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Role of on-board discharge in shock wave drag reduction and plasma cloaking*

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In the present paper, a physical model is proposed for reducing the problem of the drag reduction of an attached bow shock around the nose of a high-speed vehicle with on-board discharge, to the problem of a balance between the magnetic pressure and gas pressure of plane shock of a partially ionized gas consisting of the environmental gas around the nose of the vehicle and the on-board discharge-produced plasma. The relation between the shock strength and the discharge-induced magnetic pressure is studied by means of a set of one-fluid, hydromagnetic equations reformed for the present purpose, where the discharge-induced magnetic field consists of the electron current (produced by the discharge)-induced magnetic field and the partially ionized gas flow-induced one. A formula for the relation between the above parameters is derived. It shows that the discharge-induced magnetic pressure can minimize the shock strength, successfully explaining the two recent experimental observations on attached bow shock mitigation and elimination in a supersonic flow during on-board discharge [Phys. Plasmas **9** (2002) 721 and Phys. Plasmas **7** (2000) 1345]. In addition, the formula implies that the shock elimination leaves room for a layer of higher-density plasma rampart moving around the nose of the vehicle, being favourable to the plasma radar cloaking of the vehicle. The reason for it is expounded.

Keywords: attached bow shock and magnetohydrodynamic (MHD) drag reduction, on-board discharges, plasma cloaking, MHD and fluid equation

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1. Introduction

As is well known, shock waves have always been the main obstacle for the development of supersonic / hypersonic vehicles. When a vehicle flies with a supersonic / hypersonic velocity, flow perturbations caused by the flying vehicle will propagate slower than the vehicle and thus accumulate continuously to form shock wave. The shock wave moves upstream in the form of an attached bow shock around the nose of the high-speed vehicle. The very high wave drag associated with the shock wave formation imposes some restrictive requirements on vehicle shape, which limit the flexibility of the vehicle design. In addition, the bow shock formed around the nose of the supersonic / hypersonic vehicle has the side effect of producing a sonic boom on the ground, and the faster the vehicle is flying, the bigger the sonic boom will be, thus raising various environmental problems. Therefore, researchers have always been looking for the ways to mitigate and eliminate the attached bow shock waves.[1−10] Plasmas have been suggested possibly to offer a certain effect on shock waves. A pioneering experiment by Gordeev *et al*[1] was carried out by injecting discharge plasma into the flow through a nozzle of a cone-cylinder model placed in a supersonic stream, drawing a conclusion of significant drag reduction. Subsequently, Ganguly *et al*[2] and Baryshnikov *et al*[3] have also carried out similar experiments, separately. Riggins *et al*[4] have devoted numerical simulations to the study of plasma effect on shock waves in a supersonic flow, separately. Although these aforementioned researchers have studied the same subject (plasma effect on shock waves in a supersonic flow), their respective results are not identical. For instance, the nonthermal effect on shock waves was observed in Ref.[3], whereas the thermal effects were observed in the numerical simulations by Riggins *et al.*[4] Then, it is necessary to further study plasma effects on shock waves. Kuo *et al*[5] have carried out an experimental

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study on the influence of a plasma on the structure of an attached conical shock front appearing at the front end of a missile-shaped model in a Mach-2.5 flow. In the experiment the tip and the body of the model are designed as the cathode and anode respectively for on-board discharge (which produces a spray-like plasma moving around the tip); such an arrangement permits electron current, produced by the discharge, to pass through the shock front into the lower pressure region, where the discharge occurs, before returning to the body of the model as the anode. It is observed that the plasma has caused the shock front to separate from the model, and then the shock wave moves upstream in the form of a detached bow shock a sensible distance away from the model tip. The detached shock front appears to be highly dispersed in its new location. The significant result of the experiment is that as the discharge current increases (it means that the discharge-induced magnetic field increases), experimental evidence shown in the video further reveals a distinct state of the flow without the presence of any shock wave, namely shock wave elimination. In other words, the experimental result indirectly indicates that the discharge-induced magnetic field (or pressure) is a key factor leading to shock wave elimination. By the way, Hu and Hu's numerical result also shows that the effect of magnetic field weakens the plane shock wave in a non-magnetized (i.e., without applied magnetic field) plasma.[11] In Ref.[6], Kuo and Bivolaru presented additional experimental evidence obtained from experiments, conducted prior to those presented in Ref.[5]. The experimental evidence supports the nonthermal plasma effect on shock waves in a supersonic flow. More recently, Bivolaru and Kuo[7] have carried out another experimental study on the effect of plasma on shock waves in a supersonic flow. In this experiment, a wind-tunnel model in the shape of a 30° half-angle truncated-cone is designed to generate a strong bow shock behind a weak conical (oblique) shock wave attached to the tip of a protruding central-electrode, in a non-ionized supersonic flow. Plasma is generated between two shocks by an on-board discharge. The results show that the plasma spike has drastically modified the original complicated shock structure to a simple structure having only a single conical shock attached to the tip of the model, namely shock wave mitigation. In summary, the experimental observation presented in Ref.[5] demonstrates that plasma can eliminate simple bow shock and the experimental observation in Ref.[7] demonstrates that plasma can mitigate complicated bow shock. The exact cause of drag reduction mechanism of plasma is, however, not clear. Therefore we are interested in how to interpret theoretically the two experimental observations. This is the motive to make the present investigation.

On the other hand, for plasma radar cloaking applications[12−15] it has been tried to enhance the density of on-board discharge plasma. However this trial may bring about an undesired consequence of considerable reduction in the ratio of engine thrust to weight of vehicle because the discharge device for generating the higher-density plasmas increases the weight of the vehicle; but, as mentioned above, the discharge also can bring about a considerable reduction of external wave drag associated with the attached bow shock wave formed around the nose of the supersonic / hypersonic vehicle, namely considerable improvement in the ratio of engine thrust to external drag. In other words, it is possible that the improvement in the ratio of engine thrust to external drag makes up for the reduction in the ratio of engine thrust to vehicle weight. Therefore it is desirable to develop an on-board discharge technology that may be used to accomplish the two goals: attached bow shock wave drag reduction and plasma radar cloaking. The present work will make a theoretical analysis of it.

2. Physical model and basic assumptions

As is known, the attached bow shock layer formed around the nose of the supersonic / hypersonic vehicle is very thin, for example, at pressure of 1.8×10^4 Pa (the experimental value in Ref.[5]) the shock front thickness is approximately 10^{-2} cm. Thus, although wholly the thin shock is bowed, locally we can consider each section of it a thin plane shock. In other words, the thin bow shock can be imagined as a series of piecewise plane shocks with the same thin thickness. So we can take any one of them as a representative to analyse our problem, as shown in Fig.1. As stated above, in addition to this reason, the discharge-induced magnetic pressure plays a leading role in the attached bow shock mitigation and elimination. And the scalar character of magnetic pressure allows us to take any one of a series of piecewise plane shocks with the same thin thickness to analyse our problem.

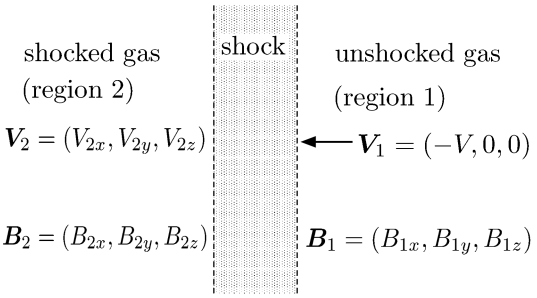


Fig.1. Plane shock (one of a series of piecewise plane shocks with the same thin thickness) in the reference frame moving with shock velocity. The unshocked (partially ionized) gas (region 1) flows into the front of the shock and the shocked (partially ionized) gas (region 2) flows out from the back of the shock.

In the present problem we can make the following assumptions: (a) We focus on the study of the relation between the shock strength and the discharge-induced magnetic pressure. Therefore, for simplicity we might as well assume both the environmental gas around the nose of vehicle and the discharge-produced plasma (we give them a general designation 'partially ionized gas') to be perfect. (b) As stated above, the shock front thickness is very thin. So we might as well assume there is not enough time for the non-adiabatic processes (such as heat conduction) to occur inside the shock layer. (c) Because of the mitigation of the discharge-induced magnetic pressure, the temperature T_2 behind the shock (region 2 in Fig.1) cannot become high enough to cause the ionization of the environmental gas, besides the discharge-caused ionization. So taking the thinness of the shock layer into account again we can assume both the degree of ionization $i_{\rm d}$ and the ion-species in regions 1 (before the shock front) and 2 to be the same during the discharge, namely $i_{\rm d2}/i_{\rm d1} \cong 1$ and $\rho_2/\rho_1 \cong n_2/n_1$, where ρ is the mass-density and n the number-density. (d) For simplicity, we restrict our analysis to plane shocks moving in a direction normal to the plane of shock. Let this be the x direction; then all the variables involved in the present problem are assumed to be functions only of x both inside and outside the plane shock, and furthermore only the temperature outside the plane shock (in regions 1 and 2 in Fig.1) is assumed to be constant. This relaxes the requirements of the usual treatment of plane shock waves in plasmas (see, e.g., Ref.[16]) where all the variables are assumed to be constant outside the plane shock. It is convenient to use a reference frame moving with the shock; if in the laboratory frame the unshocked gas (region 1) is at rest, then in the moving frame (Fig.1) it enters the front of the shock with a velocity $\boldsymbol{V}_1 = (-V, 0, 0)$, where V is the speed of the shock. Then the steady-state condition is valid for the present problem. That is, all the variables involved in the present problem are independent of time. Assumptions (a)–(d) greatly facilitate theoretical treatment of the present problem, as stated in the following sections of this paper, but these assumptions have no influence on the main physical pictures.

For the kind of problem under consideration, we are interested most in the values of variables at the interfaces (the interface between region 1 and the plane shock and / or the interface between the plane shock and region 2). So we study the relation between the shock strength (P_2/P_1, where P is the pressure of the partially ionized gas during the discharge) and the discharge-induced magnetic pressure ($B_1^2/8\pi$). Moreover, as shown at the end of Section 3 of the present paper the relation is independent of the case with these two quantities present inside the plane shock. That is, the present problem does not involve the discussion of the physical phenomena inside the plane shock.

In summary, firstly we propose a physical model to reduce the problem of the drag reduction of an attached bow shock wave around the nose of a high-speed vehicle with on-board discharge, to the one of balance between the magnetic pressure and gas pressure of the plane shock wave of a partially ionized gas consisting of the environmental gas around the nose of the vehicle and the on-board discharge-produced plasma. Then we reform a set of the steady-state one-fluid, hydromagnetic equations[16] applicable to the plane shock wave in a non-magnetized (i.e., without applied magnetic field) plasma for using it to describe the present problem.

3. Governing equations and analytical development

Under assumptions (a)–(d) in Section 2, the reformed set of steady-state one-fluid, hydromagnetic equations (all notations are conventional, unless otherwise specified) can be written as follows.

The mass conservation equation,

$$\frac{{\rm d}(\rho V_x)}{{\rm d}x} = 0. \tag{1}$$

The momentum conservation equation,

$$\frac{\mathrm{d}\Pi_{xx}}{\mathrm{d}x}=0,\quad \frac{\mathrm{d}\Pi_{xy}}{\mathrm{d}x}=0,\quad \frac{\mathrm{d}\Pi_{xz}}{\mathrm{d}x}=0, \tag{2}$$

where

$\Pi_{ij}=\rho V_iV_j+P_{ij}-T_{ij}$ is the total momentum flux, $\boldsymbol{T}_{ij}=\dfrac{1}{4\pi}(B_iB_j+E_iE_j)-\dfrac{\delta_{ij}}{8\pi}(B^2+E^2)$ is the electromagnetic stress tensor.

The energy conservation equation,

$$\frac{\mathrm{d}S_x}{\mathrm{d}x}=0, \tag{3}$$

where

$$\boldsymbol{S}=\frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})+\boldsymbol{Q}+\boldsymbol{P}\cdot\boldsymbol{V}+\left(\frac{3}{2}N\kappa T+\frac{1}{2}\rho V^2\right)\boldsymbol{V}$$

is total energy flux, $\boldsymbol{Q}$ is the heat flux vector and $\boldsymbol{Q}=0$ in regions 1 and 2 in Fig.1, because the temperature is assumed to be constant in regions 1 and 2.

Ampere's law,

$$\boldsymbol{j}=\frac{c}{4\pi}\left(0,-\frac{\mathrm{d}B_z}{\mathrm{d}x},\frac{\mathrm{d}B_y}{\mathrm{d}x}\right). \tag{4}$$

Ohm's law in regions 1 and 2,

$$\boldsymbol{E}_1+\frac{\boldsymbol{V}_1\times\boldsymbol{B}_1}{c}=\frac{\boldsymbol{j}_1}{\sigma_1},\quad \boldsymbol{E}_2+\frac{\boldsymbol{V}_2\times\boldsymbol{B}_2}{c}=\frac{\boldsymbol{j}_2}{\sigma_2}. \tag{5}$$

In what follows, $\boldsymbol{B}_1$ is called the initial magnetic field. Here we emphasize that in the present paper the magnetic field (e.g., $\boldsymbol{B}_1$) consists of the electron current (produced by the on-board discharge)-induced magnetic field and the partially ionized gas (due to the discharge) flow-induced magnetic field since ionized gas flow energy ($\propto V_1^2$) can be converted into magnetic energy as well as heat.[16] As observed in experiment in Ref.[5], the electron current produced by the discharge passes through the shock front into the lower pressure region, where the discharge occurs, before returning to the body of the model serving as an anode. Because both the electron current-induced magnetic field and the ionized gas flow-induced magnetic field are induced by the discharge, we give these two magnetic fields a general designation 'the discharge-induced magnetic field'. But for convenience of following discussions we call the former one an applied magnetic field, because it is irrelevant to the ionized gas flow

Absence of magnetic monopoles,

$$\nabla\cdot\boldsymbol{B}=0, \tag{6}$$

Faraday's law,

$$\nabla\times\boldsymbol{E}=0. \tag{7}$$

At first, from Eqs.(6) and (7) we obtain

$$\frac{\mathrm{d}B_x}{\mathrm{d}x}=0,\quad \frac{\mathrm{d}E_y}{\mathrm{d}x}=0,\quad \frac{\mathrm{d}E_z}{\mathrm{d}x}=0. \tag{8}$$

Integrating Eq.(8) and using Eqs.(4) and (5), we have

$$[B_x]_1^2=0, \tag{9}$$

$$[V_xB_y-V_yB_x]_1^2=0, \tag{10}$$

$$[V_xB_z-V_zB_x]_1^2=0, \tag{11}$$

where $[F]_1^2\equiv(F_2-F_1)$. From now on, let us bear in mind that F_1 represents the value of variable F at the interface between region 1 and the shock and F_2 the value at the interface between the shock and region 2, unless otherwise specified. This is different from the treatment in Ref.[16] on shock waves in plasmas where F_1 represents the value of F in region 1 and F_2 the value in region 2, since all the variables in Ref.[16] are assumed to be constant in regions 1 and 2 shown in Fig.1. Similarly, integrating Eqs.(1)–(3) and using the approximations $P_{ij}=P\delta_{ij},\boldsymbol{Q}=0,E\ll B$ (which apply in regions 1 and 2), we find

$$[\rho V_x]_1^2=0, \tag{12}$$

$$[\rho V_x^2+P+(B_y^2+B_z^2)/8\pi]_1^2=0, \tag{13}$$

$$[\rho V_xV_y-B_xB_y/4\pi]_1^2=0, \tag{14}$$

$$[\rho V_xV_z-B_xB_z/4\pi]_1^2=0, \tag{15}$$

$$\Big[V_x\rho I+PV_x+\frac{1}{2}\rho V_xV^2+V_x(B_y^2+B_z^2)/4\pi -B_x(B_yV_y+B_zV_z)/4\pi\Big]_1^2=0. \tag{16}$$

Here we have replaced $(3/2)N\kappa T$ by ρI in $\boldsymbol{S}$, where

$$I=P/(\gamma-1)\rho \tag{17}$$

with the ratio of specific heats γ, is the internal energy.[16]

We see that the validity of relations (9)–(16) is independent of how the variables (except the temperature) vary in regions 1 and 2 in Fig.1, greatly relaxing the limits in which the relations (9)–(16) may apply. This is why in assumption (d) in Section 2 of the present paper we do not assume, except the temperature, that all the variables are constant in regions 1 and 2 (like the treatment in Ref.[16]). Therefore, these relations can be called the jump conditions across the shock. Before giving the solutions of the shock equations (9)–(16), we derive the Hugoniot relation for the partially ionized gas. Although the derivation process is considerably lengthy, the main procedures are more

or less the same as in Ref.[16]. Here we only give the final result

$$[I]_1^2+\frac{1}{2}(P_2+P_1)\left[\frac{1}{\rho}\right]_1^2+\frac{1}{16\pi}\{[\boldsymbol{B}]_1^2\}^2\left[\frac{1}{\rho}\right]_1^2=0. \tag{18}$$

Equation (18) gives the relation among the kinetic pressure, mass density and magnetic field on either side of the shock and may be referred to as the Hugoniot relation for the partially ionized gas.

4. Solutions of shock equations

We now look for particular solutions of the shock equations (9)–(16) for the following two cases, namely the initial magnetic field $\boldsymbol{B}_1$ parallel and perpendicular to the shock propagation. For the general case in which $\boldsymbol{B}_1$ is neither parallel nor perpendicular to the shock propagation, we recall that the initial magnetic field $\boldsymbol{B}_1$ can always be divided into a tangential and a normal components. So the two cases are most representative and hence worth considering first.

4.1. The initial magnetic field parallel to the shock propagation

Letting the normal direction be in the shock propagation direction, namely in the x direction, the initial magnetic field $\boldsymbol{B}_1$ may be written as

$$\boldsymbol{B}_1=(B_1,0,0). \tag{19}$$

In general, the magnetic field after the shock has passed may have both a normal and a tangential components; so, choosing the y-axis to coincide with its tangential component, we have

$$\boldsymbol{B}_2=(B_x,B_y,0). \tag{20}$$

Similarly, $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ may be written as

$$\boldsymbol{V}_1=(V_1,0,0)=(-V,0,0) \tag{21}$$

and

$$\boldsymbol{V}_2=(V_x,V_y,V_z). \tag{22}$$

From Eqs.(19)–(22) and Eqs.(11)–(17), we have

$$\rho_2V_2=\rho_1V_1, \tag{23}$$

$$\rho_2V_2^2+P_2=\rho_1V_1^2+P_1, \tag{24}$$

$$\frac{\gamma P_2}{(\gamma-1)\rho_2}+\frac{1}{2}V_2^2=\frac{\gamma P_1}{(\gamma-1)\rho_1}+\frac{1}{2}V_1^2. \tag{25}$$

We see that in this particular solution, the magnetic field does not play a part. Therefore, for our purpose we should look for the particular solution of the shock equations (9)–(16) in the other case.

4.2. The initial magnetic field perpendicular to the shock propagation

In this case, the y-axis may be chosen such that

$$\boldsymbol{B}_1=(0,B_1,0), \tag{26}$$

so using Eq.(9), $\boldsymbol{B}_2$ may be written as

$$\boldsymbol{B}_2=(0,B_y,B_z). \tag{27}$$

In terms of Eqs.(21) and (22), Eq.(12) gives

$$V_x=V_1/r, \tag{28}$$

where $r\equiv\rho_2/\rho_1$. Then, substituting Eqs.(26)–(28) into Eqs.(10), (14), and (15) yields $B_z=0, V_y=0$, and $V_z=0$, thus from Eqs.(11) and (28), Eqs.(22) and (27) become

$$\boldsymbol{V}_2=(V_1/r,0,0), \tag{29}$$

$$\boldsymbol{B}_2=(0,B_y,0)=(0,rB_1,0). \tag{30}$$

Equation (30) shows that the magnetic field is constant in its direction and increased in magnitude by the same ratio as the density because $r\equiv\rho_2/\rho_1$. Finally, Eqs.(13) and (16) become

$$\rho_2V_2^2+P_2+B_2^2/8\pi=\rho_1V_1^2+P_1+B_1^2/8\pi$$

and

$$\frac{\gamma P_2}{(\gamma-1)\rho_2}+\frac{V_2^2}{2}+\frac{B_2^2}{4\pi\rho_2}=\frac{\gamma P_1}{(\gamma-1)\rho_1}+\frac{V_1^2}{2}+\frac{B_1^2}{4\pi\rho_1}.$$

Defining the shock strength $R, R\equiv P_2/P_1$, and the Mach number $M, M\equiv V_1/c_{\rm s1}$, where $c_{\rm s1}=\sqrt{\gamma P_1/\rho_1}$, these two equations may be rewritten as

$$\gamma M^2(1-1/r)=(R-1)+\beta(r^2-1) \tag{31}$$

and

$$\gamma M^2\left(1-\frac{1}{r^2}\right)=\frac{2\gamma}{(\gamma-1)}\left(\frac{R}{r}-1\right)+4\beta(r-1), \tag{32}$$

where $\beta\equiv B_1^2/8\pi P_1$.

5. Results and discussion

Since $B_1^2/8\pi$ is the magnetic pressure, β is the ratio of magnetic to gas pressure ahead of the shock, from Eq.(31) we obtain

$$R = 1 + \gamma M^2 \left(1 - \frac{1}{r}\right) - \beta(r^2 - 1). \tag{33}$$

Equation (33) is, indeed, a formula desired for the relation between the bow shock strength and the on-board discharge-induced magnetic pressure (included in β), because from Eq.(33) we can see that the second term ($\propto M^2 \propto$ the kinetic energy of vehicle) on its right-hand side enhances the bow shock strength R and the third term ($\propto \beta$) mitigates the bow shock strength R till the bow shock is eliminated with the increase of the discharge-induced magnetic pressure. In other words, physically Eq.(33) is reasonable though it is derived under the four approximate assumptions (a)–(d) in Section 2. Therefore, Eq.(33) might be used to explain the two recent experimental observations on the attached bow shock mitigation and elimination in a supersonic flow during on-board discharge, carried out by Bivolaru and Kuo[7] and Kuo *et al*[5] separately.

Because the value of the on-board discharge-induced magnetic pressure (included in β) can be changed artificially, we can make the value of the third term ($\propto \beta$) on the right-hand side of Eq.(33) large enough to cancel the value of the second term, leading to $R = 1 \rightarrow p_2 = p_1$, namely the shock front disappears. In this case Eq.(33) becomes the third-order algebraic equation of r

$$(r - 1)(r^2 + r - \gamma M^2/\beta) = 0. \tag{34}$$

It has a positive root

$$r_{\rm p} = (1/2)[(1 + 4\gamma M^2/\beta)^{1/2} - 1], \tag{35}$$

a negative root without physical meaning, and a trivial root $r_{\rm t} = 1$. It is easy to prove $r_{\rm t} = 1$ to be a trivial root. In fact, when $r = 1$, namely $\rho_2 = \rho_1$, it follows from Eq.(18) that $I_2 = I_1$, in spite of the presence or absence of the magnetic energy, and hence [recall Eq.(17)] $P_2 = P_1$. This is the limiting case of no shock. Therefore the positive root, Eq.(35), alone is a real solution of the shock equations in the case of the initial magnetic field perpendicular to the shock propagation. Similarly, Eq.(32) with $R = 1$ has a positive root

$$r_{\rm p} = \frac{1}{2}\left\{\sqrt{\left[\frac{\gamma M^2}{4\beta} + \frac{2\gamma}{4\beta(\gamma-1)}\right]^2 + \frac{\gamma M^2}{\beta}} + \left[\frac{\gamma M^2}{4\beta} + \frac{2\gamma}{4\beta(\gamma-1)}\right]\right\}, \tag{36}$$

a negative root without physical meaning, and a trivial root $r_{\rm t} = 1$. From now on, the subscript of $r_{\rm p}$ can be dropped. It should be noted that in the absence of the applied magnetic field induced by the electron current (produced by the on-board discharge) only two parameters M and P_1 (included in β) in Eq.(35) or (36) are independent and the other two parameters r and B_1 (included in β) must satisfy Eq.(35) or (36). In the meantime r is limited by the following inequalities[16]

$$\frac{\gamma - 1}{\gamma + 1} < r < \frac{\gamma + 1}{\gamma - 1}, \tag{37}$$

namely the upper limit of r is 4, if $\gamma = 5/3$ (as usual). It is because in the absence of the applied magnetic field Eqs.(1)–(37) are self-consistent each and every. However, in the present case the applied magnetic field which is induced by the electron current (produced by the on-board discharge) breaks the self-consistency, namely the electron current-induced magnetic field is the third independent parameter in the system. Therefore, in the present case there is a new relation $r = B_2/B_1$ [see Eq.(30)] besides Eq.(37), where both B_2 and B_1 include the applied magnetic field as well as the partially ionized gas flow-induced magnetic field. Moreover, the applied magnetic field in B_2 is much more stronger than that in B_1, because the electron current produced by the on-board discharge, as observed in the experiment in Ref.[5], passes through the shock front into the lower pressure region, where the discharge occurs, before returning to the body of the model serving as the anode. Then, in the present case the upper limit of r should be able to become much higher than 4. Now, on the assumption that in Eq.(35) $r = 10$ (the subscript of $r_{\rm p}$ has been omitted), $M = 2.5$ (the experimental value in Ref.[5]), and $\gamma = 5/3$ (as usual), from Eq.(35) and the definition of β we have $B_1^2/8\pi \cong 0.1P_1$. The significance of this result is that the higher the altitude at which the vehicle is flying (i.e., the lower the P_1), the lower the discharge-induced magnetic pressure necessary for eliminating the attached shock. If P_1 is 1.8×10^4 Pa (the experimental value in Ref.[5]), $B_1 \cong 0.067$ T. In the same way, from Eq.(36) we have $B_1 \cong 0.134$ T. We might as well take the major one between 0.067 T and 0.134 T. This value (0.134 T) of B_1, even if it is

induced completely by the electron current (produced by the on-board discharge), is available by means of existing on-board discharge technology.

From the approximate assumption (c) in Section 2, we have

$$r \equiv \rho_2/\rho_1 \cong n_2/n_1 \cong n_2 i_{\mathrm{d}2}/n_1 i_{\mathrm{d}1} \cong n_{\mathrm{e}2}/n_{\mathrm{e}1},$$

where n_{e} is electron density, hence $r = 10$ means that $n_{\mathrm{e}2} \cong 10 n_{\mathrm{e}1}$. That is, if P_1 is 1.8×10^4 Pa, $B_1 \cong 0.134$ T will be able to eliminate the attached bow shock, and the shock elimination leaves room for a layer of higher-density (10-fold the initial density) plasma rampart moving around the nose of a vehicle, being favourable to the plasma radar cloaking of the vehicle. The reason for it may be as follows: When the attached bow shock strength is cancelled by the on-board discharge-induced magnetic pressure, the energy ($\propto$ the kinetic energy of vehicle) contained originally in the shock layer is consumed for changing the distribution of the density $n_{\mathrm{e}2}$ of electrons tightly behind the shock front, thus greatly raising the $n_{\mathrm{e}2}$-value (10-fold the initial density). In other words, although the attached shock is eliminated by the on-board discharge-induced magnetic pressure, the kinetic energy of the vehicle is not wasted much—a part of it is spent on raising the $n_{\mathrm{e}2}$-value, being favourable to the plasma radar cloaking of the vehicle.

6. Concluding remarks

In the present paper, we have firstly proposed a physical model to reduce the problem of the drag reduction of an attached bow shock around the nose of a high-speed vehicle with on-board discharge, to the problem of a balance between the on-board discharge-induced magnetic pressure and gas pressure of plane shock of a partially ionized gas consisting of the environmental gas around the nose of the vehicle and the on-board discharge-produced plasma. Then we have reformed a set of the steady-state one-fluid, hydromagnetic equations applicable to the plane shock wave in a non-magnetized (i.e., without applied magnetic field) plasma to describe the present problem. Under the approximate assumptions (a)–(d) in Section 2, but without assuming all the variables, except the temperature, to be constant outside the shock, we have derived a formula for the relation between the bow shock strength and the on-board discharge-induced magnetic pressure. The formula allows us to explain successfully the two recent experimental observations on bow shock mitigation and elimination in a supersonic flow during on-board discharge, carried out by Bivolaru and Kuo[7] and Kuo *et al*[5] separately. This shows, as indirectly indicated by the experimental result in Ref.[5], that the on-board discharge-induced magnetic pressure is a key factor leading to shock wave elimination. In addition, the formula implies that the attached bow shock elimination leaves room for a layer of higher-density plasma rampart moving around the nose of the vehicle, being favourable to the plasma radar cloaking of the vehicle. The reason for it has been expounded.

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