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A new method of asymmetric Abel inversion for magnetic equilibrium configuration state in tokamaks

Tian Chong-Li(田崇利)†, Zhou Yan(周 艳),
Shi Zhong-Bing(石中兵), and Li Ying-Liang(李英量)

Southwestern Institute of Physics, Chengdu 610041, *China*

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It is difficult to obtain the asymmetrical factor along the observation direction parallel to the plasma mid-plane when the detected radiation is also in the mid-plane. This paper considers the magnetic surfaces and Grad–Shafranov shift, and develops a new method for inverse asymmetric electron density information, during magnetic equilibrium configuration in a tokamak.

Keywords: asymmetric Abel inversion, electron density, Grad–Shafranov shift, matrix method

PACC: 5270, 5225L, 5255

1. Introduction

Abel inversion numerical methods for circularly symmetric radiation sources were developed by Barr,[1] Bockasten[2] and Cho *et al.*[3] The asymmetrical factor along the observation direction parallel to the plasma mid-plane can be obtained by the method of separation of variables[4] when the detected radiation direction is perpendicular to the plasma mid-plane. It is always difficult to obtain the asymmetrical information along the plasma mid-plane when the data are measured on chords parallel to the plasma mid-plane. In this paper, a new method is developed for solving the asymmetric Abel inversion with magnetic surfaces and Grad–Shafranov shift.

The numerical method using matrix inversion, developed by Cho, will also be applied to inverse asymmetrical data with a Grad–Shafranov shift.

2. Cho matrix method

In this paper, the direction parallel to the plasma mid-plane is defined as the x-direction, and the direction perpendicular to the plasma mid-plane is the y-direction. Therefore, the symmetric Abel inversion equation can be formulated in discrete form as follows:

$$I(y)\Delta y\Delta z = \sum_{-x_0}^{x_0} \varepsilon(r)\Delta x\Delta y\Delta z. \tag{1}$$

Where $I(y)$ denotes the intensity of radiation in the y direction, $\varepsilon(r)$ is the emission at radius r, z denotes the symmetry axis and the observation direction parallel to the x-direction.

Since the intensity of radiation multiplied by local area of measurement is the same as the summation of all the emissions multiplied by the local volumes, with the same local thickness, elimination of Δz from both sides gives the following equation:

$$I(y)\Delta y = \sum_{-x_0}^{x_0} \varepsilon(r)\Delta x\Delta y. \tag{2}$$

The left hand side of the equation is the measured intensity multiplied by the measuring interval and right hand side is the summation of local emission multiplied by local area in the same direction. When I_1, I_2, $\cdots$, I_n denote the measured data at each pixel of the measuring apparatus, they can be expressed as follows (see Fig.1):

$$\begin{aligned} I_1 &= 2(S_{11}\varepsilon_1 + S_{12}\varepsilon_2 + \cdots + S_{1n}\varepsilon_n)/d, \\ I_2 &= 2(S_{22}\varepsilon_2 + S_{23}\varepsilon_3 + \cdots + S_{2n}\varepsilon_n)/d, \\ &\vdots \qquad\qquad \vdots \\ I_n &= 2(S_{nn}\varepsilon_n)/d. \end{aligned} \tag{3}$$

Where ε_j means the local emission at the jth radius, S_{ij} is the area of cross-position of the ith measurement line and the jth radius, and d denotes the interval of measuring pixels. It is easy to transform Eq.(3) to the

†E-mail: tianchongli@163.com

form of a matrix, and Cho has developed the method to calculate the S_{ij}. So the emission of plasma can be found by a linear matrix equation as follows:

$$[\varepsilon] = \frac{d}{2}[S]^{-1}[I]. \tag{4}$$

In the following part of this paper, the new method will be introduced, which solves the asymmetric Abel inversion with a Grad–Shafranov shift. Therefore, it is necessary to introduce magnetic equilibrium equations.[5]

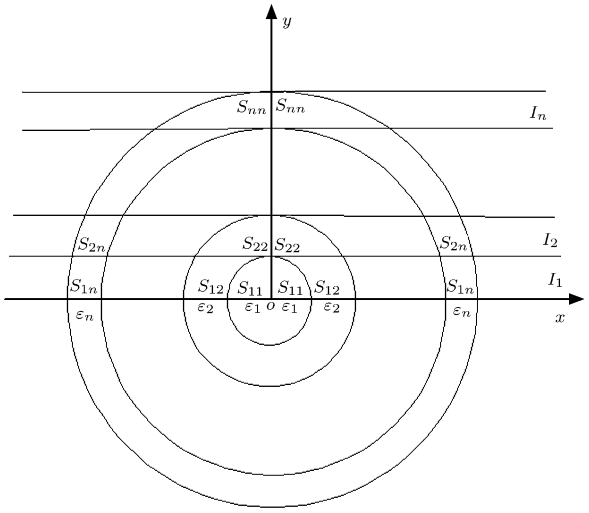


Fig.1. The matrix analysed with this graph of symmetric geometry.

3. Magnetic surfaces and Grad–Shafranov shift

Magnetic equilibrium equations can be defined as follows:

$$\begin{aligned} \nabla p &= \frac{1}{c}\boldsymbol{J} \times \boldsymbol{B}, \\ \nabla \times \boldsymbol{B} &= \frac{4\pi}{c}\boldsymbol{J}, \\ \nabla \cdot \boldsymbol{B} &= 0. \end{aligned} \tag{5}$$

Using particle conservation equations, electron and ion thermal conservation equations, we can obtain the magnetic pressure by the sophisticated calculation. The plasma pressure is expressed as follows:

$$p = n(T_{\mathrm{e}} + T_{\mathrm{i}}). \tag{6}$$

The following equation can be obtained from magnetic equilibrium equations, which is described as follows:

$$\boldsymbol{B} \cdot \nabla p = 0. \tag{7}$$

Equation (7) indicates that magnetic pressure is constant on the same magnetic surfaces. Electron and ion temperatures are constant on the same magnetic surfaces, because of intensive heat conductivity along with magnetic lines of flux. Therefore, particle density is constant on the magnetic surfaces. The magnetic axis is not always located on the geometrical centre of plasma on tokamak, the Grad–Shafranov shift between the centre of each magnetic surfaces and the geometrical centre of plasma can be calculated from Grad–Shafranov equations. At the equilibrium, electron density surfaces are the same as the magnetic surfaces. So the Grad–Shafranov shift is the same as the shift of electron density surfaces.

4. New method of asymmetric Abel inversion

Matrix S_{ij} is obtained by Cho's method for the condition of circularly symmetry. The circularly asymmetric case is divided into two geometrical components, the right-part and the left-part. And the area of each element is calculated with symmetrical matrix technology as follows:

$$\begin{aligned} S_{ij}^{+} &= S_{ij} + \left[\frac{1}{2}\mathrm{d}^2\theta_{ij} + \frac{1}{2}\Delta_j d\cos(\theta_{ij})\right], \\ S_{ij}^{-} &= S_{ij} - \left[\frac{1}{2}\mathrm{d}^2\theta_{ij} + \frac{1}{2}\Delta_j d\cos(\theta_{ij})\right]. \end{aligned} \tag{8}$$

Where S_{ij}^{+} denotes the area of cross-position of the ith measurement line and the jth radius on the right of magnetic axis, and S_{ij}^{-} denotes the area of cross-position of the ith measurement line and the jth radius on the left of magnetic axis, Δ_j denotes the Grad–Shafranov shift on the jth radius. The intersection position of the jth density surface and the y-axis is defined as κ_{ij}. h_j is the intersection position of the centre of the jth density surface and x-axis, and z_{ij} is the intersection position of the centre of the jth density surface and the ith detected chords. So the θ_{ij} as the angle between the line connecting the κ_{ij} with the origin and the line connecting the z_{ij} with the h_j. Therefore, following equation can be obtained:

$$\theta_{ij} = \arctan\left(\frac{\Delta_j}{\mathrm{i}d}\right). \tag{9}$$

The premise of Cho's method is that the slice Δx and the area S_{ij} of same density surfaces are distributed equally on both sides of geometrical centre.

Each equal density surface has been represented in the above section, despite the geometrical symmetry. But the new Δx and S_{ij} are unequal on the right and left sides of magnetic axis in the asymmetrical model, so that the surfaces are not assumed equal on the two sides of magnetic axis. It is assumed that the density has the same value in the left and right parts of S_{ij}. Using Eqs.(8) and (3), we can express the signals to the detector in the asymmetric model as follows (see Fig.2):

$$I(i) = \sum_{i}^{M} \sum_{j}^{N} [S_{ij}^{+} \varepsilon^{+}(j) + S_{ij}^{-} \varepsilon^{-}(j)]/d, \qquad (10)$$

where point 'o' denotes plasma geometric centre, different line-types denote the centres of the (j-1)th density surface in Fig.2. Therefore, the x coordinates have different forms on the two sizes of the magnetic axis. The right-x and left-x are described as follows:

$$\begin{aligned} x_j^{+} &= x_j + \Delta_0(j-1), \\ x_j^{-} &= x_j - \Delta_0(j-1). \end{aligned} \qquad (11)$$

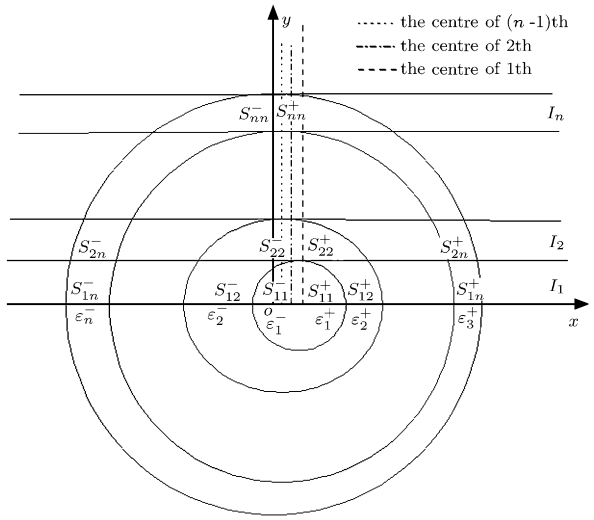


Fig.2. The matrix analysed with this graph of asymmetric geometry.

After applying the matrix technology, asymmetric inversion is expressed as follows:

$$\begin{bmatrix} S_{nn}^{-} & \cdots & S_{12}^{-} & S_{11}^{-} & S_{11}^{+} & S_{12}^{+} & \cdots & S_{nn}^{+} \\ S_{2n}^{-} & \cdots & S_{22}^{-} & 0 & 0 & S_{22}^{+} & \cdots & S_{2n}^{+} \\ & & \vdots & & & \vdots & & \\ S_{nn}^{-} & \cdots & 0 & 0 & 0 & 0 & \cdots & S_{nn}^{+} \end{bmatrix} \begin{bmatrix} \varepsilon_n^{-} \\ \vdots \\ \varepsilon_1^{-} \\ \varepsilon_1^{+} \\ \vdots \\ \varepsilon_n^{+} \end{bmatrix} = d \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}. \qquad (12)$$

The matrix of Eq.(12) can be written as linear matrix equation as follows:

$$[\varepsilon^{-}, \varepsilon^{+}] = d[S^{-}, S^{+}]^{-1}[I]. \qquad (13)$$

The discrete value of density can be obtained from the matrix, and the density of asymmetric inversion along with x-direction can be fitted from $[x_j^{-}, x_j^{+}]$ and $[\varepsilon^{-}(j), \varepsilon^{+}(j)]$ easily.

5. Conclusion and discussion

In this paper, the method of asymmetric Abel inversion was developed for circular surfaces of electron density, based on magnetic equilibrium in a tokamak. The Grad–Shafranov shift, obtained with equilibrium fit technology, can reconstruct the equilibrium magnetic surfaces. Calculation of the matrix in the asymmetric model is introduced in this paper. The density distribution of the asymmetric surfaces can be obtained with least-square matrix inversion method.

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