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An improved formula for the surface resistance of normal metals at millimetre wavelengths*

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A number of researchers have reported discrepancies between surface resistance (SR) measurements and classical theoretical predictions in normal metals for millimetre wavelengths (MW). In this paper, a rigorous model is developed for analysing SR of normal metals. This model is based on quantum mechanical analysis of the spatial dispersion within the metal. We use the model to predict SR and eliminate the discrepancies between SR measurements and classical theoretical predictions in normal metals for MW. Moreover, we have compared the results of this model with that of the classical skin-effect model and classical relaxation-effect model. Our analysis shows that the conductivity is not only frequency- but also wave-vector-dependent for MW. We demonstrate that our model has good quantitative agreement with the published experimental data for the room temperature surface resistance of normal metals for MW.

Keywords: surface resistance, spatial dispersion, millimeter wavelengths

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1. Introduction

Theoretical studies of the surface resistance (SR) in metallic ferromagnets have been based on solving Maxwell's equations for electromagnetic fields with appropriate boundary conditions at sample's surfaces.[1−7] Measurements of the SR provide an accurate tool for the investigation of the Fermi wavevector. Accurate values of the SR of conductors are very important to the efficient utilization at millimetre wavelengths (MW) region which is under development for applications in radar, communications, and radio astronomy at present.

However, a number of researchers have reported discrepancies between SR measurements and classical theoretical predictions at MW.[8] In this paper, we have develop a rigorous model for analysing SR of normal metals which is used to evaluate the SR in normal metals at MW. The model includes spatial dispersion within the metal. The SR of this model is quite different from the SR of the classical model.

2. Theoretical model

In this section we provide the quantum expression for the dielectric function for normal metals, within the frame of spatial dispersion theory and use perturbation theory to find the change in the exciton wavefunction caused by an electric field with frequency ω and wave vector $\boldsymbol{q}$. We consider the simplest model of a metal as a free-electron gas. The dynamical dielectric function $\varepsilon(\boldsymbol{q},\omega)$ in MKS units can be calculated from the quantum expression of the free-electron gas as given by Ref.[3]:

$$\varepsilon(\boldsymbol{q},\omega) = 1 + \frac{2e^2}{q^2\varepsilon_0}\frac{1}{V}\sum_k \frac{f(k) - f(k+q)}{E(k+q) - E(k) - (\hbar\omega + \mathrm{i}\eta)}. \tag{1}$$

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where $f(k)$ is the Fermi–Dirac distribution function, $E(k) = \hbar^2 k^2/2m$ and $E(k+p) = \hbar^2(k+p)^2/2m$ denote the energy, V is the volume. All other variables have their usual meaning. We start from this basic equation by performing the replacement $\boldsymbol{k}+\boldsymbol{q} \to -\boldsymbol{k}$ in the term containing $f(k+q)$ (then $\boldsymbol{k}'$ is relabelled as $\boldsymbol{k}$),

$$\begin{aligned}\varepsilon(\boldsymbol{q},\omega) = & \ 1 + \frac{2e^2}{q^2\varepsilon_0}\frac{1}{V}\sum_k \frac{f(k+q)-f(k)}{E(k)-E(k+q)-(\hbar\omega+\mathrm{i}\eta)} \\ = & \ 1 + \frac{2e^2}{q^2\varepsilon_0}\frac{1}{V}\sum_k \frac{f(k)-f(k+q)}{E(k+q)-E(k)+(\hbar\omega+\mathrm{i}\eta)}. \end{aligned} \tag{2}$$

From Eqs.(1) and (2), the dynamical dielectric function $\varepsilon(\boldsymbol{q},\omega)$ can also be written as

$$\varepsilon(\boldsymbol{q},\omega) = 1 + \frac{e^2}{q^2\varepsilon_0}\frac{1}{V}\sum_k \left[\frac{f(k)-f(k+q)}{E(k+q)-E(k)-(\hbar\omega+\mathrm{i}\eta)} + \frac{f(k)-f(k+q)}{E(k+q)-E(k)+(\hbar\omega+\mathrm{i}\eta)}\right]. \tag{3}$$

One can rewrite $f(k)-f(k+q)$ as

$$f(k)-f(k+q) = f(k)(1-f(k+q)) - f(k+q)(1-f(k)). \tag{4}$$

Performing the replacement $\boldsymbol{k}+\boldsymbol{q} \to -\boldsymbol{k}$, $\boldsymbol{k} \to -\boldsymbol{k}-\boldsymbol{q}$ for $f(k+q)(1-f(k))$,

$$\begin{aligned}\varepsilon(\boldsymbol{q},\omega) = & \ 1 + \frac{2e^2}{q^2\varepsilon_0}\frac{1}{V}\sum_k f(k)(1-f(k+q))\left[\frac{1}{E(k+q)-E(k)-(\hbar\omega+\mathrm{i}\eta)} + \frac{1}{E(k+q)-E(k)+(\hbar\omega+\mathrm{i}\eta)}\right] \\ = & \ 1 + \frac{2e^2}{q^2\varepsilon_0}\frac{1}{V}\sum_k f(k)\left[\frac{2(E(k+q)-E(k))}{(E(k+q)-E(k))^2-(\hbar\omega+\mathrm{i}\eta)^2}\right]. \end{aligned} \tag{5}$$

We also use the dynamical dielectric function in the long wavelength limit. We notice that $\frac{1}{V}\sum_k f(k) = \frac{n}{2}$, $\frac{1}{V}\sum_k \left(\frac{\hbar k}{m}\right)^2 f(k) = \frac{3}{5}v_{\mathrm{F}}^2\frac{n}{2}$, and the DC conductivity $\sigma_0 = \dfrac{ne^2}{m}\tau$, where the Fermi velocity $\boldsymbol{v}_{\mathrm{F}} = \hbar\boldsymbol{k}_{\mathrm{F}}/m$, and $2\eta/\hbar = 1/\tau$, τ is the electron collision time and n is the electron density. We obtain

$$\varepsilon(\boldsymbol{q},\omega) = 1 + \frac{\sigma_0}{\mathrm{i}\omega\varepsilon_0\left(1+\mathrm{i}\omega\tau\left(\dfrac{3v_{\mathrm{F}}^2q^2}{5\omega^2}\right)\right)}. \tag{6}$$

Because of the inclusion of spatial dispersion, the dynamical dielectric function $\varepsilon(\boldsymbol{q},\omega)$ is not only frequency but also wave vector dependent.

The surface impedance Z is defined as the ratio of the (complex) electric and magnetic field components in the direction parallel to the surface of incidence, evaluated just inside the surface. This concept gives a convenient description of the optical properties of a material half-space. The real part of Z, which is called the 'surface resistance', is proportional to the energy absorbed from the wave and it is useful in determining the shapes of the Fermi surfaces of metals. The imaginary part of Z, called the 'surface reactance', determines the frequency shift of a resonant cavity bounded by the conductor. Our definition of the surface impedance in MKS units is

$$Z = E(0)/B(0) = \mu c/n = \mu c/\sqrt{\varepsilon}. \tag{7}$$

The propagation of electromagnetic waves through matter is determined by solving the Maxwell equations, in which the dielectric function ε, equal to the square of the index of refraction, carries information about the material properties. From Eqs.(6) and (7), we obtain

$$Z = \sqrt{\frac{\mathrm{i}\omega\mu\mu_0}{\sigma_{\mathrm{s}}+\mathrm{i}\omega\varepsilon_0}}. \tag{8}$$

So the SR is

$$\begin{aligned} R_{\mathrm{sd}} = & \ \mathrm{Re}\left[\sqrt{\frac{\mathrm{i}\omega\mu_0\mu_{\mathrm{r}}}{\sigma_{\mathrm{sd}}+\mathrm{i}\omega\varepsilon_0}}\right], \\ \sigma_{\mathrm{sd}} = & \ \frac{\sigma_0}{1+\mathrm{i}\omega\tau\left(1-\dfrac{3v_{\mathrm{F}}^2q^2}{5\omega^2}\right)}. \end{aligned} \tag{9}$$

We have thus developed a rigorous model that is called the 'spatial dispersion model'. The classical skin-effect model $R_0 = \mathrm{Re}\left\{\sqrt{\frac{\mathrm{i}\omega\mu_0\mu_\mathrm{r}}{\sigma_0 + \mathrm{i}\omega\varepsilon_0}}\right\}$, $\sigma_0 = \frac{ne^2}{m}\tau$; and the classical relaxation-effect model $R_\mathrm{R} = \mathrm{Re}\left\{\sqrt{\frac{\mathrm{i}\omega\mu_0\mu_\mathrm{r}}{\sigma_\mathrm{R} + \mathrm{i}\omega\varepsilon_0}}\right\}$, $\sigma_\mathrm{R} = \frac{\sigma_0}{1 + \mathrm{i}\omega\tau}$; are cited from Refs.[3] and [4]. In Fresnel optics spatial dispersion is neglected. This simple free-electron form of $\varepsilon(\omega)$ is, however, often not sufficient for real metals. In the 'spatial dispersion model', we includes spatial dispersion within the metal in the approximation. The following analysis shows that using this model leads to good quantitative agreement with the published experimental data for the room temperature excess conduction loss of normal metals in the MW range.

3. Results of calculation and discussion

In this section we use the spatial dispersion model to predict the excess SR in normal metals in MW range. We have compared the results of the model with the results of the classical skin-effect model and classical relaxation-effect model. As a lot of experimental data for the room temperature SR of high electrical conductivity metals: silver, copper, aluminium and gold, are available, we calculate the SR in this work only for these metals, and it is expected that this model should be equally applicable to other normal metals. Calculation is based on the values of q given in Ref.[3], other values are given in Table 1 noted from other sources.[9]

Table 1. Solid-state parameters for different metals (20 °C).[8,9]

Parameter / Metal	element density/(g/cm^3)	atomic weight/(g/mole)	τ/fs	$\sigma_0/(\times 10^7\,\mathrm{S/m})$
silver	10.49	107.868	38.182	6.301
copper	8.92	63.546	25.018	5.959
gold	19.32	196.967	27.135	4.517
aluminium	2.70	26.982	7.407	3.774

The results obtained by using the spatial dispersion model for copper, silver, aluminium and gold are shown in Tables 2–5, which also include the results from the classical skin-effect model and the classical relaxation-effect model.[8]

An extracted value of R_s is given by Tischer[10] in Table 2. The SR for both classical SR models was reported by Ref.[6]. Using the spatial dispersion model, we obtained a value for the SR. It is clearly seen that discrepancies between SR measurements and theoretical predictions are: 11.55%, 11.91% and 12.86% for the classical skin-effect model; 11.91%, 12.27% and 13.04% for the classical relaxation-effect model; and 4.51%, 4.87% and 5.79% for the spatial dispersion model at the conductivity of 5.7551×10^7, 5.8×10^7 and $5.9595 \times 10^7\,\mathrm{S/m}$, respectively.

Table 2. Surface resistance for copper at 35 GHz.

$\sigma_0/\times 10^7\mathrm{S/m}$	R_s/Ω	R_0/Ω	R_R/Ω	R_sd/Ω
5.7551	0.0554	0.0490	0.0488	0.0529
5.8000	0.0554	0.0488	0.0486	0.0527
5.9595	0.0552	0.0481	0.0480	0.0520

In Table 3 an extracted value of R_s was given by Tischer.[11] The SR for both classical SR models was reported by Ref.[6]. Using the spatial dispersion model, we obtained a value for the SR. The discrepancies between SR measurements and theoretical predictions are: 17.44% and 18.54% for the classical skin-effect model; 17.91% and 19.02% for the classical relaxation-effect model; and 14.59% and 15.67% for the spatial dispersion model at the conductivity of 5.7551×10^7 and $5.9595 \times 10^7\,\mathrm{S/m}$, respectively.

Table 3. Surface resistance for copper at 70 GHz.

$\sigma_0/(\times 10^7\mathrm{S/m})$	R_s/Ω	R_0/Ω	R_R/Ω	R_sd/Ω
5.7551	0.0843	0.0696	0.0692	0.0720
5.9595	0.0836	0.0681	0.0677	0.0705

The results of SR calculations from the three models are given in Table 4.[12] It is clearly seen that discrepancies between SR measurements and theoretical predictions are: 39.68%, 42.35% and 30% for the classical skin-effect model; 40.69%, 43% and 30.23% for the classical relaxation-effect model; and 11.14%, 24.71% and 23.98% for the spatial dispersion model of

silver, copper and aluminium, respectively.

Table 4. Surface resistance at 145 GHz.

	R_s/Ω	R_0/Ω	R_R/Ω	R_{sd}/Ω
silver	0.158	0.0953	0.0937	0.1404
copper	0.170	0.0980	0.0969	0.1280
aluminium	0.176	0.1232	0.1228	0.1338

In Table 5 an extracted value of R_s was given by Batt.[13] The SR for both classical SR models was reported by Ref.[6]. Using the spatial dispersion model, we obtained a value for the SR factor. The discrepancies between SR measurements and theoretical predictions are: 55.04% and 56.74% for the classical skin-effect model; 57.98% and 59.84% for the classical relaxation-effect model; and 48.84% and 50.86% for the spatial dispersion model at the conductivity of 4.1667×10^7 S/m and 4.5167×10^7 S/m, respectively.

Table 5. Surface resistance for gold at 890 GHz.

$\sigma_0/(\times 10^7 \mathrm{S/m})$	R_s/Ω	R_0/Ω	R_R/Ω	R_{sd}/Ω
4.1667	0.645	0.290	0.271	0.330
4.5167	0.645	0.279	0.259	0.317

4. Summary

In this paper, we have developed and demonstrated a rigorous model applied to normal metals in the MW range. We have used the model to predict the SR in normal metals in the MW range. We have compared the experiment data with predictions of SR for silver, gold, aluminium for three models. Our analysis shows that conductivity is not only frequency- but also wave-vector-dependent. We demonstrate that our spatial dispersion model has good quantitative agreement with the published experiment data for the room temperature SR of normal metals in the MW range.

It can be clearly seen from Tables 2–5 that there are very large deviations in classical theory predictions of the SR for copper from the lower frequency measurement results. The worst case measured was: 12.86%, 18.54%, 42.35% and 56.74% for the classical skin-effect model; 13.04%, 19.02%, 43% and 59.84% for the classical relaxation-effect model; and 5.79%, 15.67%, 24.71%, and 50.86% for the spatial dispersion model at the frequency of 35 GHz, 70 GHz, 145 GHz and 890 GHz, respectively, above the predicted. We demonstrate that our spatial dispersion model has good quantitative agreement with the published experimental data for the room temperature SR of normal metals in the MW range.

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