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# Dynamics of erbium-doped fibre laser with optical delay feedback and chaotic synchronization*

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The dynamical behaviour of the erbium-doped fibre single-ring laser with an optical delay feedback is discussed. Simulation shows that as the delay rate increases, the lasing light displays period-doubling which leads to chaos and via reverse period-doubling route returns from chaos to periodic. At a particular delay rate the intermittently chaotic route to chaos is also observed. The identical synchronization based on chaos in this ring laser is demonstrated by numerical simulation.

**Keywords:** erbium-doped fibre laser, bifurcation, chaotic synchronization
**PACC:** 0545, 0547, 4255N

## 1. Introduction

Erbium-doped fibre lasers have attracted considerable attention for their applications in various fields, such as secure communications, sensing and medicine.[1] Owing to the very fast decay time of its polarization variables, erbium-doped fibre laser can be treated as class B lasers. It cannot perform in chaos because it has only two dynamical equations with two degrees of freedom (DOF). Additional DOF must be supplied to ensure the possibility of chaotic behaviour. Up to now, there have been a great deal of schemes to add the DOF, for instance, adding a single-mode fibre in the Fabry–Perot cavity when the polarization of the lasing light was not controlled.[2] In 1998, Liguo Luo *et al* presented a single-ring fibre laser which could show chaotic behaviour by modulating the pump laser intensity.[3] The dynamical behaviour of an erbium-doped fibre laser, such as quasi-periodic route to chaos, self-pulsing, period-doubling route to chaos and the intermittently chaotic routes to chaos were observed.[2–7] These additional DOF also can be added by a delayed feedback loop acting on a time scale comparable with that of the other two internal DOF.[8] In this paper we theoretically investigate an erbium-doped fibre single-ring laser system. The required DOF in our system is introduced by the optical delayed feedback. As we know, the erbium-doped single-mode fibre laser is CW without the delayed optical signal injection into the system.[3] However when the appropriate delayed optical signal is injected into the system, chaos can be generated via the route of period-doubling or intermittent chaos. In addition the chaos can be also changed to periodic as the rate of injection of delayed optical signal is further increased.

Many researchers have carried out theoretical and experimental investigations on the chaotic synchronization in erbium-doped fibre lasers.[9–15] But few of them have studied the synchronization in erbium-doped fibre laser systems with optical delay feedback. In this paper we investigate the probability of chaotic synchronization using these chaotic fibre lasers. We duplicate an erbium-doped fibre laser system as the driven system. The output of the driving system lasing light is directly injected into the driven system. When the coefficient is suitable in the driving and driven systems, the chaotic synchronization can be realized.

## 2. Theoretical model and dynamical behaviour of the erbium-doped fibre-ring laser

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The setup of erbium-doped fibre single-ring laser with one polarization is shown in Fig.1, which consists of a wavelength-division-multiplexing (WDM) coupler, an erbium-doped fibre, a delay-line, two isolators and a polarization controller. $I_{\rm p}$ is the pump intensity of the ring from the semiconductor. The WDM coupler allows 94% of the pump power at 980nm from the semiconductor laser to go into the resonator but only 7% of the lasing light at 1550nm to exit from the resonator. A coupler C1 is inserted immediately after the output of the laser field. It divides the laser field into two parts: one is the real output $E$, and the other is delayed for a retardation of $\tau_0$ before injection into the ring through coupler $C_2$. The output of single-mode laser light is obtained with single-mode fibre in the system.

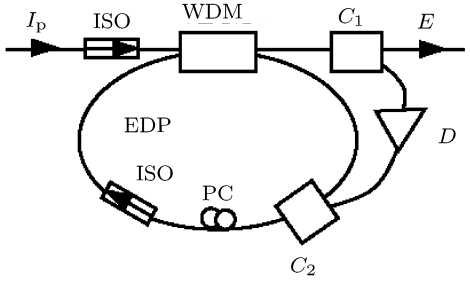



**Fig.1.** Erbium-doped fibre-ring laser system with optical delay feedback. $I_{\rm p}$, pump intensity; $E$, output lasing field; EDF, erbium-doped fibre; $D$, delay-line; PC, polarization controller.

As is known, the erbium-doped fibre laser has generally been treated as a three-level laser system. From Ref.[3], we can see that when the pump intensity $I_{\rm p}$ is CW the output of laser field is also CW without the delayed feedback term. But in this paper the fibre laser system has a delay feedback lasing field, which thus acts as a multidimensional system and shows the chaotic behaviour if condition is suitable. Then the equations for the lasing field $E$ and the population inversion $D$ on this scheme can be described as follows:[3]

$$\dot{E}(\tau) = -kE(\tau) + gE(\tau)D(\tau) - \varepsilon E(\tau - \tau_0),$$
$$\dot{D}(\tau) = -[I_{\rm p} + 1 + |E(\tau)|^2]D(\tau) + I_{\rm p} - 1, \qquad (1)$$

where the dot denotes $\mathrm{d}/\mathrm{d}\tau$ and $\tau = \gamma t$. For an erbium-doped fibre laser, $\gamma = 1/\tau_2$ and $\tau_2$ is the decay time of the lasing upper level and is about 10 ms. $k$ and $g$ are the decayed rate and the gain coefficient of the lasing field, respectively. $\varepsilon$ and $\tau_0$ are the delay rate and retardation of the delayed feedback term, respectively. Obviously the equations are normalized by the decay time of the lasing upper level $\tau_2$, which is taken as 10 ms in the simulation.

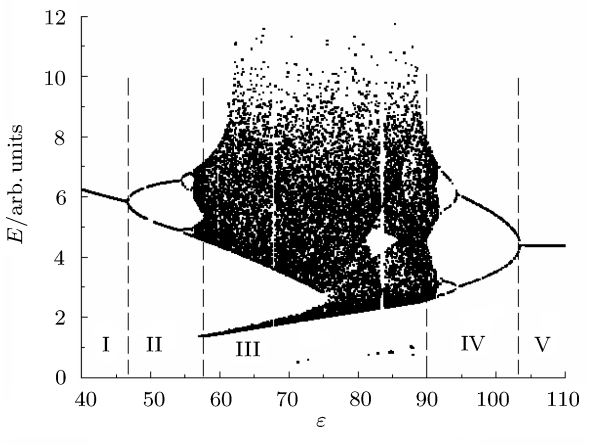



**Fig.2.** The bifurcation of erbium-doped fibre ring laser with delayed feedback by sweeping the delay rate $\varepsilon$.

We choose the appropriate system parameters for equation (1) as follows: $k = 1000$, $g = 4700$, $I_{\rm p} = 4$, $\tau_0 = 0.01$ i.e. the retardation is 100 $\mu$s. With this model we numerically simulate the dynamical behaviour of the erbium-doped fibre single-ring laser system. Figure 2 shows the bifurcation of the lasing field with the delay rate of the delay feedback term $\varepsilon$ increasing from 40 to 110 . We have divided the bifurcation into five areas. When $\varepsilon$ is less than 47, period-one output lasing light is generated. From 47 to 57.5, there occurs the period-doubling and finally becomes chaotic when $\varepsilon$ is further increased. However, there exist two branches of period-doubling routes to chaos. As the different initial value of the system it can be stabilized in different branch and each branch includes a period-doubling route to chaos. Figure 3 shows the period-two attractor projected on the $E$-$D$ plane with different initial values. On the other hand, in the higher region from 90 to 103 a reverse period-doubling behaviour is obtained, i.e. it is period-one for $\varepsilon = 98$ while it is chaotic for $\varepsilon = 91$, which also includes two branches of reverse period-doubling. The output of lasing light in the erbium-doped fibre laser system can also be stabilized at different branches according to different initial value. In the higher region when $\varepsilon > 103$, the lasing light is period-one whatever the initial value is and the developed chaos exists in the large area III. Intermittently chaotic route is also found in the marginal region near 67.5 and 83 as shown in Fig.4. The period-doubling and intermittently chaotic routes to chaos are the typical scenarios in many chaotic systems. All the simulation results

are good guidance for real experiments and communication using this erbium-doped fibre single-ring laser model. From this simulation model we see that this single-ring class B laser field can be chaotic in a very wide region.

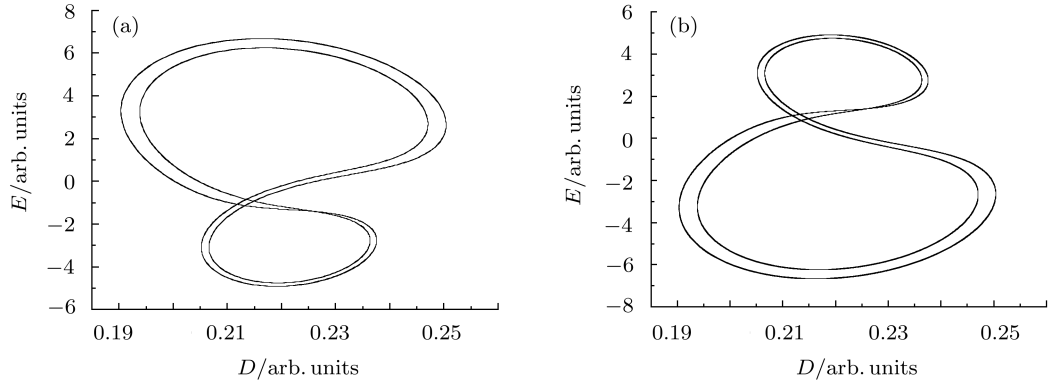



**Fig.3.** Period-two attractors in the phase $E$-$D$ plane.

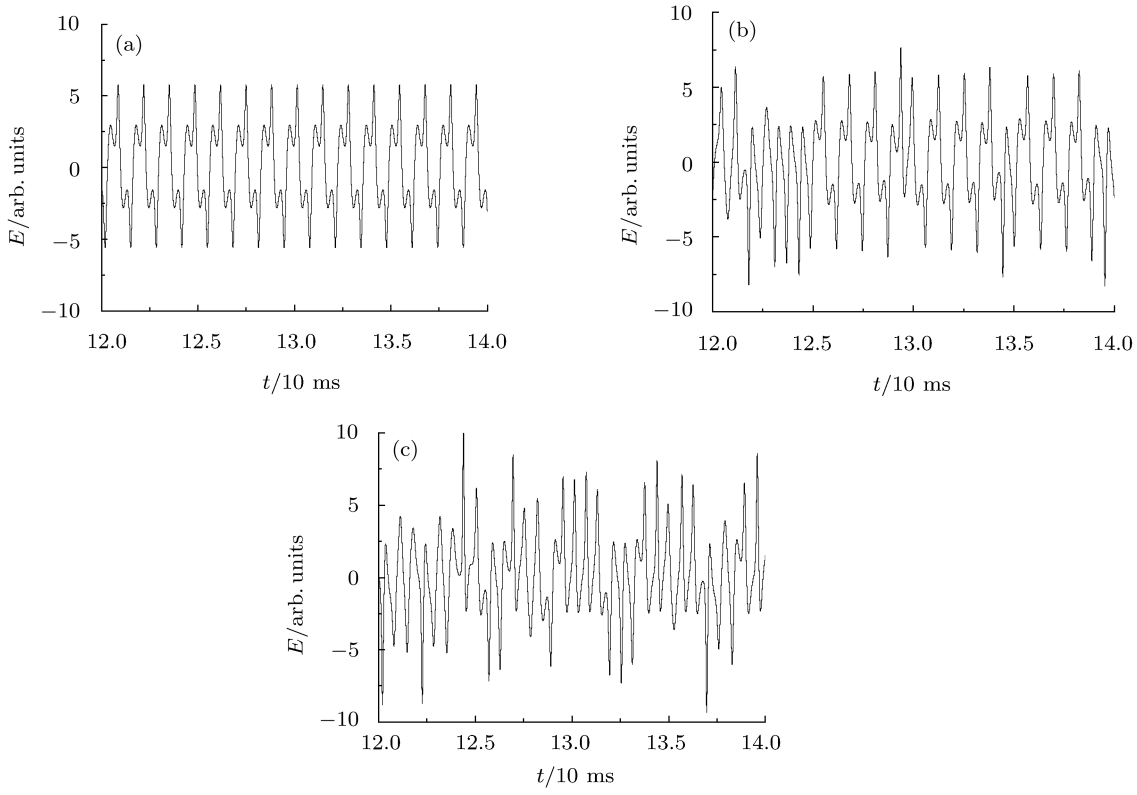



**Fig.4.** The approach with intermittently chaotic route (a) $\varepsilon = 83$, (b) $\varepsilon = 83.5$, (c) $\varepsilon = 84$.

From the dynamic behaviour discussed above, we know chaos can be obtained in the fibre ring laser by providing appropriate coefficients. The strange attractor of chaos is shown on the $E$-$D$ plane in Fig.5(a), and the time series of $E$ in Fig.5(b). We can find that the value seems to change at random, but it is different from irregular random noise signals and seems to change with some regularity.

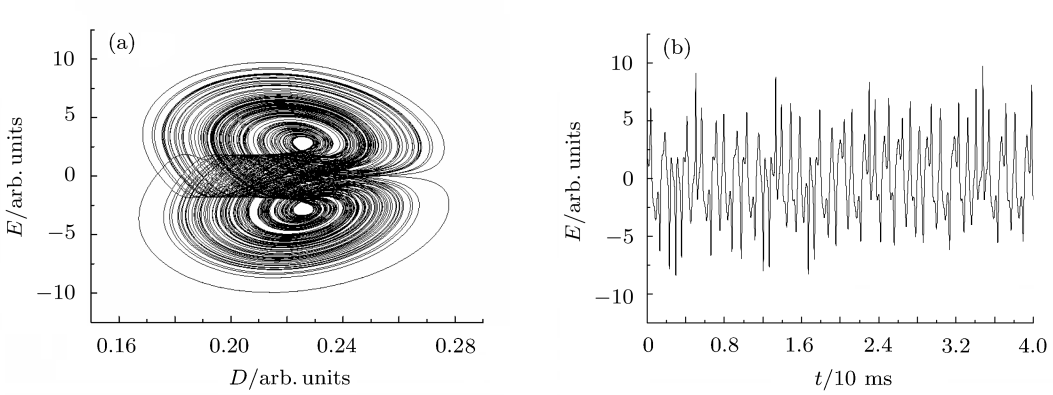



**Fig.5.** (a) The strange attractor projected on the $E$-$D$ plane and (b) the time series of $E$. The system parameters are: $k = 1000$, $g = 4700$, $I_{\rm p} = 4$, $\tau_0 = 0.01$, $\varepsilon = 70$.

## 3. Chaotic synchronization

Chaotic synchronization is an important application of the chaos theory. Many researchers have realized chaotic synchronization in the erbium-doped fibre dual-ring lasers using various schemes.[12−14] But the synchronization system discussed in this paper is an erbium-doped fibre single-ring laser, which is simpler than the dual-ring system. It can realize the chaotic synchronization through a unidirectional coupler,[16] and the synchronized signal can be obtained from the driven subsystem.

We consider a master-slave chaotic synchronization system using erbium-doped fibre single-mode ring lasers as shown in Fig.6.[9] Here the fibre lasers in the master and slave subsystems are identical except for the decay rate of the lasing field. The pump intensities $I_{\rm pm}$ and $I_{\rm ps}$ for the subsystems are constant and large enough to generate the chaotic output. The couplers $C_1$ and $C_6$ are introduced in the master and slave subsystems, respectively, to obtain the output of chaotic laser field $E_{\rm m}$ and $E_{\rm s}$.

The master subsystem can be described by the following equations:

$$\dot{E}_{\rm m}(\tau) = -k_{\rm m}E_{\rm m}(\tau) + g_{\rm m}E_{\rm m}(\tau)D_{\rm m}(\tau) - \varepsilon E_{\rm m}(\tau-\tau_0),$$
$$\dot{D}_{\rm m}(\tau) = -[I_{\rm pm} + 1 + |E_{\rm m}(\tau)|^2]D_{\rm m}(\tau) + I_{\rm pm} - 1. \quad (2)$$

The output lasing light $E_{\rm m}$ from the master subsystem is added to the input of the slave subsystem. Similarly, the slave subsystem is expressed by the following equations:

$$\dot{E}_{\rm s}(\tau) = -k_{\rm s}[E_{\rm s}(\tau) - b_0E_{\rm m}(\tau)] + g_{\rm s}E_{\rm s}(\tau)D_{\rm s}(\tau) - \varepsilon E_{\rm s}(\tau-\tau_0),$$
$$\dot{D}_{\rm s}(\tau) = -[I_{\rm ps} + 1 + |E_{\rm s}(\tau)|^2]D_{\rm s}(\tau) + I_{\rm ps} - 1, \quad (3)$$

where $b_0$ accounts for the introduction of the transmitted signal through coupler $C_4$, and its value equals $(k_{\rm s} - k_{\rm m})/k_{\rm s}$, where $k_{\rm m}$ and $k_{\rm s}$ are decay rates in the master and the slave subsystems, respectively.

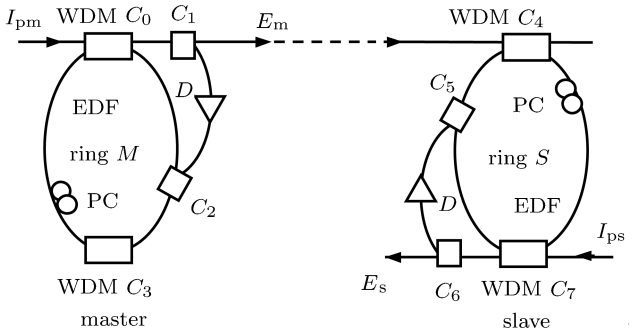



**Fig.6.** Erbium-doped fibre-ring laser system for chaotic synchronization with the master-slave scheme.

The laser field $E_{\rm s}$ in the slave subsystem is driven by $E_{\rm m}$. In order to synchronize the chaos between master and slave subsystems, only a small combined signal is needed to drive the slave subsystem through WDM coupler $C_4$. Then the synchronized chaos signal $E_{\rm s}$ can be obtained from the output of $C_6$. We have numerically obtained the chaotic synchronization between the lasing light of the master subsystem and that of the slave subsystem with the following system parameters: $k_{\rm m} = 1000$, $k_{\rm s} = 1052.6$, $g_{\rm m} = g_{\rm s} = 4700$, $I_{\rm pm} = I_{\rm ps} = 4$, $\tau_0 = 0.01$, $\varepsilon = 70$ and $b_0 = 0.05$, as shown in Fig.7. Figure 7(a) and 7(b) indicate the output of lasing light from the master and the slave subsystems, respectively, with different initial values. The chaotic synchronization can be achieved in a very short time, which can be seen in Fig.7(c). If the amplitude of transmission message is negligibly small, we can realize chaotic communications using this system and the message can be demodulated from the slave subsystem. Hence all these may have potential applications in real secure communications.

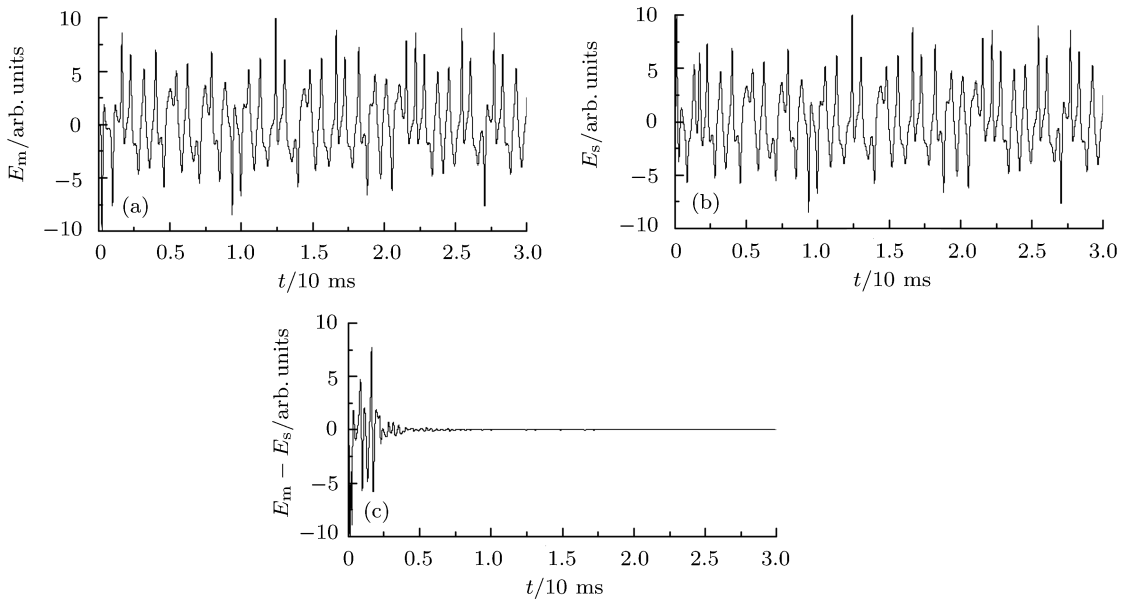



**Fig.7.** Numerical simulation of the chaotic synchronization. (a) $E_m$, (b) $E_s$, (c) $E_m - E_s$.

# 4. Conclusions

In this paper we study the erbium-doped fibre single-mode ring laser with optical delay feedback. With the delay rate of feedback term varying continuously, the period-doubling, the intermittent chaotic routes to chaos and the reverse period-doubling from chaos to periodic are obtained. We have achieved the chaotic synchronization by using the corresponding identical parameters in the two erbium-doped fibre ring laser systems based on the master-slave scheme. The slave subsystem can be identically synchronized to the master subsystem immediately, which provides the possibility of application in secure communications.

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