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Frequency bands of negative refraction in finite one-dimensional photonic crystals*

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We have discussed theoretically the negative refraction in finite one-dimensional (1D) photonic crystals (PCs) composed of alternative layers with high index contrast. The frequency bands of negative refraction are obtained with the help of the photonic band structure, the group velocity and the power transmittance, which are all obtained in analytical expression. There shows negative transverse position shift at the endface when negative refraction occurs, which is analysed in detail.

Keywords: photonic crystal, negative refraction, group velocity

PACC: 4270Q, 4225B, 7820

1. Introduction

Negative refraction has been the subject of considerable interest in recent years, which may provide the possibility of a variety of novel applications.[1−7] There have been many studies about negative refraction phenomena in meta-materials that have simultaneous negative permittivity ε and permeability μ, i.e. have a negative index of refraction n.[8−13] And this fantastic phenomenon may also occur in ordinary materials (positive-n), which are called photonic crystals (PCs).[14−16]

PCs are artificial structures, which have periodic dielectric structures with high index contrast, the resultant photonic dispersion exhibits a band nature analogous to the electronic band structure in a solid, and the propagation of electromagnetic waves are forbidden in the photonic bandgap. There also shows an extraordinary strong nonlinear dispersion at wavelengths close to the bandgap, and PCs can refract abnormally light at these wavelengths under certain conditions as if they had a negative refractive index, which is referred to as negative refraction effect.[17−23]

In PCs, light travels as Bloch waves, which travel through crystals with a definite propagation direction despite the presence of scattering. In the long-wavelength regime, the average direction of energy propagation can be shown to be the same as the direction of group velocity.[24] Most of the theoretical studies of negative refractive in PCs make use of the equifrequency surfaces, which are contour diagrams in the wavevector space for a constant frequency and get numerical results by computer simulation.[17−23] By contrast, we will give rigorous analytical expressions to describe negative refraction and their properties.

Boedecker and Henkel[16] mentioned that the simple one-dimensional Kronig–Penney model provided an exactly soluble example of a photonic crystal with negative refraction, but the discussion was not expanded. Therefore, in this paper, we investigate the negative refractive behaviours in finite 1D PCs by the Bloch theory and transfer matrix treatment. With the help of the group velocity and the power transmittance, we get frequency bands of negative refraction. When negative refraction occurs in PCs, the transmission wave will shift adversely from the incidence point

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in x axis at the endface, as will be discussed in detail presently.

2. Analytical dispersion relation and group velocity

Let us consider a 10-period structure composed of alternating layers of two materials with different relative permittivities ε_1, ε_2 and permeabilities μ_1, μ_2 as shown in Fig.1. d_1 and d_2 are the widths of the two layers respectively, and $a = d_1 + d_2$ is the lattice constant.

Suppose a quasimonochromatic electromagnetic wave in the long-wavelength limit in space (ε_0, μ_0) falls obliquely on the interface of a 1D PC with an incidence angle θ_0, here we concern ourselves with the TE waves, and the treatment of the TM waves is similar. A point source may also be considered, and a Gaussian wave packet can be expressed as a sum of many plane waves with different frequencies by fast Fourier transform (FFT), such that every wave could be dealt with in the same way. Without loss of generality, we assume that the wave vector has components only in the x and z directions.

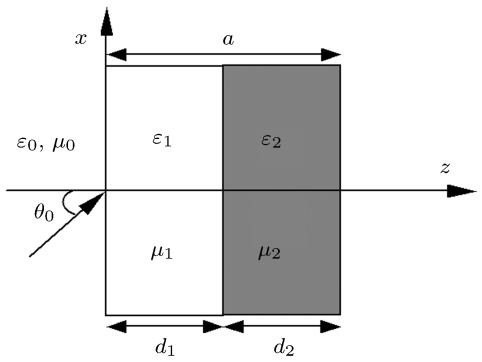


Fig.1. One period of 1D structure consisting of alternate layers.

According to the Block wave theory, the dispersion relation for this periodic dielectric layers is given by[25]

$$\cos(Ka) = \cos(k_{1z}d_1)\cos(k_{2z}d_2) - \gamma\sin(k_{1z}d_1)\sin(k_{2z}d_2), \tag{1}$$

where $\gamma = \frac{1}{2}\left(\frac{k_{1z}}{k_{2z}} + \frac{k_{2z}}{k_{1z}}\right)$, K is the amplitude of the Block wave vector in the z direction of the periodicity, $k_{iz} = [(n_i\omega/c)^2 - \beta^2]^{1/2} = n_i\omega/c\cos\theta_i$, $n_i = \sqrt{\varepsilon_i\mu_i}$, $\theta_i = \sin^{-1}(n_0\sin\theta_0/n_i)(i = 1, 2)$ are the refractive angles in layer 1 and 2, respectively, and $\beta = \omega n_0\sin\theta_0/c$ is the propagation constant parallel to layers in the x axis.

Employing the transformations $t_1 = d_1/a$, $t_2 = d_2/a$ and normalizing the frequency $\varpi = \omega a/c$, we obtain the straightforward expression for the dispersion relation:

$$Ka = \arccos[\cos(n_1t_1\cos\theta_1\varpi)\cos(n_2t_2\cos\theta_2\varpi) - \gamma\sin(n_1t_1\cos\theta_1\varpi)\sin(n_2t_2\cos\theta_2\varpi)], \tag{2}$$

where $\gamma = \frac{1}{2}\left(\frac{n_1\cos\theta_1}{n_2\cos\theta_2} + \frac{n_2\cos\theta_2}{n_1\cos\theta_1}\right)$.

From Eq.(2), we get the corresponding band structure of this PC for each fixed θ_0. Figure 2 depicts the band structure when $n_0 = n_1 = 1$, $n_2 = 4.2$, $t_1 = 0.85$, $t_2 = 0.15$ for the oblique incidence case of $\theta_0 = \pi/4$. The bandwidth of odd number bandgap is wide, but the bandwidth of even number bandgap is very narrow. It is noticeable here that the lattice constant a is arbitrary; thus, the result obtained here is valid for arbitrary wavelengths and the existence of bandgap is possible as long as $a \cong \lambda$.

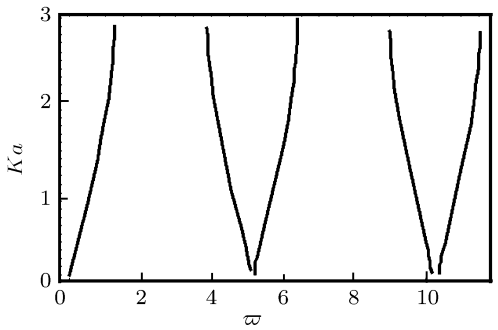


Fig.2. The photonic band structure for the oblique incidence ($\theta_0 = \pi/4$) with $n_0 = n_1 = 1$, $n_2 = 4.2$, $t_1 = 0.85$, $t_2 = 0.15$ and $\varpi = \omega a/c$.

As mentioned above, the group velocity governs the energy flow of a light beam, and the Bloch wave vector in PCs is given by $\boldsymbol{k} = \beta\hat{x} + K\hat{z}$. So the group velocity in PCs is expressed as[26]

$$\boldsymbol{v}_{\mathrm{g}} = \nabla_{\boldsymbol{k}}\omega(\boldsymbol{k}) = v_{gx}\hat{x} + v_{gz}\hat{z}, \tag{3}$$

$$v_{\mathrm{g}x} = \frac{\partial\omega}{\partial\beta} = -\frac{\partial K/\partial\beta}{\partial K/\partial\omega}, \tag{3a}$$

$$v_{\mathrm{g}z} = \frac{\partial\omega}{\partial K} = \frac{1}{\partial K/\partial\omega}, \tag{3b}$$

where $v_{\mathrm{g}x}$ and $v_{\mathrm{g}z}$ are the components of the group velocity in the x and z axis, respectively. When $v_{\mathrm{g}x} > 0$, the wave propagates along the positive direction of x axis, and the beam refracts to the other side of the normal according to the classical Snell law; $v_{\mathrm{g}x} = 0$ is associated with the normal propagation; but $v_{\mathrm{g}x} < 0$ means the transmitting beam will bend to the 'wrong' side, that is the same side as the incident beam, which means the negative refraction if the beam passes through the PC.

Substituting Eq.(1) into Eq.(3a), the expression of $v_{\mathrm{g}x}$ is calculated as:

$$\begin{aligned} v_{\mathrm{g}x} = & \left\{\beta\left[\sin\phi_1\cos\phi_2\left(\frac{d_1}{k_{1z}}+\frac{d_2}{k_{2z}}\gamma\right)+\sin\phi_2\cos\phi_1\left(\frac{d_2}{k_{2z}}+\frac{d_1}{k_{1z}}\gamma\right)-2\sin\phi_1\sin\phi_2\frac{\gamma^2-1}{k_{1z}k_{2z}}\right]\right\}\Big/ \\ & \left\{\omega/c^2\left[\sin\phi_1\cos\phi_2\left(\frac{d_1}{k_{1z}}n_1^2+\frac{d_2}{k_{2z}}n_2^2\gamma\right)+\sin\phi_2\cos\phi_1\left(\frac{d_2}{k_{2z}}n_2^2+\frac{d_1}{k_{1z}}n_1^2\gamma\right)\right.\right. \\ & \left.\left.+(1/2)\sin\phi_1\sin\phi_2\frac{(k_{1z}^2-k_{2z}^2)(n_1^2k_{2z}^2-n_2^2k_{1z}^2)}{k_{1z}^3k_{2z}^3}\right]\right\}, \end{aligned} \tag{4}$$

where $\phi_1 = d_1k_{1z}$ and $\phi_2 = d_2k_{2z}$.

In Fig.3 the x component of group velocity v_{gx} as a function of the normalized frequency ϖ is plotted. Comparing Figs.3 and 2, we can see that there is the strong group velocity dispersion at the bandgap edge. v_{gx} decreases from positive value to negative value with the increase of frequency, falls off sharply to the negative minimum at the edge of band and then jumps to the positive maximum, afterwards, $v_{\mathrm{g}x}$ decreases again, and cycles in this manner. Figures 4(a) and 4(b) give enlargements of the second and fourth bandgaps. We have seen that $v_{\mathrm{g}x} < 0$ indicates the energy flow may tend to the negative direction of x axis, so negative refraction phenomenon might occur at some frequency in the bandgap. From Eq.(4), we also find $v_{\mathrm{g}x}$ always equals to zero when $\theta_0 = 0$, so the oblique incidence of wave is the necessary condition for negative refraction. In bandgaps, the Bloch wave is evanescent, so it will decay exponentially to zero in infinite periodic medium, therefore, only the finite sized PCs are considered in the following discussion.

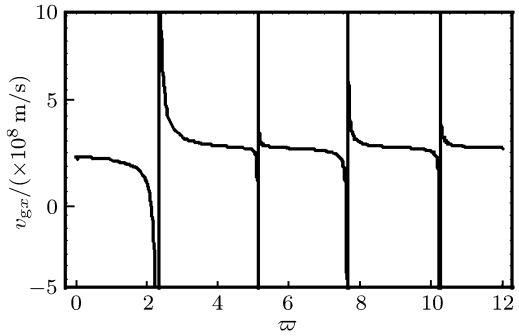


Fig.3. The x component of group velocity v_{gx} vs ϖ with $\theta_0 = \pi/4$, $n_0 = n_1 = 1$, $n_2 = 4.2$, $t_1 = 0.85$, $t_2 = 0.15$.

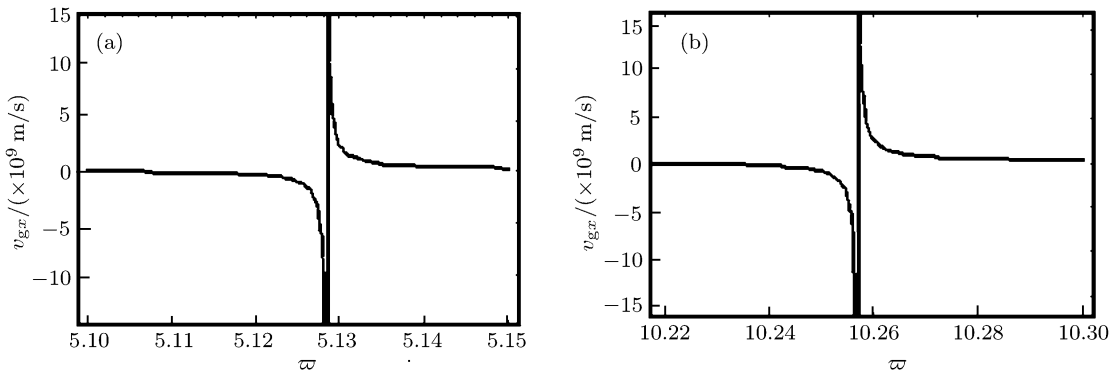


Fig.4. The enlargements of the curves near the second bandgap (a) and the fourth bandgap (b).

3. Discussion on negative refraction

By applying the transfer matrix theory,[27] we obtain the matrix equation that describes the beam passing through the PC structure shown in Fig.1:

$$\begin{bmatrix} E_1 + E_2 \\ \eta_0(E_1 - E_2) \end{bmatrix} = M \begin{bmatrix} E_3 \\ \eta_0 E_3 \end{bmatrix}, \tag{5}$$

where E_1, E_2 and E_3 are the complex amplitudes of incidence, reflection and transmission electric field, respectively; and $\eta_0 = n_0 \cos\theta_0$ is the effective index of refraction, M is the translation matrix, and

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}^N, \tag{6}$$

where

$$m_{11} = -\eta_2/\eta_1 \sin\alpha_1 \sin\alpha_2 + \cos\alpha_1 \cos\alpha_2, \tag{6a}$$

$$m_{12} = \mathrm{i} \sin\alpha_1 \cos\alpha_2/\eta_1 + \mathrm{i} \cos\alpha_1 \sin\alpha_2/\eta_2, \tag{6b}$$

$$m_{21} = \mathrm{i} \sin\alpha_1 \cos\alpha_2 \eta_1 + \mathrm{i} \cos\alpha_1 \sin\alpha_2 \eta_2, \tag{6c}$$

$$m_{22} = -\eta_1/\eta_2 \sin\alpha_1 \sin\alpha_2 + \cos\alpha_1 \cos\alpha_2, \tag{6d}$$

where N denotes the period number, and $\alpha_i = k_0 n_i d_i \cos\theta_i$, $\eta_i = n_i \cos\theta_i (i = 1, 2)$.

The translation matrix M for the whole structure is given by using Cayley–Hamilton theorem,

$$M = \begin{bmatrix} m_{11}u_{s-1}(x) - u_{s-2}(x) & m_{12}u_{s-1}(x) \\ m_{21}u_{s-1}(x) & m_{22}u_{s-1}(x) - u_{s-2}(x) \end{bmatrix}, \tag{7}$$

where $u_s(x)$ is the type II Chebshev Polynomial, and $x = (m_{11} + m_{22})/2$.

Substituting Eq.(7) into Eq.(6), the analytical expression of the translation matrix can be derived. And then we can obtain the power coefficient

$$t = \frac{E_3}{E_1} = \frac{2\eta_0}{\eta_0 M[1,1] + \eta_0^2 M[1,2] + M[2,1] + \eta_0 M[2,2]} \tag{8}$$

and the power transmittance can be calculated by using the formula $T = tt^*$. Figure 5 depicts the transmittance T as a function of ϖ through the 10-period structure shown in Fig.1.

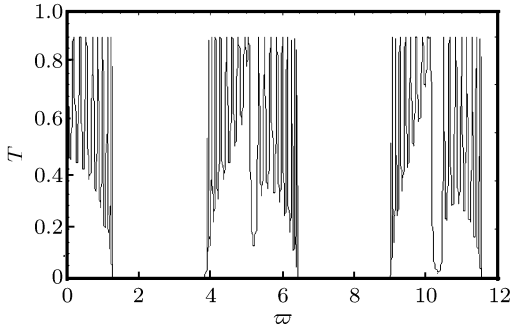


Fig.5. The power transmittance T as the function of ϖ with $\theta_0 = \pi/4$, $n_0 = n_1 = 1$, $n_2 = 4.2$, $t_1 = 0.85$, $t_2 = 0.15$.

From Figs.2 and 5, it is observed that the transmittance is zero in the first and third bandgap, that is, the waves at these frequencies are completely reflected and do not pass through the PC. But for finite layers of PC, the transmittance is not zero in the second and fourth bandgaps, which means part of wave can pass. Relating with Fig.3, we can see there exists the negative group velocity in the low frequency edge of the second bandgap, at the same time, the transmittance is not zero, so the transmission wave will bend to the negative direction of x axis, which is the negative refraction phenomenon. According to the parameters in Fig.5, the normalized frequency band for negative refraction lies between 5.1016 and 5.1283. The case in the fourth bandgap is similar, and the negative refraction band is in the range 10.2034 and 10.2569, but the transmission energy here is less than 5%. And that the frequencies of negative refraction band in the fourth bandgap is just integer times of i.e. twice that in second bandgap as the result of the band folding in photonic crystals.

Although some group velocity in the first and

third bandgaps is negative in Fig.2, there is no transmission of energy and the negative refraction cannot happen. In the following, we will give further discussion with the help of the phase velocity in z direction, and $v_{\varphi z} = \omega/K$. According to Eq.(2), we plot the phase velocity $v_{\varphi z}$ in Fig.6. The phase velocity in the second and fourth bandgaps is infinite, we think it is because the wave here is evanescent, the wave number is complex and only imaginary part exists, and in theory the evanescent wave decays to zero in infinite PC, so the phase of wave does not exist, and therefore the phase velocity is no meaning. But the photo tunnelling can still happen in finite PC. In the first and third bandgaps, the wave number is complex but the real part is not zero, so the phase velocity exists, and the phase of wave varies because of the reflection at interface boundary, the wave are all reflected and can not pass through PC. Therefore, we can conclude that negative refraction may occur in the frequency near the low frequency edge of the second and fourth bandgap because of the strong group velocity dispersion. And there also exists negative group velocity in the sixth bandgap, but the transmittance is close to zero, so negative refraction phenomenon is difficult to be observed there.

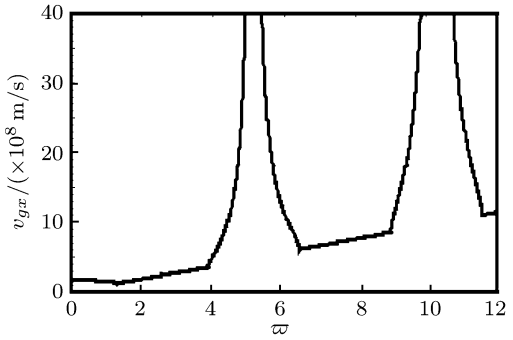


Fig.6. The z component of group velocity $v_{\varphi z}$ vs ϖ with $\theta_0 = \pi/4$, $n_0 = n_1 = 1$, $n_2 = 4.2$, $t_1 = 0.85$, $t_2 = 0.15$.

The negative refraction is a distinct abnormal spatial dispersion, how do we characterize it? The x component of the group velocity v_{gx} may cause the transverse position shift S_x along x axis after a beam passes through PC, and S_x is defined as[28]

$$S_x = v_{gx}\tau \tag{9}$$

where τ is the group delay time through the structure, and

$$\tau = \frac{\partial}{\partial\omega}\arctan(\mathrm{Im}t/\mathrm{Re}t). \tag{10}$$

Then we obtain the group delay time by substituting Eq.(9) into Eq.(10), as depicted in Fig.7.

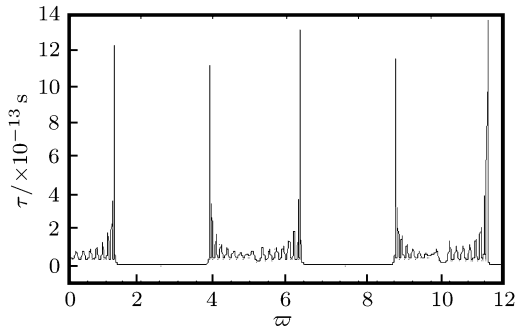


Fig.7. The group delay time τ through 10-period PC structure as the function of ϖ with $\theta_0 = \pi/4$, $n_0 = n_1 = 1$, $n_2 = 4.2$, $t_1 = 0.85$, $t_2 = 0.15$.

From Fig.7, we can see clearly that the group delay time in the first and third bandgap is zero, but in the second and fourth bandgaps it is obvious. Thus, the position shift S_x can be obtained by the definition of Eq.(10), and Fig.8 represents the plot of the position shift S_x versus frequency ϖ in the second bandgap. Because the magnitude of v_{gx} in most part of the bandgap is too small, so the shift at these frequency are not obvious. Only in an extremely narrow frequency band, the beam shows great shift. And in Fig.9, we show the logarithm of absolute value of S_x, with 10 as the base, in the frequency band of negative refraction, and we can find the shift for most frequency is nearly 10^{-5}, but the absolute value of S_x rises rapidly with the increase of ϖ at the high frequency edge of this band, and the magnitude order can even reach 10^{-1} in theory. When the shift reaches the negative minimum, it will jump to the positive maximum suddenly, that means, the negative refraction switches to the normal refraction, which is depicted in Fig.8. The discussion on negative refraction in the fourth bandgap can also be done likewise.

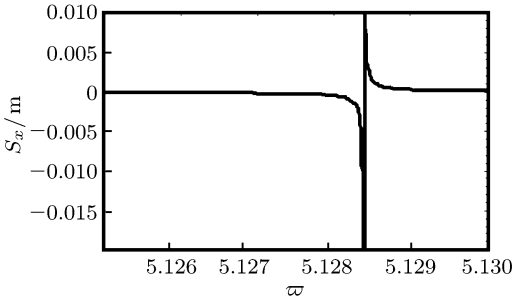


Fig.8. The plot of the position shift S_x versus frequency ϖ in the second bandgap.

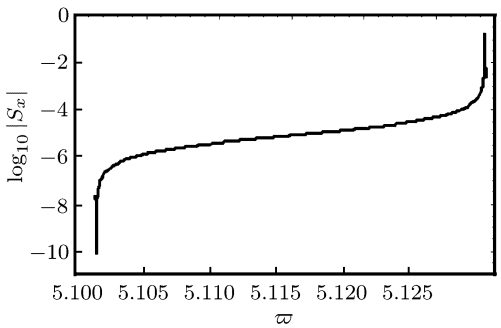


Fig.9. The logarithm of absolute value of S_x as the function of ϖ in negative refraction frequency band.

4. Conclusion

In summary, we demonstrated theoretically that negative refraction may occur near the low frequency edge of the second and fourth bandgaps in finite 1D PCs for an oblique incidence of wave. Additionally, we calculate the transverse position shift through PC that can characterize negative refraction more explicitly. These unique properties of refracting Bloch photons have the potential to perfect the design of the integrated photonic systems.

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