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# Entanglement swapping of continuous variable using squeezed vacuum states*

Yan Wei(闫 伟)[a)b)†] and Zhang Wei-Jun(张为俊)[a)]

[a)] *Laboratory of Environmental Spectroscopy, Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei* 230031, *China*

[b)] *Department of Physics, Qufu Normal University, Qufu* 273165, *China*

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We present a realistic scheme for the entanglement swapping of continuous variable, in which a two-mode squeezed vacuum state serves as a quantum channel. The position sum and momentum difference of two local modes are measured. By taking the input entangled state also as a two-mode squeezed vacuum state, we investigate the average fidelity and the von Neumann entropy of the output state. The results show that the perfect teleportation can be achieved by increasing the squeezing of the quantum channel and that any nonzero squeezing in both the quantum channel and the input entangled state is sufficient to swap the entanglement.

**Keywords:** entanglement swapping, continuous variable, squeezed vacuum states
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Entanglement is one of the basic resources in quantum information processing, which has a number of applications such as a secret key distribution,[1] quantum teleportation[2,3] and superdense coding.[4−6] Initially, quantum communication dealt almost exclusively with two-valued quantum variables, or qubits. The first demonstration of teleportation was reported by Bouwmeester *et al*,[7] where the polarization state of a single photon was teleported using a pair of polarization entangled photons. Their scheme was close to the theoretical proposal by Bennett *et al*.[2] Later, Vaidman[8] and Braunstein and Kimble[3] proposed the possibility of teleportation of continuous variables such as the quadrature amplitudes of the electromagnetic field. Continuous variable quantum teleportation of arbitrary coherent states has been realized experimentally with bipartite entanglement.[9] The discrete variable and continuous variable teleportations were unified in the framework of canonical quantum teleportation.[10] Since then, much attention has been paid to the study of continuous variables as they might be an unconditional teleportation.[11]

Entanglement swapping may be considered as the quantum teleportation of a part of an entangled state. It means to entangle two quantum systems that have never directly interacted with each other. Hence the entanglement swapping is one of the important methods in quantum communication technology which aims at quantum information network. With single photons, it has already been demonstrated experimentally.[12] Practical uses of entanglement swapping have been suggested[13−17] and the entanglement swapping has also been generalized to multiparticle systems.[18] As for continuous variables entanglement swapping, there have been several theoretical proposals about experiment,[19,20] but so far, no experimental demonstration has been reported.

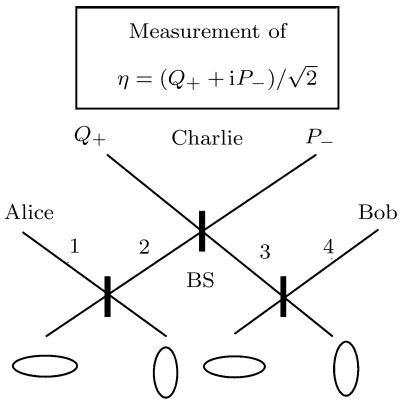



**Fig.1.** Entanglement swapping using four squeezed vacuum states. Before the detection, mode 1 is entangled with mode 2 and mode 3 is entangled with mode 4. Initially suppose that $|\phi\rangle_{12}$ is an unknown entangled state of mode 1 and mode 2.

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†E-mail: yanweiqf@126.com

In this paper, a realistic scheme for continuous variables entanglement swapping using squeezed vacuum states is proposed. With a two-mode squeezed vacuum state (TMSV) used as the input and finite squeezing, we derive the average fidelity of the output entangled state with respect to the input TMSV state in the entanglement swapping process.

Now we introduce 'Alice','Bob', and 'Charlie' to illustrate the whole protocol and subsequence (see Fig.1). Alice and Bob each possess a mode of electromagnetic field, and their modes of electromagnetic field are labelled by 1 and 4 respectively, while Charlie possesses two modes of electromagnetic field, labelled by 2 and 3. Charlie and Bob share a TMSV state of mode 3 and 4

$$|\psi\rangle_{34} = |\mathrm{TMSV}\rangle = \sqrt{1-q^2}\exp[-q\hat{a}_3^\dagger\hat{a}_4^\dagger]|0,0\rangle_{34} = \sqrt{1-q^2}\sum_{n=0}^{\infty}(-q)^n|n\rangle_3|n\rangle_4, \tag{1}$$

where $q$ is a parameter that is a measure of the degree of entanglement, which can be produced by combining two squeezed vacuum modes at a beam splitter (BS).[21] Alice and Charlie share an unknown entangled state $|\phi\rangle_{12}$ of mode 1 and 2. Hence the state for the whole system is $|\Psi\rangle_{1234} = |\phi\rangle_{12}|\psi\rangle_{34}$. In order that Alice and Bob share the unknown state (modes 1 and 4), Charlie has only to make a local measurement of the position sum $Q_2+Q_3$ and the momentum difference $P_2 - P_3$ on his two modes, where

$$Q_2 = \frac{\hat{a}_2+\hat{a}_2^\dagger}{\sqrt{2}}, \quad Q_3 = \frac{\hat{a}_3+\hat{a}_3^\dagger}{\sqrt{2}};$$

$$p_2 = \frac{\hat{a}_2-\hat{a}_2^\dagger}{\sqrt{2}\,\mathrm{i}}, \quad P_3 = \frac{\hat{a}_3-\hat{a}_3^\dagger}{\sqrt{2}\,\mathrm{i}}. \tag{2}$$

The common eigenstates $\{|\eta\rangle\}$ of these two commuting observables are given by the following entangled states of two-mode optical field:[22]

$$\begin{aligned}|\eta\rangle_{23} &= \exp\Big[-\frac{|\eta|^2}{2}+\eta\hat{a}_2^\dagger+\eta^*\hat{a}_3^\dagger-\hat{a}_2^\dagger\hat{a}_3^\dagger\Big]|0\rangle_2|0\rangle_3\\ &= \sum_m \exp[\hat{a}_2^\dagger\eta-\hat{a}_2\eta^*](-1)^m|m\rangle_2|m\rangle_3\\ &= D_2(\eta)\sum_m(-1)^m|m\rangle_2|m\rangle_3,\end{aligned} \tag{3}$$

in which $\eta=\eta_1+\mathrm{i}\eta_2$ is an arbitrary complex number. And we have $(Q_2+Q_3)|\eta\rangle = \sqrt{2}\eta_1|\eta\rangle$, and $(P_2-P_3)|\eta\rangle = \sqrt{2}\eta_2|\eta\rangle$. Because the state $|\eta\rangle$ is complete, i.e. $\int \mathrm{d}^2\eta/\pi|\eta\rangle\langle\eta| = 1$, and orthonormal i.e. $\langle\eta'|\eta\rangle = \pi\delta(\eta-\eta')(\eta^*-\eta'^*)$, it is appropriate to be considered as a continuous Bell basis. After the measurement made by Charlie with outcome $\eta = (Q_+ + \mathrm{i}P_-)/\sqrt{2}$ where $Q_+ = Q_2+Q_3$ and $P_- = P_2-P_3$, the state shared by Alice and Bob reads

$$|\Psi\rangle_{14} = {}_{23}\langle\eta|\phi\rangle_{12}|\psi\rangle_{34}. \tag{4}$$

With the help of the completeness relation of coherent states

$$\frac{1}{\pi}\int|\alpha\rangle\langle\alpha|\mathrm{d}^2\alpha = 1 \ \text{ and } \ \frac{1}{\pi}\int|z\rangle\langle z|\mathrm{d}^2z = 1,$$

one can have

$$\begin{aligned}|\Psi\rangle_{14} &= \frac{1}{\pi^2}\int \mathrm{d}^2\alpha\mathrm{d}^2z\,{}_{23}\langle\eta|\alpha\rangle_1|z\rangle_2|\psi\rangle_{342}\langle z|_1\langle\alpha|\phi\rangle_{12}\\ &= \frac{1}{\pi^2}\int \mathrm{d}^2\alpha\mathrm{d}^2z\sum_m{}_3\langle m|_2\langle m|(-1)^mD_2^\dagger(\eta)|z\rangle_2\sqrt{1-q^2}\sum_n(-q)^n|n\rangle_3|n\rangle_4|\alpha\rangle_{12}\langle z|_1\langle\alpha|\phi\rangle_{12}\\ &= \frac{1}{\pi^2}\int \mathrm{d}^2\alpha\mathrm{d}^2z\sqrt{1-q^2}\sum_n q^n|n\rangle_{44}\langle n|D_4^\dagger(\eta)|z\rangle_4|\alpha\rangle_{14}\langle z|_1\langle\alpha|\phi\rangle_{14} = \sqrt{1-q^2}q^{\hat{N}_4}D_4^\dagger(\eta)|\phi\rangle_{14},\end{aligned} \tag{5}$$

where $\hat{N}_4 = \hat{a}_4^\dagger\hat{a}_4$ is a particle number operator acting on mode 4. In the limit of infinite squeezing ($q\to 1$) the TMSV state which serves as the quantum channel tends to Einstein-Podolsky-Rosen (EPR) state $|\psi_{\mathrm{E}}\rangle_{34} \propto \exp[-\hat{a}_3^\dagger\hat{a}_4^\dagger]|0\rangle_3|0\rangle_4$. As discussed above, we can obtain

$$|\Psi_{\mathrm{E}}\rangle_{14} = {}_{23}\langle\eta|\phi\rangle_{12}|\psi_{\mathrm{E}}\rangle_{34} \propto D_4^\dagger(\eta)|\phi\rangle_{14}. \tag{6}$$

We see that $D_4^\dagger(\eta)$ is a unitary transformation. If Charlie now sends the result of measurement $\eta$ to Bob via a classical channel, Bob is able to perform the unitary operation $D_4(\eta)$ on mode 4 and $|\Psi_{\mathrm{E}}\rangle_{14}$ is then in the state $|\phi\rangle_{14}$. However, without Charlie's measurement results, Alice and Bob cannot use mode 1 and mode 4 for subsequent quantum teleportation because

they cannot even know whether mode 1 and mode 4 are entangled.

To consider the effects of finite squeezing, we also take the unknown state shared by Alice and Charlie as a TMSV state parametrized by $p$ as

$$|\phi\rangle_{12} = \sqrt{1-p^2}\exp[-p\hat{a}_1^\dagger \hat{a}_2^\dagger]|00\rangle_{12}. \tag{7}$$

Because the normalized probability of finding the outcome $\eta$ after the measurement is

$$p(\eta) = (1/\pi)\ {}_{14}\langle \Psi|\Psi\rangle_{14},$$

the fidelity $F(\eta)$ of the output state with respect to the input TMSV state and the average fidelity $\bar{F}$, respectively, read

$$F(\eta) = \frac{1}{\pi P(\eta)}|{}_{14}\langle \Psi_{\rm E}|\Psi\rangle_{14}|^2, \tag{8}$$

$$\bar{F} = \int \mathrm{d}^2\eta P(\eta)F(\eta) = \int \frac{\mathrm{d}^2\eta}{\pi}|{}_{14}\langle \Psi_{\rm E}|\Psi\rangle_{14}|^2. \tag{9}$$

From expression (5) we obtain

$$\begin{aligned}{}_{14}\langle \Psi_{\rm E}|\Psi\rangle_{14} &= \sqrt{1-q^2}\,{}_{14}\langle\phi|D_4(\eta)q^{\hat{N}_4}D_4^\dagger(\eta)|\phi\rangle_{14}\\ &= \sqrt{1-q^2}(1-p^2)\sum_{m,n}{}_4\langle m|_1\langle m|(-p)^m\\ &\quad\times D_4(\eta)q^{\hat{N}_4}D_4^\dagger(\eta)(-p)^n|n\rangle_1|n\rangle_4\\ &= \frac{\sqrt{1-q^2}(1-p^2)}{1-p^2q}\\ &\quad\times\exp\Big[-\frac{(1-q)(1-p^2)}{1-p^2q}|\eta|^2\Big]. \end{aligned}\tag{10}$$

As a result

$$\bar{F} = \frac{(1+q)(1-p^2)}{2(1-p^2q)}. \tag{11}$$

It is easy to see that as $q\to 1$, $\bar{F}\to 1$, i.e. a perfect teleportation is performed.

To characterize quantitatively the entanglement of the output state $|\Psi\rangle_{14}$, we compute its von Neumann entropy of reduced density matrix[23]

$$E_{\rm P} = S(\rho_{_A}) = S(\rho_{_B}) = -\mathrm{tr}(\rho_{_A}\ln\rho_{_A}) = -\mathrm{tr}(\rho_{_B}\ln\rho_{_B}) = -\sum_{i=1}^{m}\lambda_i\ln\lambda_i, \tag{12}$$

where $\rho_{_A} = \mathrm{tr}^{(B)}(|\psi\rangle_{AB\,AB}\langle\psi|)$, and $\rho_{_B} = \mathrm{tr}^{(A)}(|\psi\rangle_{AB\,AB}\langle\psi|)$. Using expression (5), the reduced density matrix $\rho_4$ is then given by

$$\rho_4 = \mathrm{tr}^{(1)}(|\tilde{\Psi}\rangle_{14}\ {}_{14}\langle\tilde{\Psi}|) = q^{\hat{N}}D^\dagger(\eta)p^{2\hat{N}}D(\eta)q^{\hat{N}}/\mathrm{tr}(p^{\hat{N}}D(\eta)q^{2\hat{N}}D^\dagger(\eta)p^{\hat{N}}), \tag{13}$$

where $|\tilde{\Psi}\rangle_{14} = |\Psi\rangle_{14}/\sqrt{{}_{14}\langle\Psi|\Psi\rangle_{14}}$ is a normalized state. Substituting expression (13) into expression (12), direct calculation yields

$$E_p(|\tilde{\Psi}\rangle_{14}) = -\frac{p^2q^2\ln(p^2q^2)}{1-p^2q^2} - \ln(1-p^2q^2). \tag{14}$$

Note that if $q, p \neq 0$, then $E_p > 0$. This means that for any nonzero squeezing in both of two initial sources, there always exists some entanglement between two modes 1 and 4.

In order to compare the entanglement of the output state with that of the input entangled state, we now calculate the entanglement of the input TMSV state as shown below:

$$E_p(|\mathrm{TMSV}\rangle) = -\frac{p^2\ln p^2}{1-p^2} - \ln(1-p^2). \tag{15}$$

Comparing expression (14) with expression (15), we find that for the same $p$ and $q$, $E_p(|\mathrm{TMSV}\rangle) > E_p(|\Psi\rangle_{14})$, which implies that the entanglement decreases after the entanglement swapping as expected because only local operations and classical communications are involved.

In conclusion, we have shown how a two-mode squeezed vacuum state can be used for entanglement swapping of arbitrary quantum state by measuring the joint quadrature phase quantities, $Q_1+Q_2$ and $P_1-P_2$. If the entanglement source is squeezed infinitely, a perfect teleportation can be achieved; if the entanglement source is squeezed finitely and the input state is also a TMSV state we derive the average fidelity $\bar{F}$ (which tends to 1 in the case of infinitely squeezing) and the entanglement of the output state. It turns out that the entanglement swapping is always successful for any nonzero squeezing of the initial sources with a lower entanglement. Entanglement swapping entangles two modes that have never directly interacted with each other. Hence the distribution of entangled states between distant nodes is of great importance in the future large-scale realization of quantum communication protocols.

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