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Anomalous scaling in a non-Gaussian random shell model for passive scalars*

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In this paper, we have introduced a shell-model of Kraichnan's passive scalar problem. Different from the original problem, the prescribed random velocity field is non-Gaussian and δ correlated in time, and its introduction is inspired by She and Lévêque (Phys. Rev. Lett. **72**, 336 (1994)). For comparison, we also give the passive scalar advected by the Gaussian random velocity field. The anomalous scaling exponents $H(p)$ of passive scalar advected by these two kinds of random velocities above are determined for structure function with values of p up to 15 by Monte Carlo simulations of the random shell model, with Gear methods used to solve the stochastic differential equations. We find that the $H(p)$ advected by the non-Gaussian random velocity is not more anomalous than that advected by the Gaussian random velocity. Whether the advecting velocity is non-Gaussian or Gaussian, similar scaling exponents of passive scalar are obtained with the same molecular diffusivity.

Keywords: scaling, shell model, She and Lévêque (SL) model, non-Gaussian, passive scalar
PACC: 0340G, 4700

1. Introduction

Over the past decades much attention has been paid to the statistical properties of the turbulence of passive scalars, such as temperature and weak magnetic fields, advected by a fluid.[1−4] The passive scalar shell model is very popular in analysis and numerical simulations.[5−7] Fundamental results have been obtained when the passive scalar shell model is driven by the velocity field of the Gledzer–Ohkitani–Yamada (GOY) Model.[5−9] When the advecting velocity field is assumed to be Gaussian and delta-correlated in time, the so-called Kraichnan model, anomalous scaling of passive scalar, can also be obtained.[10−13] It is recognized that advection does not fully determine the scalar structure functions in the inertial range of spatial separations. Molecular diffusion plays an important direct role. But the advecting velocity field is still a factor that affects the passive scalar. The properties of passive scalar advected by non-Gaussian random velocity field have not been studied before. Here we discuss this kind of problem by using the passive scalar shell model.

The time evolution of the passive scalar field $\Theta(\boldsymbol{x},t)$ advected by an incompressible velocity field $\upsilon(\boldsymbol{x},t)$, is given by the partial differential equation

$$\partial_t \Theta + (v\cdot\nabla)\Theta = \kappa\Delta\Theta + f, \tag{1}$$

where κ is the molecular diffusivity, and f is an external force that acts on a length scale L. In the inertial range where there exists neither external force nor molecular damping, the increment of moment of θ is

$$S_p(\boldsymbol{r}) = \langle|\Theta(\boldsymbol{x}) - \Theta(\boldsymbol{x}+\boldsymbol{r})|^p\rangle \sim l^{H(p)}, \tag{2}$$

where $l = |\boldsymbol{r}|$. By analogy with the K41 theory,[14] we can accurately obtain $H(p) = p/3$. However, experiments show that $H(p)$ strongly deviates from $p/3$ for $p > 3$. This is referred to as anomalous scaling or intermittency. Statistics of the passive scalar is strongly related to the properties of the advecting velocity fields. As is known, the intermittent passive scalar can be induced by a nonintermittent velocity field. Our goal is to use an intermittent velocity field, and then to find whether the passive scalar is more intermittent than that advected by the nonintermittent velocity field.

The rest of this paper is organized as follows. In Section 2, we introduce a passive scalar shell model

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driven by a non-Gaussian random velocity which follows the probability distribution function given by She and Lévêque. In Section 3, Monte Carlo simulations of the stochastic different equation are presented. Scaling exponents of structure functions for passive scalar are shown with advecting non-Gaussian random velocity. At the same time, the scaling exponents of passive scalar advected by Gaussian random velocity are given for comparison. Concluding remarks and suggestions for further work are given in Section 4.

2. The shell model and its stochastic differential equation

Our passive scalar shell model, introduced by Jensen,[5] is defined by the following equations ($n = 1, 2, \ldots, N$):

$$\begin{aligned}&\Big[\frac{\mathrm{d}}{\mathrm{d}t}+\kappa k_n^2\Big]\theta_n(t)\\&=\mathrm{i}\{e_n[\theta^*_{n+1}u^*_{n-1}-\theta^*_{n-1}u^*_{n+1}]\\&\quad+g_n[\theta^*_{n-1}u^*_{n-2}+\theta^*_{n-2}u^*_{n-1}]\\&\quad+h_n[\theta^*_{n+2}u^*_{n+1}+\theta^*_{n+1}u^*_{n+2}]\}+f\delta_{n,1},\end{aligned}\tag{3}$$

where * denotes the complex conjugate, and κ is the molecular diffusion coefficient. A possible choice for the coefficients is

$$e_n=\frac{k_n}{2},\quad g_n=-\frac{k_{n-1}}{2},\quad\text{and}\quad h_n=\frac{k_{n+1}}{2}.\tag{4}$$

The boundary conditions are as follows:

$$e_1=e_N=g_1=g_2=h_{N-1}=h_N=0.\tag{5}$$

The force term $f\delta_{n,1}$ acts only on the first shell. A shell of radius $k_n = r^n k_0$, with $r > 1$, contains the wave numbers with modulus k such that $k_n < k < k_{n+1}$ and the standard choice for the radius between shells is $r = 2$. In Ref.[5], Jensen coupled Eq.(3) with the GOY function and obtained

$$\begin{aligned}\Big(\frac{\mathrm{d}}{\mathrm{d}t}+\nu k_n^2\Big)u_n(t)=\mathrm{i}k_n(&a_nu^*_{n+1}u^*_{n+2}+b_nu^*_{n-1}u^*_{n+1}\\&+c_nu^*_{n-1}u^*_{n-2})+f_u\delta_{n,1}.\end{aligned}\tag{6}$$

The external force is added to the first shell in Eq.(6) corresponding to Eq.(3). Different from Jensen's, a kind of non-Gaussian random velocity field was used to advect the passive scalar in this paper. In the following, we will introduce the intermittent random velocity field.

She and Lévêque developed the model of intermittency in fully developed turbulence. A universal relation between scalings and successive structures is postulated, which leads to a prediction of the entire set of scaling exponents[15]

$$\epsilon_l^p\sim l^{\tau(p)},\quad \tau(p)=-\frac{2}{3}p+2\Big[1-\Big(\frac{2}{3}\Big)^p\Big];\tag{7}$$

$$\delta v_l^p\sim l^{\xi(p)},\quad \xi_{\mathrm{SL}}(p)=\frac{p}{9}+2\Big[1-\Big(\frac{2}{3}\Big)^{p/3}\Big].\tag{8}$$

The probability density function solution of the model is shown to be simply related to the log-poisson statistics of local nondimensional energy dissipation by

$$P_l(v_l)=\sum_{i=0}^{\infty}\mathrm{e}^{-\lambda}\frac{\lambda^i}{i!}\Big(\frac{l}{l_0}\Big)^{-\gamma}\beta^{-i}\cdot P_{l_0}\Big(\Big(\frac{l}{l_0}\Big)^{-\gamma}\beta^{-i}v_l\Big),\tag{9}$$

where $\beta = 2/3$, $\lambda = C\log(l/l_0)$, and $C = 2$. $P_{l_0}(x) = \mathrm{e}^{(-x^2)/\sigma^2}/\sigma\sqrt{2\pi}$ is a Gaussian distribution and we choose $\sigma = l_0$. In a shell model, the velocity variable u_n is given by the mean energy of the nth wave number shell

$$u_n(t)=\sqrt{\int_{k_n}^{k_{n+1}}2E(q,t)\mathrm{d}q}\tag{10}$$

and can also be regarded as the speed increment $|v(x) - v(x + l)|$ on an eddy of scale $l \sim k_n^{-1}$. Replacing l by k_n in expression (9), we obtain the probability distribution function of the velocity u_n on each shell

$$\begin{aligned}P_n(u_n)=&\sum_{i=0}^{\infty}\mathrm{e}^{-\lambda}\frac{\lambda^i}{i!}\Big(\frac{k_0}{k_n}\Big)^{-\gamma}\beta^{-i}\\&\times P_{k_0}\Big(\Big(\frac{k_0}{k_n}\Big)^{-\gamma}\beta^{-i}u_n\Big).\end{aligned}\tag{11}$$

We call stochastic variables that follow the probability density of expression (11) SL variables (or SL noise). Based on the rejection method, which is a powerful, general technique for generating random deviation whose probability distribution function $P(x)\mathrm{d}x$ is known and computable,[16] we can numerically generate SL variables. Combining the distribution function (11) with the shell model Eq.(3), we obtain stochastic different equations, where $u_m(t)$ is a δ-correlated in time and independent complex variable in the form of $a + \mathrm{i}b$ with a and b being independent SL variables. We call these random complex variables SL random velocity in this paper, which has the property

$$\langle(u_n(t)u_n^*(t'))^{p/2}\rangle=D_n^p\delta(t-t'),\quad p=2,4,6,\cdots\tag{12}$$

where

$$D_n^p\sim k_n^{-\xi_{\mathrm{SL}}(p)}.\tag{13}$$

We regard D_n^2 as the intensity of SL velocity.

To test and verify the SL random velocity given by using numerical rejection-method, we give the scaling exponent of the SL random velocity in Fig.1, which is obtained by making statistics 10^8 times each shell. These scaling exponents are also compared with those obtained from expression (8). Obviously, the statistical scaling exponent is very close to that obtained from expression (8). For the force term we assume that $f(t)$ is an independent complex function of Gaussian variable and δ correlated in time, with $\langle f(t)f^*(t')\rangle = F^2\delta(t-t')$, where F is the intensity of the external force.

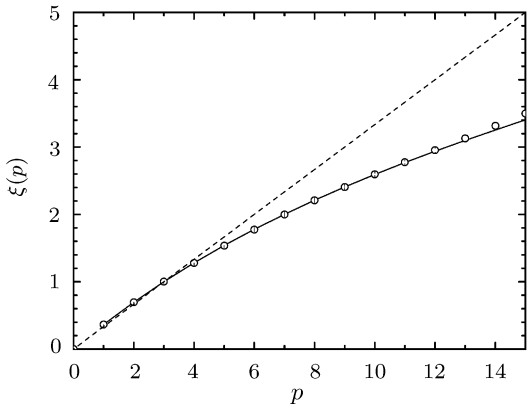


Fig.1. The structure-function exponent $\xi(p)$ (open circles) vs p. The data are obtained by making statistics of SL random velocities generated by using our numerical rejection-method with $N = 19$. The solid line is obtained from expression (8); the dashed line is for the results of Kolmogorov (1941) for $\xi(p) = p/3$.

3. Monte Carlo simulations for the stochastic shell model

In this section, we prescribe two kinds of random velocity fields, to advect the passive scalar. The first one is SL random velocity introduced in Section 2 and the second one is Gaussian random velocity with $\langle(u_n(t)u_n^*(t'))\rangle = C_n^2\delta(t-t')$, where C_n^2 is the intensity of it. Now we consider Reynolds number of the Gaussian random velocity to be equal to that of the SL's with $C_1^2 = D_1^2$.

In the shell model, the pth-order structure functions for the passive scalar is defined as

$$\langle(\theta_n\theta_n^*)^{p/2}\rangle \propto k_n^{-H(p)}, \tag{14}$$

where $H(p)$ is called the scaling exponent of order p. From a linear stochastic differential equation with white-noise coefficients, it is possible to obtain exact equations for moments of arbitrary order.[17] Equations for the second and fourth-order moments of the passive scalars are given in Ref.[13], where the velocities are Gaussian variables with scaling exponent $\xi(p) = 0.3p$. For SL random velocity, $\xi_{\rm SL}(p)$ is a nonlinear function of p, the problem becomes complicated. Here we use numerical method to study the stochastic shell model. In the following, we use subscript SL to denote the variables correlated with passive scalar advected by SL random velocity, the subscript G to denote the variables of Gaussian random velocity case, and subscript GOY to indicate the variables related to passive scalar coupled with GOY function (Eq.(6)). For example, $H_{\rm SL}(p)$ denotes the scaling exponent of pth order for passive scalar advected by SL random velocity, and $H_{\rm G}(p)$ represents the scaling exponent of Gaussian case. To find whether the scaling exponent $H_{\rm SL}(p)$ is more anomalous than $H_{\rm G}(p)$, we compare the scaling exponents of these two cases under the same condition.

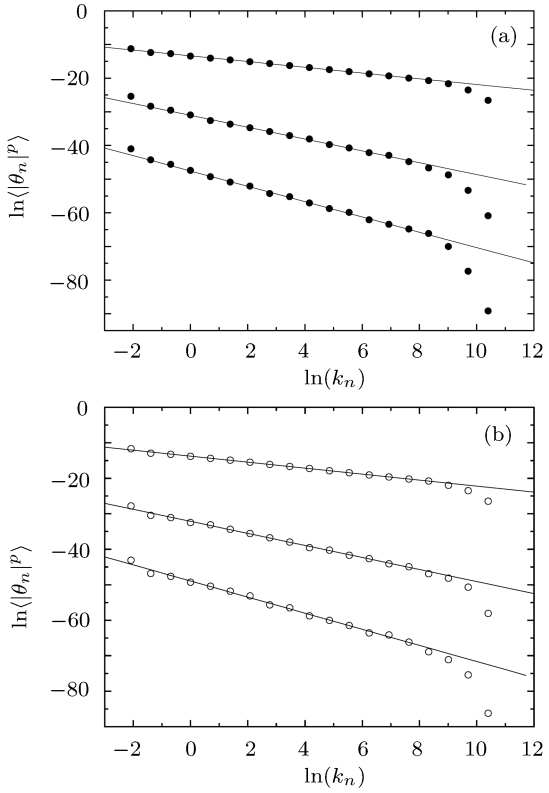


Fig.2. Scaling of the structure functions. $\ln\langle|\theta|^p\rangle$ vs $\ln k_n$ for $p = 2$, $p = 5$ and $p = 8$ from a numerical integration of Eqs. (3) with $\kappa = 7\times10^{-4}$ and $F = 5\times10^{-4}$. The advected velocities are (a) SL and (b) Gaussian. The solid lines have the slopes (a) 0.8451, 1.74464, 2.27221 and (b) 0.85453, 1.75943, 2.27523 for $p = 2$, 5, 8 respectively.

We perform the calculation of the passive scalar structure functions for positive integer p. Figure 2 shows the logarithm of pth-order structure function $\langle|\theta_n|^p\rangle$ versus the logarithm of wave number k_n with

molecular diffusion $\kappa = 7 \times 10^{-4}$ and force intensity $F = 5 \times 10^{-4}$. Apparently, there exists an inertial range from the 3rd shell to the 16th shell. With the slopes of the fitted lines for these inertial ranges we obtain the scaling exponents of passive scalars. We have $H_{\rm SL}(2) = 0.84515$, $H_{\rm SL}(5) = 1.74464$, and $H_{\rm SL}(8) = 2.27221$ in Fig.2(a) and $H_{\rm G}(2) = 0.85453$, $H_{\rm G}(5) = 1.75943$, and $H_{\rm G}(8) = 2.27523$ in Fig.2(b). Because $H_{\rm SL}(5)/H_{\rm SL}(2) < 5/2$, $H_{\rm SL}(8)/H_{\rm SL}(2) < 8/2$ and $H_{\rm G}(5)/H_{\rm G}(2) < 5/2$, and $H_{\rm G}(8)/H_{\rm G}(2) < 8/2$ in the two cases, we can draw the conclusion that the scaling of the structure function of the passive scalar is anomalous in these two cases. But it hardly shows that the scaling exponents induced by SL velocity are more "anomalous" than those of Gaussian case from these figures.

For a clear comparison between the two cases, Fig.3 shows the scaling exponents $\langle H(p)\rangle_{\rm E}$ versus p up to order $p = 15$, where the subscript E denotes ensemble, and $\langle\rangle_{\rm E}$ indicates ensemble average. We choose 64 independent initial conditions for the calculations and then average the values of $H(p)$. The error bars are obtained from these averages. Figure 3(a) shows that $\langle H_{\rm SL}(p)\rangle_{\rm E} \approx \langle H_{\rm G}(p)\rangle_{\rm E}$ with $p \leq 7$, and $\langle H_{\rm SL}(p)\rangle_{\rm E} > \langle H_{\rm SL}(p)\rangle_{\rm E}$ with $p > 7$. At the same time, the values of $|\langle H_{\rm SL}(p)\rangle_{\rm E} - \langle H_{\rm G}(p)\rangle_{\rm E}|$ each are less then their error for all the values of p. From this point of view, we believe that the scaling exponents of passive scalars are identical to each other under the condition above within the errors. That is to say, $\langle H_{\rm SL}(p)\rangle_{\rm E}$ is not more anomalous than $\langle H_{\rm G}(p)\rangle_{\rm E}$. The circles indicate scaling exponents of passive scalar ($\langle H_{\rm GOY}(p)\rangle_{\rm E}$) obtained from coupling Eq.(3) with Eq.(6), and the downmost vertical bars for that ($\langle H_{\rm e}(p)\rangle$) of experimental data. It is clear that the $\langle H_{\rm GOY}(p)\rangle_{\rm E}$ is more close to $H_{\rm e}(p)$, and there is a rather large distance between $\langle H_{\rm SL}(p)\rangle_{\rm E}$ ($\langle H_{\rm G}(p)\rangle_{\rm E}$) and the experimental data. Moreover, they cannot reach the experimental data. Because the velocities are δ correlated in time, at the same time, they lack the correlations between shells and the temporal messages of the velocity, which can be found in real fluids and the GOY model. From Fig.3(b) we can come to the same conclusion as that from Fig.3(a), though different molecular diffusions and external forces are chosen for them. During the calculation, we find that $\langle H(p)\rangle_{\rm E}$ increases with κ increasing. Figure 4 shows the scaling exponent $\langle H(p)/H(3)\rangle_{\rm E}$ using Extended Self Similarity (ESS), where $\langle\rangle_{\rm E}$ also indicates ensemble average. $\langle H_{\rm SL}(p)\rangle/\langle H_{\rm SL}(3)\rangle \approx \langle H_{\rm G}(p)\rangle/\langle H_{\rm G}(3)\rangle$ for all values of $p \leq 10$, and we can distinguish them only when $p > 10$. This strongly supports the conclusion above.

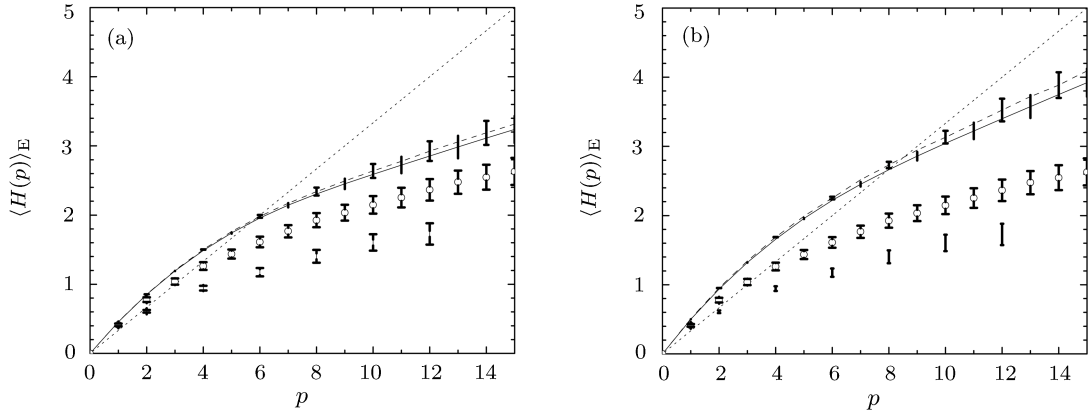


Fig.3. The structure-function scaling exponent $H(p)$ of passive scalar vs p with (a) $\kappa = 7 \times 10^{-4}$, $F = 5 \times 10^{-4}$ and (b) $\kappa = 1 \times 10^{-3}$ and $F = 1 \times 10^{-3}$. The passive scalar is advected by SL velocity (solid line) and Gaussian velocity (dashed line) respectively. The data are obtained by a numerical integration of Eq.(3). The experimental data of $H(p)$ from Ref.[18] are indicated by vertical bars. The scaling of coupling function of Eqs.(3) and (6) is shown by open circles. The dot line denotes the Obukhov–Corrsin prediction $H(p) = p/3$.

Finally we have computed the probability distribution function (PDF) of the passive-scalar increments at different scales, i.e. the PDF of θ_n at varying n, as shown in Fig.(5). $\langle P(S)\rangle_{\rm E}$ also denotes the resemble average of PDF $P(S)$, where $S = |\theta_n|/\langle|\theta_n^2|\rangle^{1/2}$, and $P_{\rm SL}(S)$ denotes the probability distribution function of passive scalar advected by SL random velocity. We find that there is no considerable difference between $\langle P_{\rm SL}(S)\rangle_{\rm E}$ and $\langle P_{\rm G}(S)\rangle_{\rm E}$, so the figures of $\langle P_{\rm SL}(S)\rangle_{\rm E}$ are shown here to compare

with the PDF of passive scalar ($P_{\rm GOY}(S)$) advected by GOY function. For k_n at the beginning of the inertial range $n = 3$ (Fig.5(a)), there is θ difference on large scale, showing that the PDF is quite close to the Gaussian form. Obviously, there is no difference between $\langle P_{\rm SL}(S)\rangle_{\rm E}$ and $\langle P_{\rm GOY}(S)\rangle_{\rm E}$. On the contrary at the end of the inertial range $n = 16$, the PDF exhibits the typical exponential tail of velocity and θ gradient in turbulence. We find that $\langle P_{\rm SL}(S)\rangle_{\rm E} < P_{\rm GOY}(S)$ when $S \in (0, 0.31)$, $\langle P_{\rm SL}(S)\rangle_{\rm E} > P_{\rm GOY}(S)$ when $S \in (0.31, 3.75)$ and $\langle P_{\rm SL}(S)\rangle_{\rm E} < P_{\rm GOY}(S)$ when $S > 3.75$, that is to say, the PDF of passive scalar induced by GOY function has a higher 'tail' than that of SL random velocity case. The factors, such as spatiotemporal coupling and energy cascade which can be found in the velocity of GOY model, are important ingredients for intermittent passive scalars. The absence of these ingredients, the scaling exponents of the structure functions for the random velocity can only weakly affect the passive scalar. So the molecular diffusion of the passive scalar is the foremost parameter for its scaling. This result in the fact that the passive scalar advected by intermittent velocity is not more intermittent than that advected by the nonintermittent velocity under the same molecular diffusion.

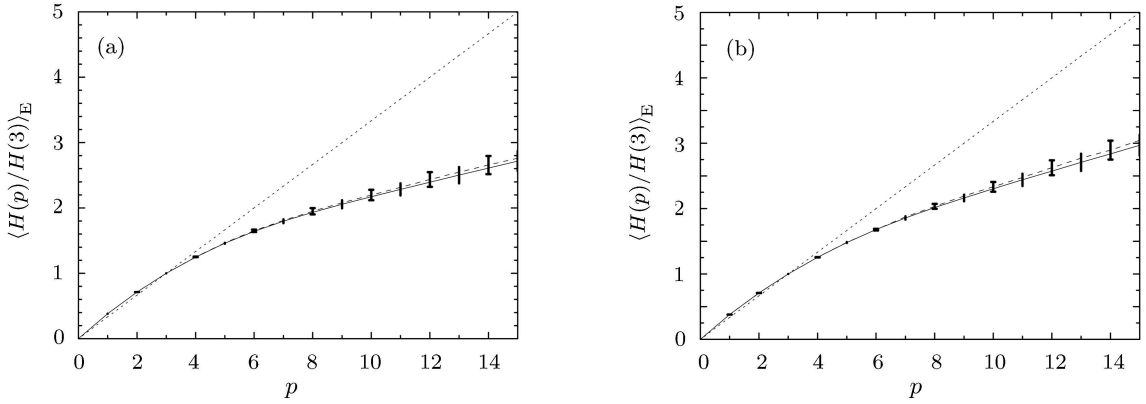


Fig.4. ESS $H(p)/H(3)$ of passive scalar vs p with (a) $\kappa = 7 \times 10^{-4}$, $F = 5 \times 10^{-4}$ and (b) $\kappa = 1 \times 10^{-3}$ and $F = 1 \times 10^{-3}$. The passive scalar is advected by SL velocity (solid line) and Gaussian velocity (dashed line) respectively. The dot line denotes the Obukhov–Corrsin prediction $H(p) = p/3$.

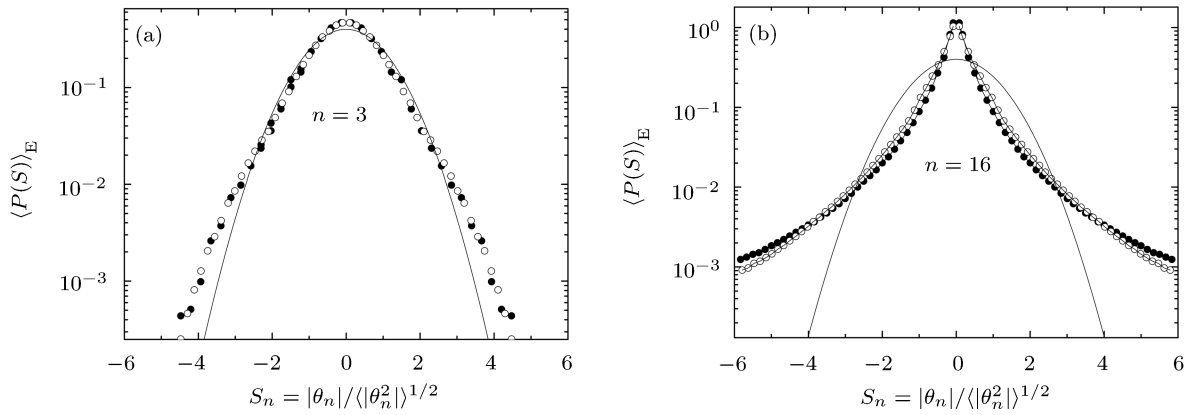


Fig.5. PDF of $S = |\theta_n|/\langle|\theta_n^2|\rangle^{1/2}$ for (a) $n = 3$ and (b) $n = 16$. The passive scalar is advected by SL velocity (open circles) and velocity of the GOY model (dots) respectively. The solid line is for the standard Gaussian shape. The PDF for negative S is drawn for clarity, and it is taken as being equal to the PDF of the corresponding positive values.

In all the numerical integrations 19 shells are used, and we use Gear scheme to solve all the functions, the details of which may be found in Appendix A. For the stochastic equation, the molecular diffusivity varies between 1×10^{-4} and 3×10^{-3}, the external force between 1×10^{-4} and 3×10^{-3}, and the

time steps is 8×10^{-6}. In the coupling functions of Eqs.(3) and (6), $\kappa \in (4 \times 10^{-8}, 5 \times 10^{-7})$, and $f \in (3\times10^{-4}, 5\times10^{-3})$, with time unit being 1×10^{-4}.

4. Conclusion and discussion

We have proposed and studied a shell model for a randomly advected passive scalar. We also introduced an SL δ-correlated-in-time velocity, which has a typical anomalous scaling exponent curve of fully developed turbulence. At the same time Gaussian random velocity is proposed to advect the passive scalar. We have compared the scaling exponents with PDF of passive scalar induced by these two kinds of velocities and not found that the intermittent random velocity can induce more intermittent passive scalar than the nonintermittent velocity. We find several properties of passive scalar. 1) Intermittence is a intrinsic property of the passive scalar, where molecular diffusion plays an important direct role. So intermittent passive scalar can be obtained with Gaussian random velocity. Intermittent random velocity will not consequentially induce any additional intermittent passive scalars. So similar anomalous scalings of $H_{\rm SL}(p)$ and $H_{\rm G}(p)$ are found in this paper. 2) The larger the dissipation (molecular diffusivity) of passive scalar is, the less anomalous the passive scaling will be. Within a certain range, external force dose not affect the scaling of passive scalar. 3) The correlation between shells (different scales) and the temporal message of the velocity are important facts. They determine the energy cascade and synergism between different scales. SL and Gaussian random velocities lake these factors. They are δ-correlated in time, so $H_{\rm SL}(p)$ and $H_{\rm G}(p)$ can never reach the experimental data. All these conclusions are obtained from our stochastic shell model, which exits in a kind of Fourier space. The anomalous properties of scaling in our model at a phenomenological level could help to develop a phenomenology of intermittent in physical space.

Appendix A Gear method

Because of the large scale difference of shell model, Eq.(3) becomes a stiff equation. The introduction of stochastic velocity add stiffness to the problem. Gear method is a implicit linear multistep scheme. It has the form[19]

$$\sum_{j=0}^{k} a_j y_{n+1} = h\beta_k f_{n+k}, \tag{A1}$$

where h is the step, and other parameters are listed in Table 1.

Table 1. Gear parameters.

k	β	a_0	a_1	a_2	a_3	a_4	a_5	a_6
1	1	-1	1					
2	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{4}{3}$	1				
3	$\frac{6}{11}$	$-\frac{2}{11}$	$\frac{9}{11}$	$-\frac{18}{11}$	1			
4	$\frac{12}{25}$	$\frac{3}{25}$	$-\frac{16}{25}$	$\frac{36}{25}$	$-\frac{48}{25}$	1		
5	$\frac{60}{137}$	$-\frac{12}{137}$	$\frac{75}{137}$	$-\frac{200}{137}$	$\frac{300}{137}$	$-\frac{300}{137}$	1	
6	$\frac{60}{147}$	$\frac{10}{147}$	$-\frac{72}{147}$	$\frac{225}{147}$	$-\frac{400}{147}$	$\frac{450}{147}$	$-\frac{360}{147}$	1

Its 6th-order form is a powerful scheme for stochastic different equations. For Eq.(3) this technique converges to the Itô integral with a convergence of order h^3. In general, the order of the stochastic convergence of a numerical method will be less than or equal to half the order of the deterministic convergence.

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