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Singular value decomposition with normalized period for magnetocaridiography signal processing*

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In this paper, we have developed an algorithm based on singular value decomposition (SVD) for matrix. And the novel SVD algorithm with normalized period of cardiac cycles is presented. The results from real magnetocardiography (MCG) data processing show that the new algorithm is better than the standard one not only in suppressing noises, but also in providing high-fidelity MCG signals.

Keywords: singular value decomposition, magnetocardiography
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1. Introduction

The progress of high-temperature Josephson junction technology provides broader applications for high-temperature superconducting quantum interference devices (SQUIDs). And now, more attention has been paid to SQUID-based magnetocardiography (MCG)—the measurement of biomagnetic and bioelectric effects in human hearts, which is one of the most important SQUID applications. Normally, the field amplitude of MCG is about 100pT. In order to obtain clear signals, the SQUIDs have to be operated in a high-quality shielded room to acquire better performances, which is costly. Researchers have completed many tests by using a simple shielded room to reduce noises, making MCG technique much cheaper and more widely applicable.[1−4]

The procedure of signal collection is perturbed by strong background noises in poor electromagnetic shielding conditions. Therefore, improving the output signal-to-noise (S/N) ratio of systems becomes one of the most important tasks in popularizing the MCG measurement.

Based on the fact that the MCG is of periodic signal, a singular value decomposition (SVD) algorithm has been proposed.[5] It is used for the detection of periodicity of and separation between the component signals. The signal is configured by the sequential extraction of consecutively weakening components with different period lengths. As the MCG is of periodic signal we can use this method to extract the MCG not only from random noises, but also from the different periodic disturbances.

In this paper, we present a method of making the SVD with normalized period. In addition, we also use it to deal with the high-temperature SQUID-based MCG data.

2. SVD algorithm

2.1. SVD principle

The SVD algorithm is based on the theory of matrix analysis.[6] Suppose that we have a series or signal $x(k) = \{x(1), x(2), \ldots, x(mn)\}$ with a period length n, then a typical set of m consecutive periods of length n can be expressed in the from of matrix as

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$$A=\begin{bmatrix} x(1) & x(2) & x(3) & \cdots & x(n) \\ x(n+1) & x(n+2) & x(n+3) & \cdots & x(2n) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x((m-1)n+1) & x((m-1)n+2) & x((m-1)n+3) & \cdots & x(mn) \end{bmatrix}, \quad (1)$$

where $k=1,2,3,\ldots,mn$. The SVD of an $m\times n$ matrix A is defined as[4]

$$A=U\begin{pmatrix} \Delta & 0 \\ 0 & 0 \end{pmatrix}V^{+}\equiv\sum_{i=1}^{r}\sigma_i u_i v_i^{+}, \quad (2)$$

where U is an $m\times m$ matrix; V is an $n\times n$ matrix and $U^{+}U=V^{+}V=I$; Δ is equal to $\mathrm{diag}\{\sigma_1,\ldots,\sigma_r\}$, depending on either $m<n$ or $m\geq n$; $r\leq\min(m,n)$; the singular values of A are represented by $\sigma_1\geq\ldots\geq\sigma_r\geq 0$; the column vectors u_i and v_i, which correspond to singular value σ_i, are called the ith left singular vector and the ith right singular vector respectively.

2.2. Determination of period length by using the singular value ratio (SVR) spectrum

The SVR[7,8] is defined as the ratio of σ_1/σ_2, which varies with the length of row (the length of period) in matrix A referring to Eq.(1). A spectrum is drawn with two variables $(\sigma_1/\sigma_2, n)$, named SVR spectrum. The value of σ_1/σ_2 will decrease if there is a disparity between the number of length n and the signal period. Therefore, the signal period can be measured. In the absence of noise, if we take the length of row as an integral multiple of signal period, then A will be a rank-one matrix (with $\sigma_1/\sigma_2=\infty$). In the presence of weak noises, σ_1/σ_2 is very high. For a random series, the value of σ_1/σ_2 approaches 1 in the sense of statistics. However, a slightly erroneous selection of row length would not produce a large σ_1/σ_2 due to the misalignment. As a result, the SVR spectrum can not only detect periodicity of signal, but also determine the length of period.

2.3. Principal pattern reconstruction

Matrix A is built with MCG data in terms of sampling frequency and rhythm of heart beating. The row length of data matrix A is variable, so that we can construct many matrices with different columns and rows. Then, we calculate the SVR of each matrix and plot an SVR spectrum. If data matrix A is formed with a varied row length n, the corresponding pattern of σ_1/σ_2 of A will peak at the values of n for which there is a dominant periodic component of period length n. In other words, the period length n is an estimated period length $\hat{l}$ of MCG signal.

Let σ_1 be the largest singular value of the data window matrix (formed using a row length determined from the SVR spectrum) and u_i and v_i be the corresponding left and right singular vectors. The rows of $\sigma_1 u_1 v_1^{+}$ may serve as possible reconstruction of the corresponding cycles. In this manner, a cycle may be estimated from each data window containing it. Then the sequence obtained by lining up the cycles may be called the principal pattern reconstruction of the signal. In Eq.(2) σ_i is the ith characteristic value of matrix A, $u_i v_i^{+}$ is an $m\times n$ matrix and also is the ith characteristic pattern. The principal pattern contributes to the reconstruction of matrix A in proportion to singular value σ_1. If the data of m rows in A matrix are all linearly independent, namely all singular values $\sigma_i\neq 0$, we can reconstruct matrix A by using all of the characteristic patterns besides the principal pattern. Contrarily, if the data of m rows in matrix A are all linearly dependent, then A will be a rank-one matrix (having only $\sigma_1\neq 0$), so we can reconstruct matrix A of principal pattern $\sigma_1 u_1 v_1^{+}$. Most energy of the signal is contained in the early characteristic patterns, because the singular values are arranged in a descending order. So we can choose a few characteristic patterns which correspond to bigger singular values to reconstruct A.

Based on the period length $\hat{l}$ determined from the SVR spectrum we can reconstruct the data window matrix as follows:

$$A_{m\times\hat{l}} = \begin{bmatrix} x(1) & x(2) & x(3) & \cdots & x(\hat{l}) \\ x(\hat{l}+1) & x(\hat{l}+2) & x(\hat{l}+3) & \cdots & x(2\hat{l}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x((m-1)\hat{l}+1) & x((m-1)\hat{l}+2) & x((m-1)\hat{l}+3) & \cdots & x(m\hat{l}) \end{bmatrix}. \quad (3)$$

If the S/N ratio of MCG signal is large and σ_1 is larger than the other singular values, then we can reconstruct the MCG signal with the principal pattern, from which most of noises have been removed.

2.4. Real MCG signal processing

2.4.1. . Standard SVD algorithm

We use a standard SVD algorithm to deal with the MCG data. The real MCG signal is shown in Fig.1 and the sampling frequency is 2 kHz. First, we take an MCG signal to construct many matrices with different columns and rows, and then plot an SVR spectrum of the signal. We estimate a period length of the MCG signal according to sampling frequency and rhythm of heart beating. An extreme point of the SVR spectrum near the estimative period length is a theoretical value of periodicity. The SVR spectrum of eight cardiac cycles in the MCG signal is plotted in Fig.2. It can be seen that the period length l is 2013.

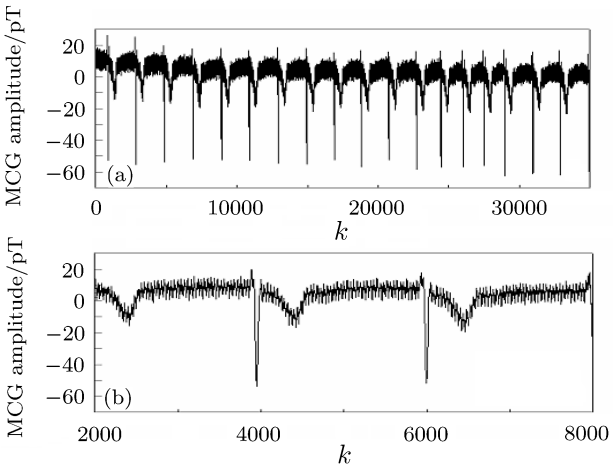


Fig.1. The real MCG signal (a) and its enlargement of the arrowed section (b).

Modifying the MCG data sequence, we construct a data matrix with $l = 2013$, which equals the length of row, and 8 columns. We use the SVD to deal with the matrix and obtain the largest singular value $\sigma_1 = 3.56 \times 10^4$. The data sequence, which is changed from singular matrix $\sigma_1 u_1 v_1^+$, is the principal pattern of the MCG. We use the same method to deal with the 10 cardiac cycles and 20 cardiac cycles of the MCG signal separately. The results are shown in Fig.3.

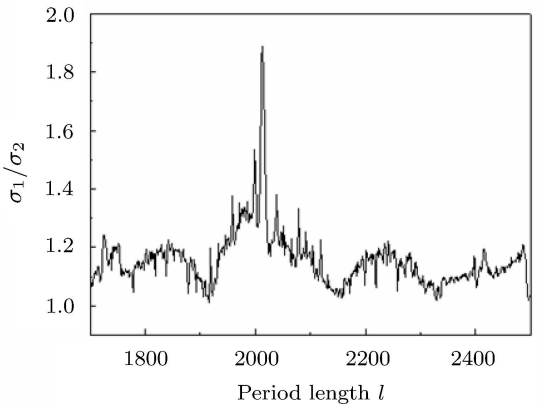


Fig.2. The SVR spectrum of MCG.

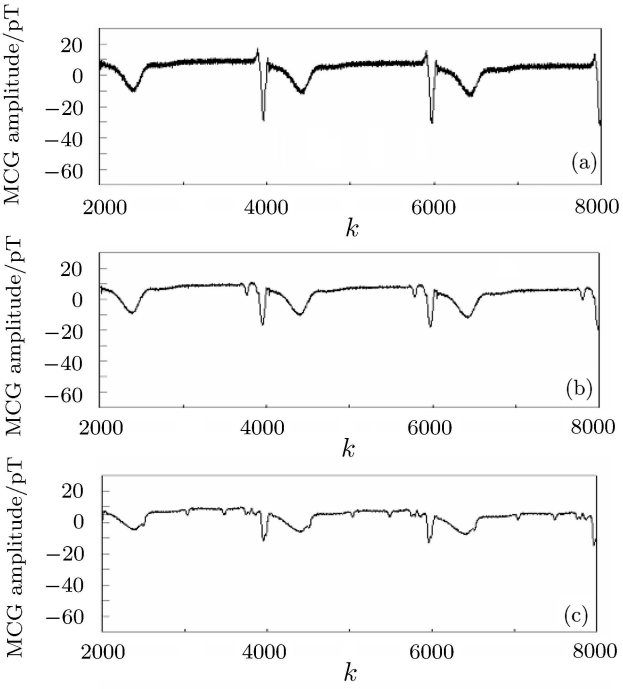


Fig.3. MCG of a standard SVD algorithm dealing with 8 (a), 10 (b) and 20 (c) cardiac cycles separately.

The results in Fig.3 show that the more cardiac cycles we choose, the less the residual noise will be. Furthermore, the amplitude of the MCG signal decreases and the distortion of the MCG signal increases, especially the peak value decreases. These results are

produced by quasi-periodic MCG signals. In Fig.1 we can see that the cardiac cycle lengths are different from each other, and so are the signal amplitudes.

2.4.2. . SVD with normalized period

The standard SVD algorithm applies only to processing exactly periodical signals. But the MCG signals are usually quasi-periodical. In order to not only suppress noises but also provide high-fidelity MCG signals, the SVD algorithm with normalized period is presented.

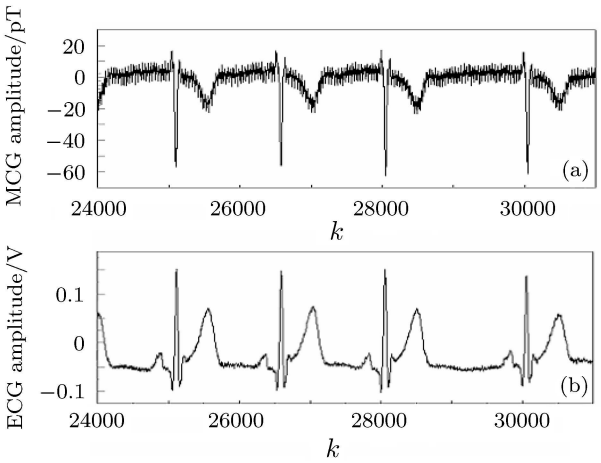


Fig.4. The real MCG signal (a) and the simultaneous ECG signal (b).

Normally, the electrocardiography (ECG) parallel to MCG is more popular to be used than MCG in diagnosis, since the ECG signals are much larger than MCG ones, and clear ECG signals are more easily acquired. Although ECG signals and MCG ones are different, their rhythms are identical for the same person. So we can use the simultaneous ECG (shown in Fig.4) curves to determine the number of data points in each cardiac cycle exactly and normalize the length of each cycle to being identical. We find the corresponding point of maximum of each R[1)]-wave in ECG curves, which is regarded as a starting point of every cardiac cycle of MCG signals. We choose the period in which most data points are presented as a standard period, and name it $\max(n_i)$. Then we complement others to meet the value of $\max(n_i)$ via adding more data points(For example, in the kth period, we add $m_k = \max(n_i) - n_k$ data points, which will be located with even interval in that period. The value is the average of neighbouring points.). By following this way, we have the periods all with the same number of data points. Thus, the quasi-period cardiac signal will be almost the same standard period signal with $l = \max(n_i)$, except for varieties of amplitudes. The data points we added are shown in Table 1, the data sequence of MCG is shown in Fig.5. And Fig.6 shows the SVR spectrum with normalized period. Compared with Fig.2, the SVR with normalized period has sharp peak, which gives proof of a better periodicity. The SVD will be performed with normalized period data. Then we achieve the principal pattern of MCG data corresponding to singular value σ_1 as shown in Fig.7.

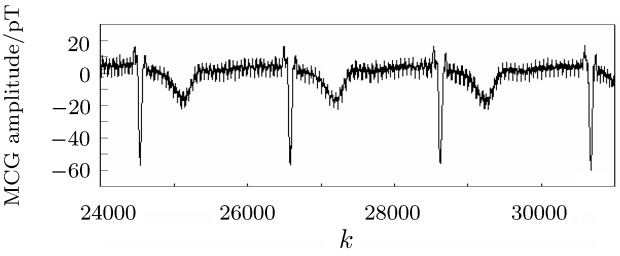


Fig.5. The MCG signal with normalized period.

In order to recover the original cardiac cycles, we remove the added points in the sequence, then we obtain a real time MCG signal after processing (shown in Fig.8).

Table 1. Complemented data of 20 cardiac cycles.

period	1	2	3	4	5	6	7	8	9	10
points	1957	2006	2046	2011	1959	2041	2016	1968	1894	1949
added points	89	40	0	35	87	5	30	78	152	97

period	11	12	13	14	15	16	17	18	19	20
points	1968	1737	1560	1479	1467	1996	1918	2035	2040	2046
added points	78	309	486	567	579	50	128	11	6	0

[1)]R-wave: the maximum value of each cardiac period refers to QRS wave.

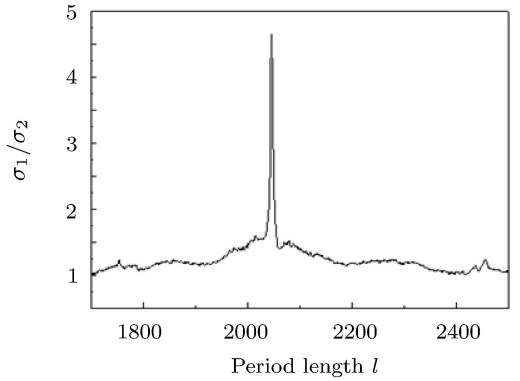


Fig.6. The SVR spectrum with normalized period.

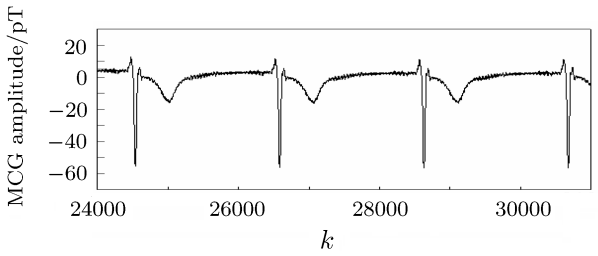


Fig.7. The MCG output data after the SVD with normalized period.

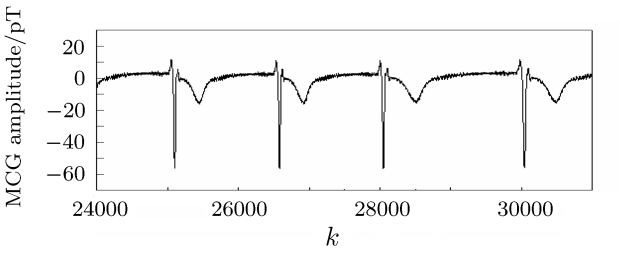


Fig.8. The real time MCG signal after processing.

Figure 9 shows that the S/N ratio of 20 cardiac cycles is better than that of 8 cardiac cycles. The MCG data do not have a distortion after the processing of SVD with normalized period, which is quite different from the regular one. The amplitude of residual noise is related to the number of cardiac cycles being processed. The more cardiac cycle we choose, the less the residual noise will be. After filtering with normalized period, the MCG signals have been well preserved, which is indicated by the peak-to-peak value, and its noise is suppressed more effectively. So the S/N ratio is improved greatly.

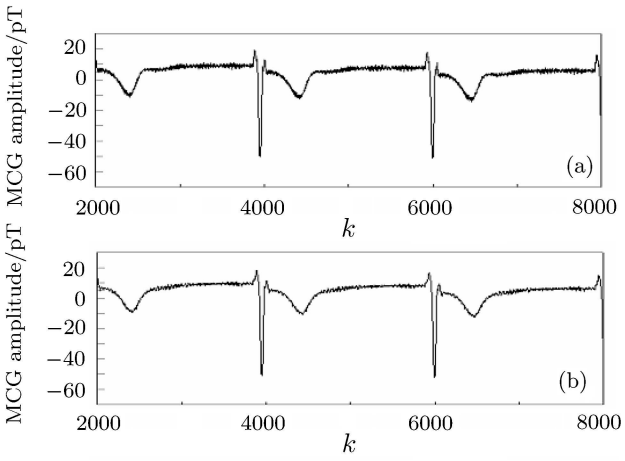


Fig.9. MCG output data of 8 cardiac cycles (a) and 20 cardiac cycles (b).

3. Discussion and conclusion

In this paper, an improved SVD algorithm with normalized period for MCG signal processing is presented. The result shows that the real MCG data processing produces low-noise and high-fidelity MCG signals by using the new algorithm. The simultaneous ECG signals are not required in the standard SVD algorithm. However, it does not fit for the processing of quasi-period signals in the SVD with normalized period.

Compared with the ordinary way of SVD, the new solution not only significantly eliminates noises in signal, but also enhances the fidelity of MCG signals.

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