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Probabilistic teleportation of an arbitrary GHZ-class state with a pure entangled two-particle quantum channel and its application in quantum state sharing*

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This paper presents a scheme for probabilistic teleportation of an arbitrary GHZ-class state with a pure entangled two-particle quantum channel. The sender Alice first teleports the coefficients of the unknown state to the receiver Bob, and then Bob reconstructs the state with an auxiliary particle and some unitary operations if the teleportation succeeds. This scheme has the advantage of transmitting much less particles for teleporting an arbitrary GHZ-class state than others. Moreover, it discusses the application of this scheme in quantum state sharing.

Keywords: quantum teleportation, arbitrary GHZ-class state, pure entanglement, quantum state sharing
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1. Introduction

Quantum teleportation, a unique thing in quantum mechanics, has no counterpart in classical physics. This technique allows two remote parties, say the sender (Alice) and the receiver (Bob), to exploit the nonlocal correlation of the quantum channel shared initially to teleport an unknown quantum state $|\chi\rangle = a|0\rangle + b|1\rangle$ from a place to another one by prearranging the share of an Einstein–Podolsky–Rosen (EPR) state and some classical information. To this end, Alice makes a joint Bell-state measurement (BSM) on the unknown quantum system and her EPR particle, and Bob can recover the unknown quantum state in his place according to the relationship between the Alice's BSM result and the state of his EPR particle.[1] Quantum teleportation plays an important role in quantum information processing, and is expected to be useful in quantum computers[2] and quantum secure direct communication.[3−8]

Researchers have devoted much attention to the study of quantum teleportation since it was first proposed by Bennett *et al*[1] in 1993. On the one hand, several groups have demonstrated experimentally the teleportation of a single qubit with entangled photons and ions.[9−14] On the other hand, theoretical schemes for teleporting an unknown state, especially an N-particle entangled state, have been proposed with different quantum channels.[15−26] For example, Gorbachev and Trubilko[15] proposed a scheme for teleporting an EPR pair with a triplet in a Greenberger–Horne–Zeilinger (GHZ) state in 1999. In 2000, Lu and Guo[18] introduced some schemes for teleporting a two-particle quantum system in a Bell-class entangled state $\alpha|00\rangle + \beta|11\rangle$. Lee[19] proposed a protocol for teleporting an entangled state $\alpha|10\rangle + \beta|01\rangle$ with the four-particle GHZ state $|\psi\rangle_{\rm L} = \frac{1}{\sqrt{2}}(|1010\rangle + |0101\rangle)$. In 2003, Yan and Yang[20] proposed an economical teleportation scheme for multi-particle state, which is obviously more convenient than Lee's scheme as

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it exploits only one EPR pair for teleporting an unknown GHZ-class state, the superposition of two product states. Moreover, Yan *et al* proposed a scheme for probabilistic teleportation of two-particle state of general formation[21] and a scheme for teleporting a quantum state from a subset of the whole Hilbert space.[22] In 2000, Yang and Guo[23] proposed a protocol for teleporting an arbitrary N-qubit entangled state with N EPR pairs acting as the quantum channel, and Rigolin[24] showed a way for teleporting an arbitrary two-qubit entangled state with two EPR pairs. Recently, Wang *et al*[27] introduced a way for teleporting a multipartite GHZ-class state $|\phi\rangle_u = c_0|00\ldots0\rangle + c_1|11\ldots1\rangle$ with only one EPR pair in the Bell state $|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB}$, similar to the Yan–Yang scheme.[20] Up to now, the teleportation of a two-qubit system has experimentally been realized with polarized photons.[28]

Quantum secret sharing is the generalization of classical secret sharing into quantum scenario.[29−36] In a secret sharing, a boss Alice divides her secret message M_A into two pieces M_B and M_C ($M_A = M_B \oplus M_C$) and sends them to her two agents, Bob and Charlie, who are at remote places, respectively, for her business. The two agents can read out the message only when they act in concert, otherwise none can get it. In this way, the honest agent can keep the potentially dishonest one from doing any damage.[29] Quantum state sharing (QSTS)[37−43] is a new branch of quantum secret sharing. It is used to share an unknown quantum state among some agents, not a classical information. In 2004, Li *et al*[37] proposed a scheme for sharing an unknown single qubit with a multipartite joint measurement. In their scheme, the boss Alice splits a qubit into m pieces for her m agents with m EPR pairs, and performs an $(m+1)$-GHZ state measurement on her qubit and the m particles in m EPR pairs controlled by her. One of the m agents can recover the unknown qubit with the help of all other agents. In 2005, Deng *et al*[38] proposed a scheme for sharing an arbitrary two-particle state with GHZ states and Bell measurements. Li *et al*[39] simplified the measurements in this scheme, and generalized it to the case for sharing an arbitrary m-qubit state. Now, people[40−43] have studied the way for sharing multipartite entanglement by using EPR pairs as the quantum channel.

In this paper, we will present a scheme for probabilistic teleportation of an arbitrary multipartite GHZ-class state with a quantum channel in a pure entangled state, following the ideas in Ref.[20]. Its obvious advantage is that the quantum channel is a pure entangled state, no maximally entangled state, which makes it more convenient in a practical application than others, and the two parties need not transmit many particles for setting up the quantum channel, which will reduce largely the entangled quantum resource in a noisy channel, similar to Refs.[20,27]. Moreover, we will discuss the application of this scheme in QSTS of a multi-particle GHZ-class state among two agents with a pure entangled three-particle quantum channel.

2. Probabilistic teleportation scheme

2.1. Teleportation of an arbitrary GHZ-class state with an EPR pair

For presenting the principle of our scheme explicitly, we first discuss it in the case that an EPR pair is used as the quantum channel, following some ideas in Refs.[20,27], and then generalize it to the case with a pure entangled two-particle state.

An EPR pair is in one of the four Bell states shown as follows:

$$|\psi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B \pm |1\rangle_A|0\rangle_B), \tag{1}$$

$$|\phi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B), \tag{2}$$

where $|0\rangle$ and $|1\rangle$ are the eigenvectors of the operator σ_z (called it the measuring basis – MB σ_z). The four unitary operations $\{U_i\}$ $(i = 0, 1, 2, 3)$ can transform one of the four Bell states into each other,

$$U_0 = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad U_1 = |1\rangle\langle 0| + |0\rangle\langle 1|,$$
$$U_2 = |0\rangle\langle 0| - |1\rangle\langle 1|, \quad U_3 = |0\rangle\langle 1| - |1\rangle\langle 0|. \tag{3}$$

Suppose the unknown arbitrary multipartite GHZ-class state teleported is

$$|\chi\rangle_u = (\alpha|i, j, \ldots, k\rangle + \beta|\bar{i}, \bar{j}, \ldots, \bar{k}\rangle)_{u_1u_2\ldots u_m}, \tag{4}$$

$$|\alpha|^2 + |\beta|^2 = 1, \tag{5}$$

where α and β are two unknown complex numbers, $i, j, \ldots, k \in \{0, 1\}$ are unknown as well, $\bar{i} = 1 - i$, $\bar{j} = 1 - j$, and $\bar{k} = 1 - k$. $u_1, u_2, \ldots, u_m$ are the m particles which compose the unknown quantum system u in the state $|\chi\rangle_u$.

For teleporting the unknown state $|\chi\rangle_u$, the sender, Alice, first shares an EPR pair with the receiver, Bob, and then determines the relation between

the state of the particle u_1 and those of the others u_l ($l = 2, 3, \ldots, m$) before she teleports the coefficients α and β to Bob with an EPR pair $|\psi^-\rangle_{AB}$. Bob can reconstruct the originally unknown state by performing some controlled-not (CNOT) operations on his EPR particle and some auxiliary particles b_l ($l = 2, 3, \ldots, m$), similar to the case in Ref.[20].

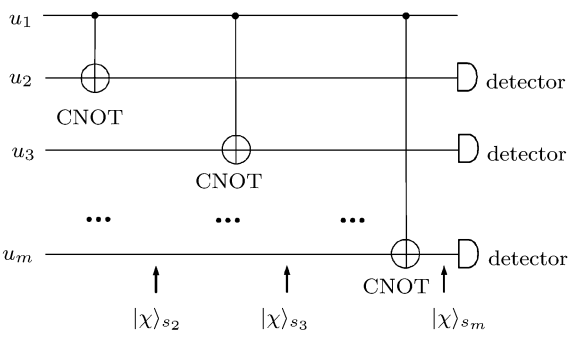


Fig.1. The schematic principle for determining the relation between the state of the particle u_1 and the others u_l ($l = 2, 3, \ldots, m$).

The principle for determining the relation between the state of the particle u_1 and those of the others u_l ($l = 2, 3, \ldots, m$) is shown in Fig.1. In detail, Alice takes a CNOT operation on the particle u_1 and the particle u_2 by using u_1 as the control qubit. After this CNOT operation, the state of the unknown quantum system composed of the particles u_l ($l = 1, 2, \ldots, m$) becomes

$$\begin{aligned}|\chi\rangle_{s_2} &= (\alpha|i, i\oplus j, \ldots, k\rangle + \beta|\bar{i}, \bar{i}\oplus\bar{j}, \ldots, \bar{k}\rangle)_{u_1u_2\ldots u_m} \\ &= (\alpha|i, \ldots, k\rangle + \beta|\bar{i}, \ldots, \bar{k}\rangle)_{u_1\ldots u_m} \otimes |i\oplus j\rangle_{u_2}. \quad (6)\end{aligned}$$

Alice repeats this process for the other particles u_l ($l = 3, \ldots, m$) and obtains the state of the unknown quantum system as

$$|\chi\rangle_{s_m} = (\alpha|i\rangle + \beta|\bar{i}\rangle)_{u_1} \otimes |i\oplus j\rangle_{u_2} \otimes \cdots \otimes |i\oplus k\rangle_{u_m}. \quad (7)$$

Alice measures the particles u_l ($l = 2, 3, \ldots, m$) with the MB σ_z and obtains the relation $i\oplus j, \ldots, i\oplus k$. After all the CNOT operations and single-particle measurements, the state of the particle u_1 is transformed into

$$|\chi\rangle_{u_1} = (\alpha|i\rangle + \beta|\bar{i}\rangle)_{u_1}. \quad (8)$$

With the standard teleportation protocol[1] for an unknown single-particle state, Alice can easily in principle teleport the coefficients α and β to Bob with an EPR pair $|\psi^-\rangle_{AB}$. One can rewrite the state of the composite quantum system composed of the three particles u_1, A and B as

$$\begin{aligned}|\chi\rangle_{u_1} \otimes |\psi^-\rangle_{AB} &= (\alpha|i\rangle + \beta|\bar{i}\rangle)_{u_1} \\ &\quad \otimes \frac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle)_{AB} \\ &= \frac{1}{2}[-|\psi^-\rangle_{u_1A}(\alpha|i\rangle + \beta|\bar{i}\rangle)_B \\ &\quad + |\psi^+\rangle_{u_1A}(-1)^i(-\alpha|i\rangle + \beta|\bar{i}\rangle)_B \\ &\quad + |\phi^+\rangle_{u_1A}(-1)^i(\alpha|\bar{i}\rangle - \beta|i\rangle)_B \\ &\quad + |\phi^-\rangle_{u_1A}(\alpha|\bar{i}\rangle + \beta|i\rangle)_B]. \quad (9)\end{aligned}$$

That is, Alice first takes a Bell-state measurement on the particle u_1 and the EPR particle A, and then tells Bob the outcome of her measurement. Bob takes a unitary operation U_0, U_1, U_2 or U_3 on the EPR particle B to reconstruct the state $|\chi\rangle_{u_1}$ according to the outcome of the Bell-state measurement $|\psi^-\rangle_{u_1A}$, $|\phi^-\rangle_{u_1A}$, $|\psi^+\rangle_{u_1A}$ or $|\phi^+\rangle_{u_1A}$, respectively.

For recovering the unknown m-particle state $|\chi\rangle_u$, Bob should first prepare m-1 auxiliary particles b_2, b_3, ... and b_m, and entangle them with the EPR particle B, see Fig.2. That is, Bob takes a CNOT operation on the EPR particle B and an auxiliary particle b_l ($l = 2, 3, \ldots, m$) by using the particle B as the control qubit. After all these CNOT operations, the state of the composite quantum system composed of the particles B and b_l ($l = 2, 3, \ldots, m$) becomes

$$|\chi'\rangle_{Bb_2b_3\ldots b_m} = (\alpha|iii\ldots i\rangle + \beta|\bar{i}\bar{i}\bar{i}\ldots\bar{i}\rangle)_{Bb_2b_3\ldots b_m}. \quad (10)$$

According to the information about the outcomes of the measurements done by Alice on the particles u_l ($l = 2, 3, \ldots, m$), Bob can reconstruct the unknown m-particle state $|\chi\rangle_u$ with a unitary operation on each auxiliary particle, i.e., $U_{i\oplus j}$, $U_{i\oplus p}$, ... and $U_{i\oplus k}$ are used on the auxiliary particles b_2, b_3, ... and b_m, respectively. Here $i\oplus j$, $i\oplus p$, ... and $i\oplus k$ are the outcomes of the measurements done by Alice on the particles u_2, u_3, ... and u_m, respectively.

In fact, this scheme for teleporting a GHZ-class state with an EPR pair is just a special example of the scheme proposed by Yan and Yang,[20] and we only give a concrete demonstration of the principle of the Yan-Yang scheme with the example for teleporting an arbitrary GHZ-class state $|\chi\rangle_u = (\alpha|i, j, \ldots, k\rangle + \beta|\bar{i}, \bar{j}, \ldots, \bar{k}\rangle)_{u_1u_2\ldots u_m}$, not the state $|\phi\rangle_u = (\alpha|00\ldots0\rangle + \beta|11\ldots1\rangle)_{u_1u_2\ldots u_m}$ discussed in Refs.[20,27]. As discussed in Ref.[27], the sender Alice need not perform some CNOT operations on her particles for teleportation of the GHZ-class state $|\phi\rangle_u$,

just Bell-state measurement and some single-particle measurements. But for the state $|\chi\rangle_u$, the approach discussed in Ref.[27] does not work, and this scheme is the optimal one.

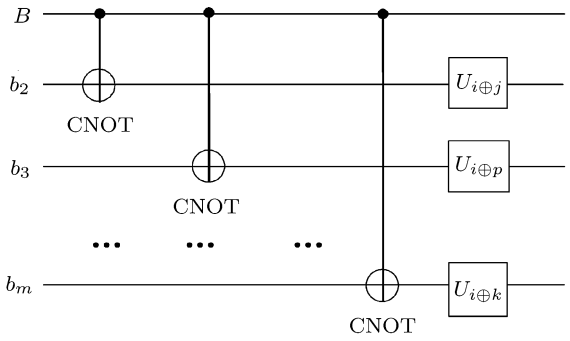


Fig.2. The schematic demonstration for reconstructing the unknown GHZ-class state $|\chi\rangle_u$.

2.2. Teleportation of an arbitrary GHZ-class state with a pure entangled two-particle quantum channel

In a practical application, the quantum channel is not in a maximally entangled state but a pure entangled state or a mixed state. The result of the teleportation with the quantum channel in a mixed state depends on the model for noise, which is not the interest of this paper. We only discuss the teleportation of an arbitrary GHZ-class state with a pure entangled two-particle quantum channel below.

Certainly, Alice and Bob should first share some pure entangled states $|\Phi\rangle_{AB} = c_0|00\rangle_{AB} + c_1|11\rangle_{AB}$ before they teleport the unknown m-particle state $|\chi\rangle_u$, and then Alice should obtain the relation between the state of the particle u_1 and the others u_l $(l = 2, 3, \ldots, m)$, as same as the case with an EPR pair discussed above. At last, the important thing is that Alice should teleport the coefficients α and β to Bob who reconstructs the originally unknown state $|\chi\rangle_u$ with CNOT operations and auxiliary particles.

The way for teleporting an arbitrary state of a single qubit $\alpha|0\rangle + \beta|1\rangle$ with a pure entangled state $|\Phi'\rangle_{AB} = c_0|00\rangle_{AB} + c_1|11\rangle_{AB}$ has been discussed in detail in Ref.[44]. We can use the same way as that in Ref.[44] to teleport the state $|\chi\rangle_{u_1} = (\alpha|i\rangle + \beta|\bar{i}\rangle)_{u_1}$. The state of the composite quantum system composed of the three particles u_1, A and B can be written as

$$\begin{aligned}|\chi\rangle_{u_1}\otimes|\Phi'\rangle_{AB} &= (\alpha|i\rangle+\beta|\bar{i}\rangle)_{u_1}\\ &\quad\otimes(c_0|00\rangle+c_1|11\rangle)_{AB}\\ &= \frac{1}{\sqrt{2}}[|\phi^+\rangle_{u_1A}(\alpha c_i|i\rangle+\beta c_{\bar{i}}|\bar{i}\rangle)_B\\ &\quad+|\phi^-\rangle_{u_1A}(-1)^i(\alpha c_i|i\rangle-\beta c_{\bar{i}}|\bar{i}\rangle)_B\\ &\quad+|\psi^+\rangle_{u_1A}(\alpha c_{\bar{i}}|\bar{i}\rangle+\beta c_i|i\rangle)_B\\ &\quad+|\psi^-\rangle_{u_1A}(-1)^i(\alpha c_{\bar{i}}|\bar{i}\rangle\\ &\quad-\beta c_i|i\rangle)_B]. \qquad (11)\end{aligned}$$

That is, Alice takes a Bell-state measurement on the particles u_1 and A, and tells Bob her outcome. Bob can obtain the state of the particle B after Alice's Bell-state measurement. The particle B collapses to the state $\alpha c_i|i\rangle + \beta c_{\bar{i}}|\bar{i}\rangle$, $\alpha c_i|i\rangle - \beta c_{\bar{i}}|\bar{i}\rangle$, $\alpha c_{\bar{i}}|\bar{i}\rangle + \beta c_i|i\rangle$ or $\alpha c_{\bar{i}}|\bar{i}\rangle - \beta c_i|i\rangle$ when the outcome of the Bell-state measurement on the particles u_1 and A is $|\phi^+\rangle_{u_1A}$, $|\phi^-\rangle_{u_1A}$, $|\psi^+\rangle_{u_1A}$ or $|\psi^-\rangle_{u_1A}$, respectively. For getting the state $|\chi\rangle_{u_1} = (\alpha|i\rangle + \beta|\bar{i}\rangle)_{u_1}$ probabilistically, Bob can carry out a general evolution on the particle B and an auxiliary qubit whose original state is $|0\rangle_{\rm aux}$ with the way introduced in Ref.[44]. That is, under the basis $\{|0\rangle_B|0\rangle_{\rm aux},\ |1\rangle_B|0\rangle_{\rm aux},\ |0\rangle_B|1\rangle_{\rm aux},\ |1\rangle_B|1\rangle_{\rm aux}\}$ the collective unitary transformation[44]

$$U_{\rm sim} = \begin{pmatrix} \frac{c_1}{c_0} & 0 & \sqrt{1-\left(\frac{c_1}{c_0}\right)^2} & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -1\\ \sqrt{1-\left(\frac{c_1}{c_0}\right)^2} & 0 & -\frac{c_1}{c_0} & 0 \end{pmatrix} \qquad (12)$$

can transform probabilistically the states $\alpha c_i|i\rangle + \beta c_{\bar{i}}|\bar{i}\rangle$, $\alpha c_i|i\rangle - \beta c_{\bar{i}}|\bar{i}\rangle$, $\alpha c_{\bar{i}}|\bar{i}\rangle + \beta c_i|i\rangle$ and $\alpha c_{\bar{i}}|\bar{i}\rangle - \beta c_i|i\rangle$ into the states $\alpha|i\rangle + \beta|\bar{i}\rangle$, $\alpha|i\rangle - \beta|\bar{i}\rangle$, $\alpha|\bar{i}\rangle + \beta|i\rangle$ and $\alpha|\bar{i}\rangle - \beta|i\rangle$, respectively, if $|c_0| > |c_1|$. In detail, if Bob measures his auxiliary particle after the unitary evolution $U_{\rm sim}$ and obtains the result $|0\rangle_{\rm aux}$, the teleportation succeeds; otherwise the teleportation fails:

$$U_{\rm sim}(\alpha c_i|i\rangle+\beta c_{\bar{i}}|\bar{i}\rangle)_B|0\rangle_{\rm aux} = c_1(\alpha|i\rangle+\beta|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-(\frac{c_1}{c_0})^2}c_0(\alpha)^{1-i}(\beta)^i|1\rangle_B|1\rangle_{\rm aux},$$

$$U_{\rm sim}(\alpha c_i|i\rangle-\beta c_{\bar{i}}|\bar{i}\rangle)_B|0\rangle_{\rm aux} = c_1(\alpha|i\rangle-\beta|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-(\frac{c_1}{c_0})^2}c_0(\alpha)^{1-i}(-\beta)^i|1\rangle_B|1\rangle_{\rm aux},$$

$$U_{\rm sim}(\alpha c_{\bar{i}}|\bar{i}\rangle+\beta c_i|i\rangle)_B|0\rangle_{\rm aux} = c_1(\beta|i\rangle+\alpha|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-(\frac{c_1}{c_0})^2}c_0(\alpha)^{i}(\beta)^{1-i}|1\rangle_B|1\rangle_{\rm aux},$$

$$U_{\rm sim}(\alpha c_{\bar{i}}|\bar{i}\rangle - \beta c_i|i\rangle)_B|0\rangle_{\rm aux} = c_1(\beta|i\rangle - \alpha|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-\left(\frac{c_1}{c_0}\right)^2}c_0(-\alpha)^i(\beta)^{1-i}|1\rangle_B|1\rangle_{\rm aux}. \quad (13)$$

When the teleportation succeeds, Bob can recover the unknown state $\alpha|i\rangle + \beta|\bar{i}\rangle$ by taking a unitary operation on the particle B. That is, the operations U_0, U_2, U_1 and U_3 can transform the states $\alpha|i\rangle + \beta|\bar{i}\rangle_B$, $\alpha|i\rangle - \beta|\bar{i}\rangle_B$, $\beta|i\rangle + \alpha|\bar{i}\rangle)_B$ and $\beta|i\rangle - \alpha|\bar{i}\rangle)_B$ into the state $\alpha|i\rangle + \beta|\bar{i}\rangle_B$, respectively. As pointed out in Ref.[44], the optimal probability of successful teleportation is $P = 2|c_1|^2$. In this way, for teleporting the unknown state $\alpha|i\rangle + \beta|\bar{i}\rangle)_B$ Alice should on average transmit $1/P$ particles to Bob in an ideal condition.

If $|c_0| < |c_1|$, Bob can choose the collective unitary transformation $U'_{\rm sim}$ to evolute the particle B and the auxiliary particle:

$$U'_{\rm sim} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c_0}{c_1} & 0 & \sqrt{1-\left(\frac{c_0}{c_1}\right)^2} \\ 0 & 0 & -1 & 0 \\ 0 & \sqrt{1-\left(\frac{c_0}{c_1}\right)^2} & 0 & -\frac{c_0}{c_1} \end{pmatrix}. \quad (14)$$

That is,

$$U'_{\rm sim}(\alpha c_i|i\rangle + \beta c_{\bar{i}}|\bar{i}\rangle)_B|0\rangle_{\rm aux} = c_0(\alpha|i\rangle + \beta|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-\left(\frac{c_0}{c_1}\right)^2}c_1(\alpha)^i(\beta)^{1-i}|1\rangle_B|1\rangle_{\rm aux},$$

$$U'_{\rm sim}(\alpha c_i|i\rangle - \beta c_{\bar{i}}|\bar{i}\rangle)_B|0\rangle_{\rm aux} = c_0(\alpha|i\rangle - \beta|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-\left(\frac{c_0}{c_1}\right)^2}c_1(\alpha)^i(-\beta)^{1-i}|1\rangle_B|1\rangle_{\rm aux},$$

$$U'_{\rm sim}(\alpha c_{\bar{i}}|\bar{i}\rangle + \beta c_i|i\rangle)_B|0\rangle_{\rm aux} = c_0(\beta|i\rangle + \alpha|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-\left(\frac{c_0}{c_1}\right)^2}c_1(\alpha)^{1-i}(\beta)^i|1\rangle_B|1\rangle_{\rm aux},$$

$$U'_{\rm sim}(\alpha c_{\bar{i}}|\bar{i}\rangle - \beta c_i|i\rangle)_B|0\rangle_{\rm aux} = c_0(\beta|i\rangle - \alpha|\bar{i}\rangle)_B|0\rangle_{\rm aux} + \sqrt{1-\left(\frac{c_0}{c_1}\right)^2}c_1(-\alpha)^{1-i}(\beta)^i|1\rangle_B|1\rangle_{\rm aux}. \quad (15)$$

Similar to the case $|c_0| > |c_1|$, Bob can obtain the unknown state $\alpha|i\rangle + \beta|\bar{i}\rangle)_B$ with a unitary operation if the teleportation succeeds. The optimal probability of successful teleportation in this time is $P' = 2|c_0|^2$. In this way, for teleporting the unknown state $\alpha|i\rangle + \beta|\bar{i}\rangle)_B$ Alice should on average transmit $1/P'$ particles to Bob in an ideal condition.

After obtaining the unknown state $\alpha|i\rangle + \beta|\bar{i}\rangle)_B$, Bob can reconstruct the unknown GHZ-class state $|\chi\rangle_u = (\alpha|i,j,\ldots,k\rangle + \beta|\bar{i},\bar{j},\ldots,\bar{k}\rangle)_{u_1u_2\ldots u_m}$ with the same way as that used in the case with an EPR pair. In essence, the scheme for teleportation of a multiparticle GHZ-class state with an EPR pair is just a special example of this generalized one.

3. Quantum state sharing of arbitrary GHZ-class states

In a QSTS scheme, there are many participants, i.e., the boss Alice and her agents.[37−42] We only discuss the case with two agents, Bob and Charlie below, as the principle of the case with m agents is the same as this one.

For sharing a sequence of arbitrary GHZ-class states, Alice, Bob and Charlie should first set up a quantum channel with a sequence of pure entangled three-particle GHZ-class states $|\psi\rangle_{ABC} = (c_0|000\rangle + c_1|111\rangle)_{ABC}$ securely.

QSTS is far more complex than quantum key distribution (QKD)[45] because the potentially dishonest agent, say Bob, is a powerful eavesdropper and he has a chance to hide his eavesdropping with a cheat.[46] The boss Alice should exploit some decoy photons to set up the entangled quantum channel between her and her agents,[47] otherwise their quantum channel can be eavesdropped freely by the dishonest agent.[46,47] When the quantum channel is in a pure entangled state, the decoy technique[8,47,48] (not decoy states,[49] just decoy photons which are used to check eavesdropping efficiently) is more important for its security.[8] The reason is that the participants cannot exploit the entanglement correlation of the quantum channel to ensure its security in this time.[47] The de-

coy photons who are randomly in one of the four states $\{|0\rangle, |1\rangle, |+x\rangle, |-x\rangle\}$ are prepared by the boss Alice and inserts into the two photon sequences S_B and S_C which are sent to Bob and Charlie, respectively.[47] Here $|\pm x\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ are the eigenvectors of the MB σ_x.

Now, let us elaborate the principle of our QSTS scheme with a sequence of pure entangled states for sharing an arbitrary GHZ-class state. It works with three steps as follows.

i. Alice sets up a pure entangled quantum channel with her agents Bob and Charlie by using some decoy photons to ensure its security.

In detail, Alice prepares N ordered pure entangled states $|\psi\rangle_{ABC} = (c_0|000\rangle + c_1|111\rangle)_{ABC}$ and then divides them into three sequences, S_A, S_B and S_C, the same as those in Ref.[3]. That is, S_A, S_B and S_C are made up of all the particles A, B and C in the N ordered pure entangled states, respectively. In order to prevent the dishonest agent from eavesdropping freely, Alice randomly replaces k particles in both S_B and S_C with decoy photons. Alice does not need an ideal single-photon source for preparing her decoy photons. She can get them with measurements and manipulations, e.g., she can measure the particles A in some entangled states $|\psi\rangle_{ABC}$ with the MB σ_z and then take one of the four unitary operations $U \equiv \{U_0, U_1, U_0 \otimes H, U_1 \otimes H\}$ on the remaining particles B and C randomly. Here H is a Hadamard operation, i.e., $H = (1/\sqrt{2})(|0\rangle\langle 0| + |1\rangle\langle 0| + |0\rangle\langle 1| - |1\rangle\langle 1|)$. After Alice takes the measurement on the particle A with the MB σ_z and operates the remaining particles B and C with two suitable operations $U_B \in U$ and $U_C \in U$, respectively, the particles B and C are in two pure single-particle states which are unknown to both Bob and Charlie. In this way, the eavesdropping of the dishonest agent will leave a trace in the outcomes obtained by measuring the decoy photons,[8,47,48] the same as the Bennett–Brassard 1984 QKD protocol.[50]

ii. Alice, Bob and Charlie perform an entanglement concentration on their quantum channel.

In this procedure, Alice wants to improve the entanglement of the quantum channel probabilistically with entanglement concentration.[44] That is, she transforms the pure entangled state $|\psi\rangle_{ABC} = (c_0|000\rangle + c_1|111\rangle)_{ABC}$ into a GHZ state $|GHZ\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{ABC}$ with a unitary transformation. To this end, Alice prepares an auxiliary particle a in the original state $|0\rangle_a$. Under the basis $\{|0\rangle_A|0\rangle_a, |1\rangle_A|0\rangle_a, |0\rangle_A|1\rangle_a, |1\rangle_A|1\rangle_a\}$, the unitary transformation $U_{\rm sim}^{Aa} = U_{\rm sim}$ (shown in Eq.(12)) on the particles A and a can transform the state $|\psi\rangle_{ABC}$ into the GHZ state $|{\rm GHZ}\rangle_{ABC}$ by measuring the auxiliary particle along the z-direction when $|c_0| > |c_1|$, i.e.,

$$\begin{aligned} & U_{\rm sim}^{Aa} \otimes I^{BC}(c_0|0\rangle_A|0\rangle_a|00\rangle_{BC} + c_1|1\rangle)_A|0\rangle_a|11\rangle_{BC} \\ &= c_1(|000\rangle + |111\rangle)_{ABC}|0\rangle_a \\ &\quad + c_0\sqrt{1 - (\frac{c_1}{c_0})}|100\rangle_{ABC}|1\rangle_a, \end{aligned} \tag{16}$$

where I^{BC} means doing nothing on the particles B and C. When Alice obtains the outcome $|0\rangle_a$ with the MB σ_z, the three particles A, B and C are in the GHZ state $|{\rm GHZ}\rangle_{ABC}$, otherwise Alice tells Bob and Charlie discarding their particles B and C. The case that $|c_0| < |c_1|$ is similar to this one with a little modification.

iii. Alice, Bob and Charlie exploit the quantum channel in the GHZ state $|{\rm GHZ}\rangle_{ABC}$ to share the quantum state $|\chi\rangle_u = (\alpha|i, j, \ldots, k\rangle + \beta|\bar{i}, \bar{j}, \ldots, \bar{k}\rangle)_{u_1 u_2 \ldots u_m}$ securely.

Similar to the teleportation of the state $|\chi\rangle_u$, Alice should first determine the relation between the state of the particle u_1 and the others $u_l (l = 2, 3, \ldots, m)$ with CNOT operations, and then let Bob and Charlie share the state $|\chi\rangle_{u_1} = (\alpha|i\rangle + \beta|\bar{i}\rangle)_{u_1}$ with the GHZ state, as same as the case in Ref.[29]. Let us suppose that Charlie will reconstruct the unknown state $|\chi\rangle_u$ with the help of Bob. The task for sharing the state $|\chi\rangle_{u_1}$ can be completed by means that Alice performs a Bell-state measurement on the particle u_1 and the particle A in the GHZ state $|{\rm GHZ}\rangle_{ABC}$ and Bob measures his particle B with the MB σ_x.

$$\begin{aligned} |\chi\rangle_{u_1} \otimes \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{ABC} = \frac{1}{2\sqrt{2}}\{ & |\phi^+\rangle_{u_1A}[|+x\rangle_B(\alpha|i\rangle + \beta|\bar{i}\rangle)_C + (-1)^i|-x\rangle_B(\alpha|i\rangle - \beta|\bar{i}\rangle)_C] \\ & + |\phi^-\rangle_{u_1A}[(-1)^i|+x\rangle_B(\alpha|i\rangle - \beta|\bar{i}\rangle)_C + |-x\rangle_B(\alpha|i\rangle + \beta|\bar{i}\rangle)_C] \\ & + |\psi^+\rangle_{u_1A}[|+x\rangle_B(\alpha|\bar{i}\rangle + \beta|i\rangle)_C - (-1)^i|-x\rangle_B(\alpha|\bar{i}\rangle - \beta|i\rangle)_C] \\ & + |\psi^-\rangle_{u_1A}[(-1)^i|+x\rangle_B(\alpha|\bar{i}\rangle - \beta|i\rangle)_C - |-x\rangle_B(\alpha|\bar{i}\rangle + \beta|i\rangle)_C]. \end{aligned} \tag{17}$$

After Alice and Bob publish their outcomes of measurements, Charlie can recover the state $|\chi\rangle_{u_1}$ with a suitable unitary operation on the particle C, shown in Table 1. Alice's outcomes of Bell-state measurements are given in the columns, whereas Bob's outcomes of single-particle measurement with the MB σ_x are given in the rows. Charlie's operations with which he can reconstruct the unknown state $|\chi\rangle_{u_1}$ appear in the boxes.

Table 1. The operations used for reconstructing the state $|\chi\rangle_{u_1}$.

		Alice			
		$\|\phi^+\rangle_{u_1A}$	$\|\phi^-\rangle_{u_1A}$	$\|\psi^+\rangle_{u_1A}$	$\|\psi^-\rangle_{u_1A}$
Bob	$\|+x\rangle$	U_0	U_2	U_1	U_3
	$\|-x\rangle$	U_2	U_0	U_3	U_1

After Charlie recovers the state $|\chi\rangle_{u_1}$ and the participants confirm that their quantum communication is secure, Alice tells Charlie the relation between the state of the particle u_1 and the others $u_l (l = 2, 3, \ldots, m)$. That is, Alice announces the information $i \oplus j$,..., $i \oplus k$. In this way, Charlie can reconstruct the unknown state $|\chi\rangle_u = (\alpha|i, j, \ldots, k\rangle + \beta|\bar{i}, \bar{j}, \ldots, \bar{k}\rangle)_{u_1u_2\ldots u_m}$ with m-1 auxiliary particles, the same as the case discussed in Section 2.1.

In fact, the security of this QSTS scheme completely depends on that of the quantum channel in the GHZ-class state $|\psi\rangle_{ABC} = (c_0|000\rangle + c_1|111\rangle)_{ABC}$. As the boss Alice exploits some decoy photons to prevent an eavesdropper from stealing the information about the quantum channel, this QSTS scheme is in principle secure, as same as the Bennett–Brassard 1984 QKD protocol.[50,51] In a practical application, the quantum systems will interact with environment, which will decrease the entanglement of the quantum channel even make it in a mixed state. In this time, Alice, Bob and Charlie should first purify their quantum channel with multiparticle entanglement purification protocol[52] and then share the unknown state.

4. Discussion and summary

It is valuable to emphasize that Alice and Bob can also first perform an entanglement concentration on the pure entangled state $|\Phi\rangle_{AB} = c_0|00\rangle_{AB} + c_1|11\rangle_{AB}$ in the teleportation of an arbitrary GHZ-class state with a pure entangled two-particle quantum channel, and then teleport the unknown state $|\chi\rangle_u$, similar to the case in QSTS. This way will reduce the probability destroying the unknown state $|\chi\rangle_u$ at the expense of storing it a long time.

In Ref.[27], the two parties Alice and Bob can exploit some special measurements to avoid the requirement that Alice should determine the relation between the first particle and the others in the unknown state $|\phi\rangle_u = (\alpha|00\ldots0\rangle + \beta|11\ldots1\rangle)_{u_1u_2\ldots u_m}$ with m CNOT operations. But when the unknown state becomes $|\chi\rangle_u = (\alpha|i, j, \ldots, k\rangle + \beta|\bar{i}, \bar{j}, \ldots, \bar{k}\rangle)_{u_1u_2\ldots u_m}$, the approach in Ref.[27] does not work. Different to Ref.[20], Bob cannot recover the unknown state $|\chi\rangle_u = (\alpha|i, j, \ldots, k\rangle + \beta|\bar{i}, \bar{j}, \ldots, \bar{k}\rangle)_{u_1u_2\ldots u_m}$ even though he gets the state $|\chi\rangle_{u_1}$ if Alice does not announce the relation $i \oplus j$, ..., $i \oplus k$. This result is very important in QSTS with an authorization verification procedure.[31] That is to say, if Alice finds that the dishonest agent (say Bob) exploits some fake information to cheat the honest agent before the authorization verification procedure, she does not tell Bob the relation $i \oplus j$, ..., $i \oplus k$. In this way, Bob cannot obtain the whole quantum information about the unknown state $|\chi\rangle_u$, which will increase the security of this QSTS scheme in a practical application. From the view of entropy, the state $|\chi\rangle_u$ contains more information than the state $|\phi\rangle_u$.

In summary, we have presented a scheme for teleportation of an arbitrary m-particle GHZ-class state by using a pure entangled two-particle quantum system as the quantum channel, following some ideas in Ref.[20]. The two parties need only share one EPR pair as their quantum channel in this scheme for teleportation of an arbitrary m-particle GHZ-class state. If the quantum channel is composed of some pure entangled states, the number of the particles transmitted between the two parties is far smaller than other schemes (for instance, those in Refs.[15–19]). In a noisy channel, this scheme is obviously more convenient than others. Finally, we discussed the application of this scheme in QSTS of an arbitrary m-particle GHZ-class state efficiently.

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