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Controlled teleportation of multi-qudit quantum information*

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We propose a scheme for realizing a controlled teleportation of random M-qudit quantum information under the control of N agents. The resource consumption includes a prearranged $(2M+N+1)$-qudit entangled quantum channel and $(2M+N+1)\log_2 d$-bit classical communication. And the quantum operations used in the teleportation process are a series of generalized Bell-state measurements, single-qudit measurements, qudit H-gates, qudit-Pauli gates and qudit phase gates. It is shown that the original state can be restored by the receiver only on condition that all the agents work in collaboration with each others. If one agent does not cooperate with the other, the original state cannot be fully recovered.

Keywords: qudit, controlled teleportation, agent
PACC: 0367, 0365

Quantum teleportation is a process that a sender (Alice) transmits an unknown quantum state to a receiver (Bob) at a distance via a previously prepared quantum channel with the help of classical communication.[1] It is the most tangible manifestation of quantum nonlocality[2] and plays an important role in quantum computer,[3,4] quantum cryptography,[5−8] quantum superdense coding,[9] and quantum secure direct communication.[10] Since Bennett *et al*[1] originally proposed the first scheme to teleport an unknown quantum state of one-particle from one place to another, quantum teleportation has become a topic of many researchers. Nowadays, quantum teleportation has well developed both theoretically and experimentally.[11−15] Theoretically, it has been extended to teleporting multi-particle systems,[16−18] continuous-variable systems,[19,20] and controlled teleportation.[21,22] Experimentally, the quantum teleportation has been realized by using photon polarized states,[11] optical coherent states,[23] and nuclear magnetic resonance.[12]

In 1998, Karlsson and Bourennane[24] showed how to teleport an arbitrary unknown quantum state of one qubit to two receivers. But only one receiver could restore the qubit state on condition that another receiver told him the measurement outcome. Later, Hillery *et al*[25] first presented a concept of quantum secret sharing—splitting a message among many spatially-separated agents so that no subset of agents, only the entire set, is sufficient to obtain the quantum message. In 2000, Zhou *et al*[26] proposed a scheme for the controlled teleporting of an unknown quantum state of one-particle. Different from the quantum secret sharing, the controlled teleportation is the process that a sender transmits quantum message to a receiver via the control of agents in a network. Recently, the quantum controlled teleportation has been investigated extensively.[21,26,27] In Ref.[21] Yang *et al* presented a scheme for teleporting an unknown multi-qubit quantum state via the control of many agents in a network. However, their method has a complicated structure and an intricate computation process. It is difficult to generalize directly their method to the teleportation of multi-particle and multi-level quantum states, as well as to give a complete and clear description. Motivated by this, we propose in this paper a method of the controlled teleportation of multi-qudit state. It is shown that the receiver can successfully restore the original state of multi-qudit on condition that all agents work in collaboration with each others.

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However, if any one of the agents does not cooperate with each others, then the original state cannot be fully recovered by the receiver. Compared with existing approaches, our method has the features of simple structure and less resource consumption, and thus it is efficient.

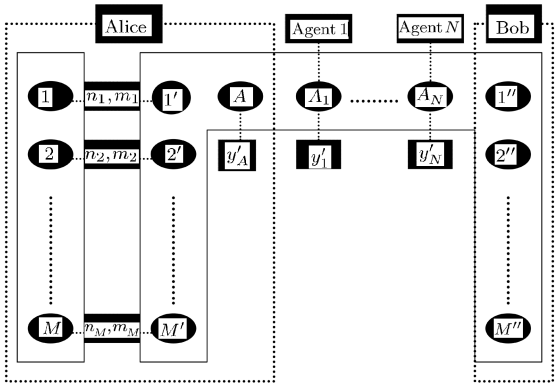


Fig.1. Diagram for controlled teleportation of multi-qudit quantum information. Each ellipse denotes a qudit, where qudits $1, 2, \ldots, M, 1', 2', \ldots, M'$ and qudit A belong to Alice, qudits $1'', 2'', \ldots, M''$ belong to Bob, $A_1, A_2, \ldots, A_N$ belong to N agents respectively. (n_l, m_l) with $l = 1, 2, \ldots, M$ denote the outcomes of Bell-measurement on qudit pair (l, l'), and $y'_A, y'_1, y'_2, \ldots, y'_N$ the outcomes of measurement on qudits $A_1, A_2, \ldots, A_N$ respectively.

In our scheme, the sender (Alice), the receiver (Bob) and N agents are spatially separated (see Fig.1). Alice has $2M + 1$ qudits that include M information qudits $1, 2, \ldots, M$, M channel qudits $1', 2', \ldots, M'$, and one control qudit A, each qudit has levels $|0\rangle, |1\rangle, \ldots, |d-1\rangle$. The information qudits are in the following unknown quantum state:

$$|\Psi\rangle_{\rm I} = \prod_{l=1}^{M} |\Psi\rangle_l = \prod_{l=1}^{M} \sum_{k=0}^{d-1} \alpha_{lk} |k\rangle_l, \tag{1}$$

where

$$\prod_{l=1}^{M} |\Psi\rangle_l = |\Psi\rangle_1 \otimes |\Psi\rangle_2 \otimes \cdots \otimes |\Psi\rangle_M$$

denotes a product state of M qudits, and the complex coefficients are random and fit normalization

$$\sum_{k=0}^{d-1} |\alpha_{lk}|^2 = 1.$$

Bob has M'' channel qudits $1'', 2'', \ldots, M''$ and each agent has one control qudit denoted by $A_1, A_2, \ldots, A_N$ respectively, all these qudits have levels $|0\rangle, |1\rangle, \ldots, |d-1\rangle$ too. Suppose that we have already established an entangled quantum channel among Alice, Bob and N agents,

$$|\Psi\rangle_L = \sum_{s=0}^{d-1} \Big\{ \Big[\sum_{z=0}^{d-1} \exp({\rm i}2\pi zs/d) |z\rangle_A |z \ldots z\rangle_N \Big] \otimes \prod_{l=1}^{M} \Big[\sum_{j=0}^{d-1} \exp({\rm i}2\pi js/d) |j\rangle_{l'} |j\rangle_{l''} \Big] \Big\}, \tag{2}$$

where $|j\rangle_{l'}$ and $|j\rangle_{l''}$ with $l = 1, 2, \ldots, M$ refer to the states of channel qudits of Alice and Bob, $|z\rangle_A$ and $|z \ldots z\rangle_N$ refer to the states of control qudits of Alice and N agents respectively. Now, Alice wants to transmit the unknown quantum state of Eq.(1) to Bob by use of this entangled quantum channel. The initial state of the combined system can be written as

$$\begin{aligned} |\Psi_0\rangle &= |\Psi\rangle_{\rm I} \otimes |\Psi\rangle_L \\ &= \sum_{s=0}^{d-1} \Big\{ \Big[\sum_{z=0}^{d-1} \exp({\rm i}2\pi zs/d) |z\rangle_A |z \ldots z\rangle_N \Big] \\ &\quad \otimes \prod_{l=1}^{M} \Big[\sum_{k=0}^{d-1} \sum_{j=0}^{d-1} \alpha_{lk} \exp({\rm i}2\pi js/d) |k\rangle_l |j\rangle_{l'} |j\rangle_{l''} \Big] \Big\}. \end{aligned} \tag{3}$$

For the convenience of description, we decompose our scheme into three steps. In the first step, Alice first performs a generalized qudit Bell-state measurement on each pair of her information-channel qudits $(1, 1'), (2, 2'), \ldots (M, M')$. Then she and each agent respectively perform a qudit H-gate and a measurement in the bases $\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$ for each of their individual control qudits. Finally, Alice and N agents transmit all of their measurement results to Bob through classical communication. In the second step, according to the information from Alice and N agents, Bob performs two unitary operations on each qudit in his hands. Then he can restore the original teleported state of Eq.(1). Finally, we discuss the controlled property of the teleportation process, and show that even if one agent does not cooperate with the others, the originally teleported state cannot fully be recovered by Bob. The detailed procedure is shown as follows.

The bases of generalized Bell-state measurement for two-qudit system of A and B are defined as

$$|\Phi_{nm}\rangle_{AB} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp({\rm i}2\pi jn/d) |j\rangle_A |j \oplus m\rangle_B, \tag{4}$$

where ${\rm i} = \sqrt{-1}$, $n, m = 0, 1, \ldots, d-1$ and $j \oplus m = (j + m) \bmod d$. It is easily tested that $|\Phi_{nm}\rangle_{AB}$ is orthonormalized and complete with respect to n and

m. Now, if Alice performs a generalized Bell-state measurement on each pair of her information-channel qudits (l, l') with $l = 1, 2, \ldots, M$, and suppose that the outcome of measurement is (n_l, m_l), then expression (3) reduces to

$$|\Psi_{nm}\rangle \propto \sum_{s=0}^{d-1}\Big\{\Big[\sum_{z=0}^{d-1}\exp(\mathrm{i}2\pi zs/d)|z\rangle_A|z\ldots z\rangle_N\Big] \otimes\prod_{l=1}^{M}\Big[\sum_{k=0}^{d-1}\alpha_{lk}\exp\Big\{\mathrm{i}\frac{2\pi}{d}[(k\oplus m_l)s-n_lk]\Big\} \times|k\oplus m_l\rangle_{l''}\Big]\Big\}, \tag{5}$$

where we use symbol '$\propto$' to indicate that $|\Psi_{nm}\rangle$ is un-normalized. Subsequently, Alice and each agent respectively perform a qudit H-gate,[28]

$$H=\frac{1}{\sqrt{d}}\sum_{x,y=0}^{d-1}\exp(\mathrm{i}2\pi xy/d)|y\rangle\langle x| \tag{6}$$

on their individual control qudits, thus we have

$$|\Psi'_{nm}\rangle \propto H_A\otimes H^{\otimes N}|\Psi_{nm}\rangle \propto \sum_{s=0}^{d-1}\Big\{\sum_{y_A=0}^{d-1}\sum_{y_1=0}^{d-1}\cdots\sum_{y_N=0}^{d-1}\sum_{z=0}^{d-1}\exp\Big[\mathrm{i}\frac{2\pi}{d}z(y_A+y_1+\cdots+y_N+s)\Big]|y_Ay_1\ldots y_N\rangle_{A,N} \otimes\prod_{l=1}^{M}\Big[\sum_{k=0}^{d-1}\alpha_{lk}\exp\Big\{\mathrm{i}\frac{2\pi}{d}[(k\oplus m_l)s-n_lk]\Big\}|k\oplus m_l\rangle_{l''}\Big]\Big\}. \tag{7}$$

Finally, Alice and N agents measure their control qudits in the bases $|0\rangle, |1\rangle, \ldots, |d-1\rangle$. With the measurement outcomes $y'_A, y'_1, y'_2, \ldots, y'_N$, the above state changes into

$$|\Psi^{\mathrm{Bob}}_{nm}\rangle \propto \langle y'_A, y'_1, y'_2, \ldots, y'_N|\otimes|\Psi'_{nm}\rangle \propto \sum_{s=0}^{d-1}f(s) \tag{8}$$

with

$$f(s)=\sum_{z=0}^{d-1}\exp\Big[\mathrm{i}\frac{2\pi}{d}z(y'_A+y'_1+\cdots+y'_N+s)\Big]\prod_{l=1}^{M}\Big[\sum_{k=0}^{d-1}\alpha_{lk}\exp\Big\{\mathrm{i}\frac{2\pi}{d}[(k\oplus m_l)s-n_lk]\Big\}|k\oplus m_l\rangle_{l''}\Big]. \tag{9}$$

For a given set of measurement outcomes of $y'_A, y'_1, y'_2, \ldots, y'_N$, there is only one value of $s = s_C$ such that $(y'_A + y'_1 + \cdots + y'_N + s) = pd$, i.e. multiples of d (p is an integer), and thus

$$\sum_{z=0}^{d-1}\exp\Big[\mathrm{i}\frac{2\pi}{d}z(y'_A+y'_1+\cdots+y'_N+s)\Big]=\begin{cases}d, & \text{when} \quad s=s_C,\\ 0, & \text{when} \quad s\neq s_C.\end{cases} \tag{10}$$

Dividing the summation of s as $\sum_{s=0}^{d-1}f(s)=f(s_C)+\sum_{s\neq s_C}f(s)$, then expression (8) reduces to

$$|\Psi^{\mathrm{Bob}}_{nm}\rangle \propto \prod_{l=1}^{M}\Big\{\sum_{k=0}^{d-1}\alpha_{lk}\exp\Big\{\mathrm{i}\frac{2\pi}{d}[(k\oplus m_l)s_C-n_lk]\}|k\oplus m_l\rangle_{l''}\Big\} \propto \prod_{l=1}^{M}\Big\{\sum_{k=0}^{d-1}\alpha_{lk}\exp\Big\{-\mathrm{i}\frac{2\pi}{d}[(k\oplus m_l)(y'_A+y'_1+\cdots+y'_N)+n_lk]\}|k\oplus m_l\rangle_{l''}\Big\}, \tag{11}$$

which now completely belongs to Bob. At this time, Alice and N agents tell Bob the values of $(n_1, m_1), (n_2, m_2), \ldots, (n_M, m_M)$ and y'_A, $y'_1, y'_2, \ldots, y'_N$ with the help of classical communication. Obviously, the classical resource needed consuming is $(2M + N + 1)\log_2 d$ bits.

In the second step, Bob will perform two unitary operations on each channel qudit in his hands. The first unitary operation depends on the result of the generalized Bell-state measurement performed by Alice. For qudit l'' of Bob, the operation only depends

on (n_l, m_l) and can be expressed as

$$\Theta_{l''} = Z_{l''}^{n_l} X_{l''}^{d-m_l}, \tag{12}$$

where $l = 1, 2, \ldots, M$ denote that $\Theta_{l''}$ performs on qudits $1'', 2'', \ldots, M''$ respectively, X and Z are the so-called generalized qudit Pauli operators which are defined as[29]

$$X|k\rangle = |k \oplus 1\rangle, \quad Z|k\rangle = \exp(\mathrm{i}2\pi k/d)|k\rangle \tag{13}$$

with $k = 0, 1, \ldots, d-1$. The properties of qudit Pauli operators were investigated in detail by Gottesman.[30] Here we only need this definition itself, thus the trivial properties are omitted. After the action of the series of single-qudit operation $\Theta_{l''}$, expression (8) becomes

$$\begin{aligned} |\Psi_m^{\mathrm{Bob}}\rangle &\propto \Big(\prod_{l=1}^{M} \Theta_{l''}\Big) |\Psi_{nm}^{\mathrm{Bob}}\rangle \\ &\propto \prod_{l=1}^{M} \Big\{ \sum_{k=0}^{d-1} \alpha_{lk} \exp\Big\{ -\mathrm{i}\frac{2\pi}{d} [(k \oplus m_l) \\ &\quad \times (y'_A + y'_1 + \cdots + y'_N)] \Big\} |k\rangle_{l''} \Big\}. \end{aligned} \tag{14}$$

The second unitary operation that Bob will perform on each of his qudits is called single-qudit phase gate, which depends not only on the result, (n_l, m_l), of generalized Bell-state measurement performed by Alice, but also on that of all the control-qudit measurements, $y'_A, y'_1, y'_2, \ldots, y'_N$, performed by Alice and each agent. For qudit l'', this phase gate may be written as

$$\begin{aligned} U_{l''} = \sum_{k=0}^{d-1} \exp\Big\{ \mathrm{i}\frac{2\pi}{d} [(k \oplus m_l) \\ \times (y'_A + y'_1 + \cdots + y'_N)] \Big\} |k\rangle_{l''l''}\langle k|. \end{aligned} \tag{15}$$

After all these operations have been done, we have

$$|\Psi^{\mathrm{Bob}}\rangle \propto \Big(\prod_{l=1}^{M} U_{l''}\Big) |\Psi_m^{\mathrm{Bob}}\rangle \propto \prod_{l=1}^{M} \sum_{k=0}^{d-1} \alpha_{lk} |k\rangle_{l''}, \tag{16}$$

which indicates that the teleported state in Alice's hands is now restored by Bob and the teleportation process has succeeded.

It is worth pointing out that the premise for the success in the above teleportation is the collaboration of all agents with Alice and Bob. If any one of the agents does not collaborate with the others, then the teleportation fails. For convenience, we suppose that the Nth agent, Charlie, does not collaborate with the others, i.e. he does not do anything. Then after Alice and $N-1$ agents have performed generalized H-gates and measurements on their individual control qudits respectively, with outcomes of measurements $y'_A, y'_1, \ldots, y'_{N-1}$, expression (5) becomes

$$\begin{aligned} |\Psi_{nm}^{\mathrm{C}}\rangle \propto \sum_{s=0}^{d-1} \Big\{ \sum_{z=0}^{d-1} \exp\Big[\mathrm{i}\frac{2\pi}{d} z(y'_A + y'_1 + \cdots + y'_{N-1} + s) \Big] |z\rangle_C \\ \otimes \prod_{l=1}^{M} \Big[\sum_{k=0}^{d-1} \alpha_{lk} \exp\Big\{ \mathrm{i}\frac{2\pi}{d} [(k \oplus m_l)s - n_l k] \Big\} |k \oplus m_l\rangle_{l''} \Big] \Big\}, \end{aligned} \tag{17}$$

where $|z\rangle_C$ is the state of Charlie, i.e. agent N. After tracing over Charlie's qudit C, the density operator for Bob's qudits $(1'', 2'', \ldots, M'')$ is

$$\rho \propto \sum_{s=0}^{d-1} \prod_{l=1}^{M} \Big[\sum_{k=0}^{d-1} \sum_{k'=0}^{d-1} \alpha_{lk} \alpha_{lk'}^{*} \exp\Big[\mathrm{i}\frac{2\pi}{d} (k - k')s \Big] \exp\Big[-\mathrm{i}\frac{2\pi}{d} n_l (k - k') \Big] |k \oplus m_l\rangle_{l''l''} \langle k' \oplus m_l| \Big], \tag{18}$$

where the delta function

$$\sum_{z=0}^{d-1} \exp[\mathrm{i}2\pi z(s - s')/d] = \delta(s - s')$$

has been used. In order to see clearly the failure of the teleportation process, we observe the density operator of any qudit t'' of Bob. After tracing over all other $M-1$ qudits (except t''), we obtain the state of qudit t'',

$$\rho_{t''} = \sum_{k=0}^{d-1} |\alpha_{tk}|^2 |k \oplus m_t\rangle_{t''t''} \langle k \oplus m_t|. \tag{19}$$

Again the delta function, and also the normalized condition $\sum_{k=0}^{d-1} |\alpha_{lk}|^2 = 1$ have been used. This is a

mixed state and certainly is non-identical to the teleported state of t-th qudit. This is to say, Bob cannot restore the original state, even if only one agent does not collaborate with the others.

In summary, we have presented a scheme for teleporting M-qudit quantum information via the control of N agents in a quantum qudit network. The resource consumed in the teleportation process is just a $(2M+N+1)$-qudit entangled quantum channel and $(2M+N+1)\log_2 d$-bit classical communication. The quantum operations used in the process comprise M generalized qudit Bell-state measurements plus $(N+1)$ single-qudit measurements, a series of qudit H-gates, qudit-Pauli gates and qudit phase gates. Our scheme has the advantages of simple structure and less resource consumption. Thus we believe that it will play an important role in the controlled teleportation field of multi-particle and multi-level states.

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