron spectra, for both long-range and short-range potentials, experience a shift, within an excellent approximation, towards the ponderomotive energy $U_{\rm p} = 0.048$ a.u.

The above results show the influence of the model potential on the coherent intensity control of atomic MPI. However, in the case of the two-colour coherent phase control, because of the different model potentials, the photoelectron spectra of the system can also be influenced significantly. So we compare the two-colour photoelectron spectra of the long-range potential model atoms with those of short-range potential ones when the relative phase between the two fields is $\phi = 0°$ and $\phi = 180°$, under the same conditions as in Fig.1(d). The results are illustrated in Fig.2.

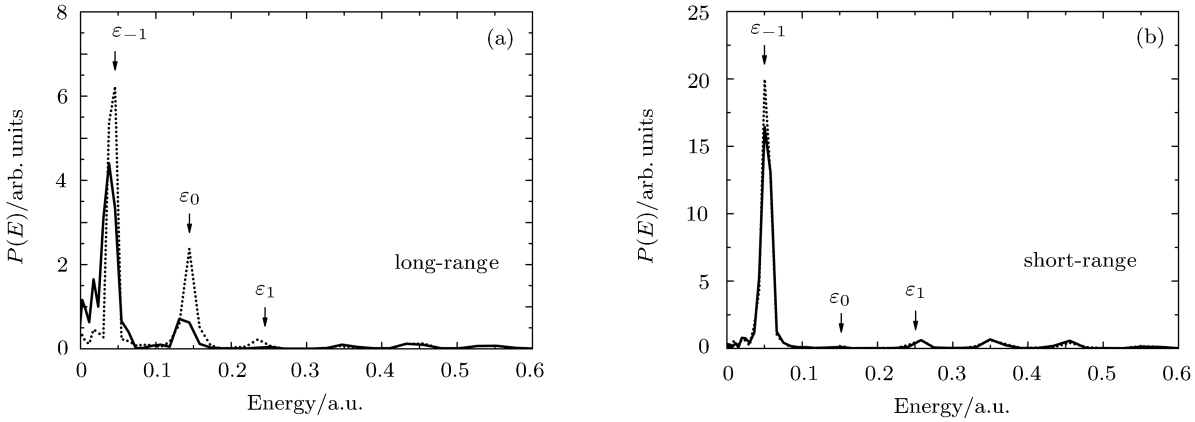


Fig.2. Photoelectron spectra obtained for two-colour coherent phase control of atomic MPI. The conditions are the same as in Fig.1(d). (a) For long-range potential model atom, dotted line, $\phi = 0°$; solid line, $\phi = 180°$. (b) Same as (a), for short-range potential model atom. The peaks with energy $\varepsilon_{\pm n}$(see text) of the photoelectron spectra are indicated by the ε_{-1}, ε_0, and ε_1.

In Figs.2(a) and 2(b), we show the typical dependence of photoelectron spectra on the relative phase between the laser and its harmonics fields for the long-range and short-range potential atoms, respectively. As expected, one observes important changes in the magnitudes of the peaks when the phase is changed. This result is identical with that in most previous studies on two-colour coherent phase control of atomic MPI. However, in contrast with Figs.2(a) and 2(b), we can see that almost all the magnitudes of the photoelectron peaks with energy $\varepsilon_{\pm n}$ for the short-range potential model atoms are higher than those for long-range potential ones when the relative phase is changed. Therefore, the different model potentials can obviously influence the coherent phase control of atomic MPI. The complicated influence of the model potential is illustrated in Fig.3, which shows the dependence of the peaks with energies ε_{-1}, ε_0, and ε_1 of the photoelectron spectra on the relative phase of the two-colour laser fields.

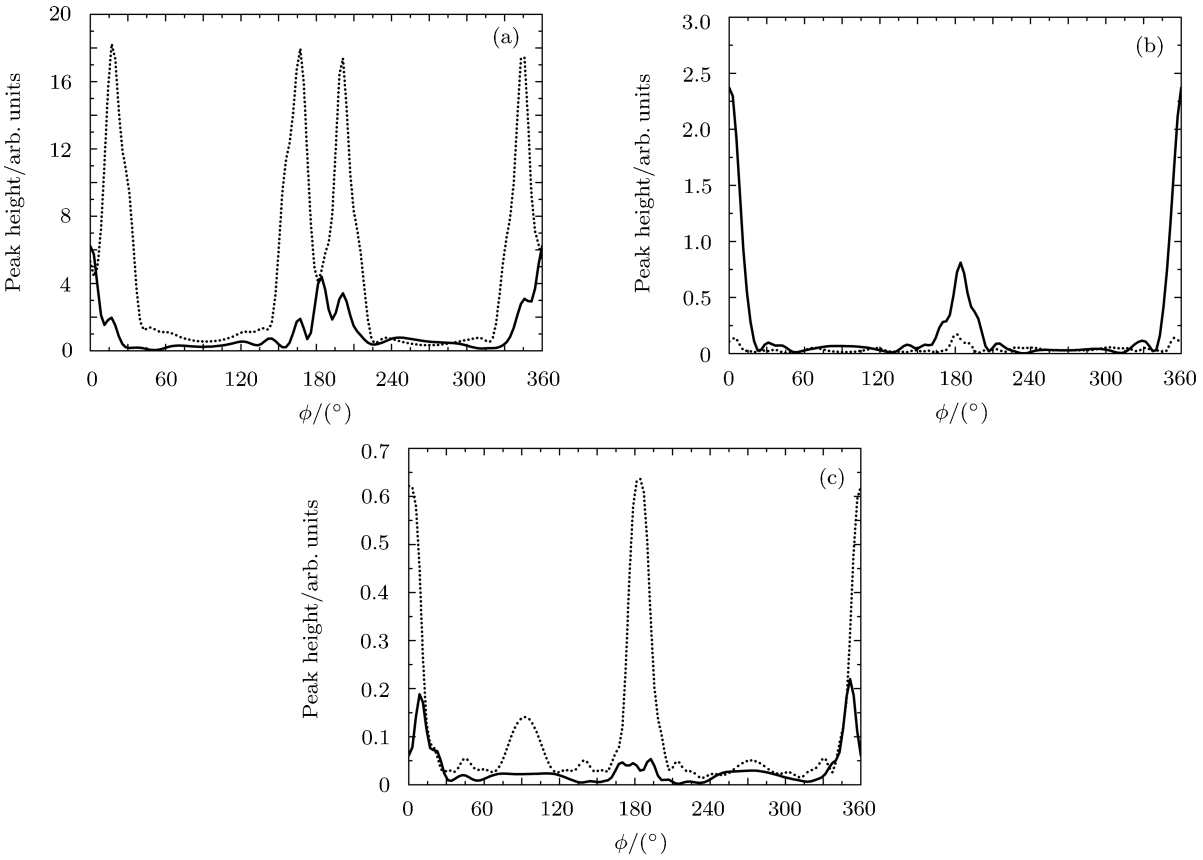


Fig.3. Peak heights of the photoelectron spectra with energy $\varepsilon_{\pm n}$ versus the relative phase of the two-colour laser fields for the long-range potential (solid line) and short-range potential (dotted line) model atoms, under the same conditions as in Fig.1(d). (a) for ε_{-1}; (b) for ε_0; (c) for ε_1.

In Fig.3(a), the peak heights of photoelectron spectra with energy ε_{-1} are shown when the relative phase is changed form $\phi = 0^\circ$ to $\phi = 360^\circ$. It is interesting that there is a slight left-right asymmetry. For short-range potential, the maxima of the peaks of the photoelectron spectra are located at $\phi = 15^\circ$, $\phi = 165^\circ$, $\phi = 195^\circ$, and $\phi = 345^\circ$. But for the long-range potential, the maxima are located at $\phi = 0^\circ$, $\phi = 180^\circ$, and $\phi = 360^\circ$. Furthermore, there is obviously a difference in the magnitudes of peaks for the two different model potentials. For short-range model potential, the peak is always higher than that for long-range potential from $\phi = 0^\circ$ to $\phi = 360^\circ$. In Fig.3(b), which corresponds to peak heights of photoelectron spectra with ε_0, there is an important change that the photoelectron peaks for the long-range potential become higher than those for the short-range potential. For the short-range potential, the photoelectron peaks have almost vanished. Indeed, from Fig.2(b) we can see that, its position is located at 0.15 a.u. Ac-

cording to the energy-conservation relation (Eq.(9)), this position corresponds to $n = 0$, implying there is no net exchange of photons with the atom system. Figure 3(c) shows the peak heights of photoelectron spectra with energy ε_1. We can see that the integrated height of the peaks of the photoelectron spectra for the short-range potential is still higher than that of the long-range potential.

4. Summary

In this paper, considering the different potential model atoms, either long-range or short-range with the same ground-state energy, we have studied the two-colour coherent control of atomic MPI. Our investigations are based on the solution of the TDSE for one-dimensional model atoms. We calculate the photoelectron spectra for the long-range and the short-range potential model atoms, which are produced by two-colour coherent intensity control and coherent phase control of atomic MPI. For two-colour coherent intensity control of atomic MPI, we find that the magnitudes of the photoelectron peaks for the short-range potential are higher than those of the long-range potential. This is because the ionization yields for the short-range potential with a few laser cycles are always higher; while in the coherent phase control case, the dependence of the peaks with energy $\varepsilon_{\pm n}$ of the photoelectron spectra on the relative phase is different for the different model potentials. Almost all peak heights of the photoelectron spectra with energy $\varepsilon_{\pm n}$ for the short-range potential are higher than those of long-range potential when the relative phase is changed. But as expected, the photoelectron peaks for the short-range potential with energy ε_0 almost vanish. Hence, our results indicate that two-colour coherent control of atomic MPI can be observed in simulations, depending on the choice of the model potentials. We believe that this set of results would motivate investigations in both theory and experiments.

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