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# Quasistable bright dissipative holographic solitons in photorefractive two-wave mixing system*

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The dynamical evolution of both signal and pump beams are traced by numerically solving the coupled-wave equation for a photorefractive two-wave mixing system. The direct simulations show that, when the intensity ratio of the pump beam to the signal beam is large enough, the pump beam presents a common decaying behaviour without modulational instability (MI), while the signal beam can evolve into a quasistable spatial soliton within a regime in which the pump beam is depleted slightly. The larger the ratio is, the longer the regime is. Such quasistable solitons can overcome the initial perturbations and numerical noises in the course of propagation, perform several cycles of slow oscillation in intensity and width, and persist over tens of diffraction lengths. From physical viewpoints, these solitons actually exist as completely rigorous physical objects. If the ratio is quite small, the pump beam is apt to show MI, during which the signal beam experiences strong expansion and shrinking in width and a drastic oscillation in intensity, or completely breaks up. The simulations using actual experimental parameters demonstrate that the observation of an effectively stable soliton is quite possible in the proposed system.

**Keywords:** spatial optical solitons, photorefractive nonlinear optics, two-wave mixing

**PACC:** 4265S, 4265J, 7820W, 4240D

## 1. Introduction

The dissipative holographic (DH) soliton was proposed for a system in which two beams (namely the pump and signal beams) were coupled via two-wave mixing (TWM) to form a interference pattern in a biased photorefractive (PR) crystal with the initial pump and signal beams having uniform and solitons-like spatial distributions, respectively.[1] The evolution of the two beams was governed by a coupled-wave equation. When an undepleted pump condition was assumed and the effect of self-focusing nonlinearity on the pump beam was ignored, the pump beam could be considered as constant and hence the coupled-wave equation degenerated into a cubic complex Ginzburg–Landau equation (CGLE) for the signal beam alone. The analytic bright and dark solitonic solutions of this cubic CGLE were obtained in Ref.[1] by use of the results achieved by many pioneers.[2−8] Consequently, the evolution of the pump beam had not been considered. These analytic solitonic solutions presented a possibility that the proposed TWM configuration could enable the signal beam to form a spatial soliton. The evolution, stability and potential application of such bright solitons were investigated numerically for the TWM process being enhanced via moving grating technique[9] and biased electric fields,[10] respectively. PR solitons have been studied widely in Hamiltonian systems.[11−26] The interference pattern formed by two beams via TWM was also used to support another kind of spatial solitons for which the periodic spatial pattern was injected into a medium to form a periodic array of dark solitons.[27−29]

However, the main claim of the previous analysis about DH solitons failed to describe the effect of pump depletion.[1] Furthermore, a well known effect was ignored in the previous analysis that a broad pump beam might be unstable under the effect of self-focusing nonlinearity, which could bring about modulational instability (MI) for the pump beam, seeming to overrule the existence of DH solitons.

Another problem in the previous analysis about DH solitons is the stability. DH solitons are existent on condition that the linear net gain is positive.[1] Under such a condition, however, the background state of a cubic CGLE is always unstable,[8,9,30] although DH solitons themselves as the solutions of this equation are stable in some cases.[31] After a long propagation distance, this instability will result in that radiation

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waves become appreciable and can distort the DH soliton itself.

In order to solve the above problems, the investigation on soliton formation in the proposed PR TMW system should be restarted with considering not only the evolution of both signal and pump beams simultaneously, but also the effects of pump depletion and MI on the soliton formation. The objective of this paper is to make a direct simulation by numerically solving the coupled-wave equation for the two beams, instead of the evolution equation for the signal beam alone. We do not endeavor to search for exact soliton solutions that are completely stable. Instead, we endeavor to search for quasistable solitary states that, in terms of possible experiments, are completely stable physical objects. Our results demonstrate that, under some suitable conditions, the signal beam can evolve into a quasistable soliton, while the pump beam exhibits a common decaying behaviour without MI.

We also perform a direct simulation with parameters pertaining to real crystals and possible experiments, which demonstrates that the signal beam can keep their spatial shape nearly unchanged in the whole propagation process, just as behaves a real soliton, signifying that the quasistable DH solitons could be observed experimentally.

It has been studied widely that a spatial optical beam with a localized structure exhibits quasistable propagation for much larger distances.[32−36] Although such a study cannot determine the complete stabilization of the localized structure, it can check whether the structure is robust or not. Because it is usually very difficult to find exact analytic solitonic solutions for a model for which solitons are expected to exist or to explore the entire parameter space of the model to find a stable numerical solution (if it exists), finding quasistable structures becomes very significative. Even though the quasistable structures correspond to unstable ones, they may be observed in the experiments,[35,36] as their instability develops at very large values of propagation distance, which would definitely exceed the size of available samples.

The rest of the paper is organized as follows. In Section 2, the model to describe quasistable DH solitons is constructed. Direct numerical simulations of the evolution of the beams are preformed in Section 3. The results are summarized in the concluding section.

## 2. Theoretical model

Consider the schematic shown in Fig.1. The PR crystal considered here is a strontium barium niobate (SNB) crystal with its optical $c$-axis oriented along the $x$-coordinate. Both signal and pump beams are ordinarily polarized. They are made to propagate in the crystal and each beam makes a small angle ($\theta$) of its $\boldsymbol{k}$ vector with the $z$-axis. The initial signal beam has a one-dimensional soliton-like spatial distribution that is confined in the $x$ direction, whereas extended (nondiffracting) in the second transverse direction, $y$. The initial pump beam is a one-dimensional broad light beam that is variational in the $x$ direction, whereas it is extended in the $y$ direction. In this configuration, the electric field should be applied along the $y$ direction to ensure that there are no focusing effects induced by the PR screening nonlinearity in $x$ direction. The two beams interact through the holographic focusing mechanism coming from the index grating that they form.

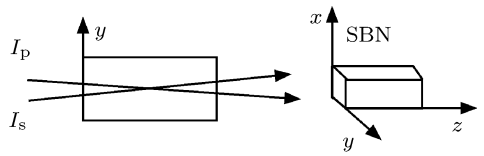



**Fig.1.** Schematic configuration of a PR two-wave mixing system that is proposed to support dissipative holographic solitons in a biased PR crystal.

Let $\phi_\mathrm{p}$ and $\phi_\mathrm{s}$ denote the slowly varying envelopes of the electric-field components of the signal and pump beams, respectively. We use a method similar to that in Ref.[34], in which the coupled-wave equation for codirectional degenerate TWM was derived. But we consider the variation of the envelopes in two directions (i.e., the propagation direction $z$ and the transverse direction $x$). In the slowly varying approximation (i.e., ignoring the terms of $\partial^2\phi_\mathrm{s}/\partial z^2$ and $\partial^2\phi_\mathrm{p}/\partial z^2$), we find $\phi_\mathrm{p}$ and $\phi_\mathrm{s}$ satisfy the following paraxial coupled-wave equations:

$$\frac{1}{2k}\frac{\partial^2\phi_\mathrm{p}}{\partial x^2}+\mathrm{i}\cos\theta\frac{\partial\phi_\mathrm{p}}{\partial z}+\left(\Gamma_0+\mathrm{i}\frac{1}{2}\Gamma\right)\frac{|\phi_\mathrm{s}|^2}{|\phi_\mathrm{s}|^2+|\phi_\mathrm{p}|^2}\phi_\mathrm{p}+\mathrm{i}\frac{1}{2}\alpha_0\phi_\mathrm{p}=0, \tag{1a}$$

$$\frac{1}{2k}\frac{\partial^2\phi_\mathrm{s}}{\partial x^2}+\mathrm{i}\cos\theta\frac{\partial\phi_\mathrm{s}}{\partial z}+\left(\Gamma_0-\mathrm{i}\frac{1}{2}\Gamma\right)\frac{|\phi_\mathrm{p}|^2}{|\phi_\mathrm{s}|^2+|\phi_\mathrm{p}|^2}\phi_\mathrm{s}+\mathrm{i}\frac{1}{2}\alpha_0\phi_\mathrm{s}=0, \tag{1b}$$

where $k = n_o k_0$ ($k_0 = 2\pi/\lambda_0$ with $\lambda_0$ being the free-space wavelength of the lightwave employed, and $n_o$ is the ordinary refractive index); $\Gamma$ and $\Gamma_0$ are the intensity and phase coupling coefficients, respectively; $\alpha_0$ is the absorption coefficient of the crystal. We now rewrite the coupled-wave equations in a dimensionless form by letting $s = x/x_0$, $\xi = z/(z_0 \cos\theta)$, $z_0 = kx_0^2$, $g_0 = z_0\Gamma_0$, $g = z_0\Gamma/2$, $\alpha = z_0\alpha_0/2$, $U_s = I_0^{-1/2}\phi_s$, and $U_p = I_0^{-1/2}\phi_p$, where $x_0$ is an arbitrary width and $I_0$ is an arbitrary intensity. Thus Eqs.(1) can be expressed in terms of $U_s$ and $U_p$ as the following:

$$i\frac{\partial U_p}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_p}{\partial s^2} + (g_0 + ig)\frac{|U_s|^2}{|U_p|^2 + |U_s|^2}U_p + i\alpha U_p = 0, \tag{2a}$$

$$i\frac{\partial U_s}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_s}{\partial s^2} + (g_0 - ig)\frac{|U_p|^2}{|U_p|^2 + |U_s|^2}U_s + i\alpha U_s = 0. \tag{2b}$$

Under the undepleted pump approximation and ignoring the effect of self-focusing nonlinearity on the pump beam, the variable $U_p(x, z)$ was taken as constant in Ref.[1]. As a result, for $|U_s|^2/|U_p|^2 \ll 1$, Eq.(2) becomes

$$i\frac{\partial U_s}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_s}{\partial s^2} + (g_0 - iG)U_s - (g_0 - ig)U_s|U_s|^2 = 0, \tag{3}$$

where $G = g - \alpha$ is the linear net gain. The above equation can be transformed into a standard cubic CGLE without the term of spatial spectral filtering. By use of the results in Refs.[2–9] directly, the bright- and dark-soliton solutions of Eq.(3) can be obtained as follows:[1]

$$U_s(s, \xi) = F\,\mathrm{sech}(Bs)\exp\{ib\ln[\mathrm{sech}(Bs)]\}\exp(-iv\xi), \tag{4a}$$

$$U_s(s, \xi) = D\tanh(Hs)\exp\{id\ln[\cosh(Hs)]\}\exp(-i\Omega\xi). \tag{4b}$$

The meaning and expression of the parameters in Eq.(4) can be found in Ref.[1]. Because both amplitudes (i.e., $F$ and $D$) are in proportion to $(G/g)^{1/2}$, these solitonic solutions are existent if $G > 0$. Although the solutions (4) reveal a possibility that TWM systems could support a kind of spatial solitons, they are, in general, unstable because the background state (namely, $U_s = 0$) is always unstable when $G > 0$, which will bring noises into the soliton and finally destroy the soliton itself after a normalized propagation distance $G^{-1}$.[8,9,30] Furthermore, the solutions (4) were obtained in the case that the pump-depletion effect and the MI process for the pump beam were not considered at all. Obviously, the model presented in Ref.[1] is oversimplified. A more accurate model for analysing soliton formation in TWM systems should also include an evolution equation for the pump beam. That is, we should use the coupled-wave equation (2) for both signal and pump beams, instead of the evolution equation (3) for the signal beam alone, to analyse the soliton formation. However, it is very difficult to find exact analytic solitonic solution of Eq.(2) (if it exists). In what follows, we do not endeavour to search for such exact analytic solutions. Instead, we search for numerical solitary states by directly solving Eq.(2). To do so, the initial signal and pump beams are assumed as

$$U_s(s, \xi = 0) = I_s^{1/2}\mathrm{sech}(s/W_s), \tag{5}$$

$$U_p(s, \xi = 0) = I_p^{1/2}\exp[-(s/W_p)^4], \tag{6}$$

where $I_s$ and $I_p$ denote the peak intensities normalized by $I_0$; $W_s$ and $W_p$ are the parameters related to the widths normalized by $x_0$. The pump beam is a super-Gauss beam and the value of $W_p$ is usually taken big enough, thus ensuring that the initial pump beam is a broad bright beam. If the width of the beams is assumed to be measured approximately from the peak intensity to where it decreases to $1/e$ of its peak value, from Eqs.(5) and (6), one can obtain the actual width of the signal and pump beams to be $W_{as} = 1.085\,W_s x_0$ and $W_{ap} = 0.84\,W_p x_0$, respectively. The diffraction length is usually defined as $z_d = 0.5\,kW_{as}^2$,[37] and hence the corresponding dimensionless diffraction length for the signal beam is $\xi_d = 0.5886\,W_s^2$.

# 3. Dynamical evolutions of the beams in PR two-wave mixing systems

We start with defining a pair of signal and pump beams via Eqs.(5) and (6), respectively, for a given set of dimensionless beam parameters (i.e., $I_s$, $I_p$, $W_s$ and $W_p$). Taking them as the initial parameters, we solve the coupled-wave equation (2) numerically for a given set of dimensionless system parameters (namely, $g$, $g_0$ and $\alpha$). The evolution of the two beams are observed over the dimensionless propagation distance $\xi$ up to tens of diffraction lengths.

## 3.1. Robustness of the signal beam as a quasistable solitonic state

For a pair of input signal and pump beams, we should select an optimum set of system parameters (i.e., $g$, $g_0$ and $\alpha$) that minimizes the oscillation of the width and peak intensity of the signal beam in the course of propagation. Figure 2 illustrates two typical examples, which show that the signal beam can keep a robust localized structure over the propagation distance up to tens of diffraction lengths. Meanwhile, the pump beam is depleted slightly and presents a common decaying behaviour without MI process (not shown here). To be specific, the propagation distance is longer than 76 and 57 diffraction lengths for the signal beam in Figs.2(a) and 2(b), respectively, indicating that the signal beam can evolve into a quasistable spatial soliton under suitable conditions. If taking $x_0 = 250\,\mu$m, the actual width of the signal beam is $W_{as} \approx 18.1\,\mu$m and $W_{as} \approx 20.9\,\mu$m for Figs.2(a) and 2(b), respectively, which are within a reasonable range for the width of spatial solitons. In Fig.2 the normalized propagation distance is $\xi = 0.2$, thus the corresponding actual propagation distance is $z = 187.5\,$cm for $k = 30\,\mu\text{m}^{-1}$ and $x_0 = 250\,\mu$m. Obviously, such a propagation distance is tremendously longer than the length of a real PR crystal which is usually of the order of one centimetre.

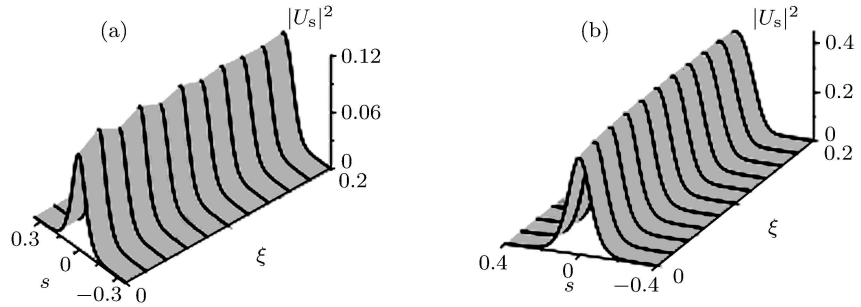



**Fig.2.** The simulations of the evolution of the signal beam in PR two-wave mixing system. (a) initial beam parameters: $I_s = 0.1$, $I_p = 10000$, $W_s = 1/15$, and $W_p = 20$; diffraction length: $\xi_d \approx 0.00348$; system parameters: $\alpha = 5$, $g = 5.4$, and $g_0 = -9870$; (b) initial beam parameters: $I_s = 0.4$, $I_p = 20000$, $W_s = 1/13$, and $W_p = 20$; diffraction length: $\xi_d \approx 0.00262$; system parameters: $\alpha = 8$, $g = 8.3$, and $g_0 = -10251$.

In order to investigate the effect of the pump depletion on the quasistable soliton formation of the signal beam, the evolution of the signal beam in Fig.2(b) is observed over a longer distance so that the pump depletion becomes evident, as shown in Fig.3. By comparing Fig.3 with Fig.2(b), one can clear see that the signal beam cannot remain a solitary state when $\xi > 0.2$ because the pump beam is depleted severely more and more after $\xi > 0.2$. A similar result has been obtained for the signal beam in Fig.2(a) over a longer distance, which indicates that the signal beam can remain in solitary state till $\xi = 0.23$.

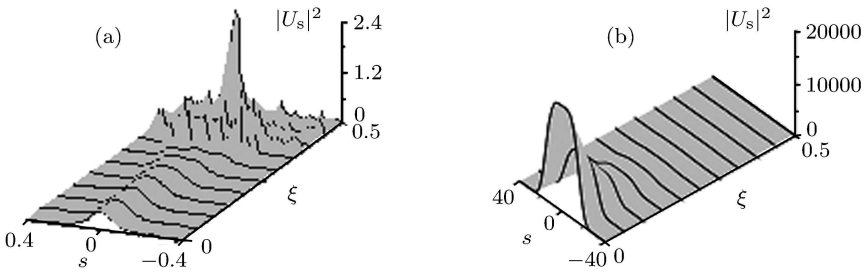



**Fig.3.** The simulations of the evolution of the signal and pump beams in PR two-wave mixing system over longer distance with the initial beam and system parameters same as that in Fig.2(b).

The results above demonstrate that, there is a regime in which the signal beam can evolve into a quasistable solitary state. A natural question to ask here is which factor mainly determines the length of the regime. Let $R$ denote the intensity ratio of the pump to signal beam, by performing a huge amount of numerical simulation, it has been found that the length mainly determined by the ratio $R$. The larger the ratio $R$ is, the longer the length is. When the propagation distance overruns the regime, the signal beam cannot retain its solitonic state and will finally break up. Therefore in the PR TWM model, although DH solitons do not exist as a general class of completely stable solitons, it could exist as a class of quasistable solitons in a regime with its length definitely exceeding the size of available PR samples if the ratio $R$ is large enough.

It should be emphasized that, because the incident signal beam is not an exact solution of Eqs.(2), this beam can remain in a quasistable solitary state in the course of very long evolution, showing that the initial perturbations, together with that directly coming from the numerical calculation, have been overcome in the process of the formation of quasistable solitons. Therefore, from the physical point of view, such a quasistable solitary state is a completely rigorous physical object.

## 3.2. Modulation instability of the pump beam

The results above demonstrate that, when the ratio $R$ is large enough, the evolution of the pump beam exhibits a common decaying process without MI. A natural question to address here is what will happen when the ratio is quite small. Although the MI process of quasi-plane-wave optical beam in biased PR crystals was investigated in great detail via both analytic and numerical approaches, in which the beam was governed by a single evolution equation,[21,22] the analytic method cannot be applied to analyse the MI process of the pump beam because this beam is governed by a coupled-wave equation, instead of a single evolution equation. Therefore, the numerical method is used here to analyse the MI process of the pump beam and the effect of the MI on the quasi soliton formation of the signal beam via directly solving Eq.(2). Figure 4 shows an example with $R = 25$. One can clearly see that the spatial filaments first appear on the top of the pump beam, and then develop towards the sides of the beam. Meanwhile, the signal beam experiences a strong expansion and shrinking in its width and a drastic oscillation in its peak intensity. Another example with $R = 2.5$ is depicted in Fig.5 with $I_{\rm p} = 1$ and the other parameters to be taken the same as that in Fig.2(b). By comparing Fig.5 with Fig.2(b) and Fig.3, one can clear see that the spatial filaments appear on the top of the pump beam at a very short propagation distance, and overspread to the whole sides of the beam rapidly. Meanwhile, the signal beam completely breaks up.

The results above demonstrate that, when $R$ is not large enough, the pump beam is apt to show MI that will result in the signal beam to be extremely unstable. Therefore, the ratio $R$ to be large enough is a necessary condition that the signal beam can evolve into a quasistable soliton within a regime. However, a natural question to raise here is whether such a condition could be realized in experiment. It will be discussed in the following subsection.

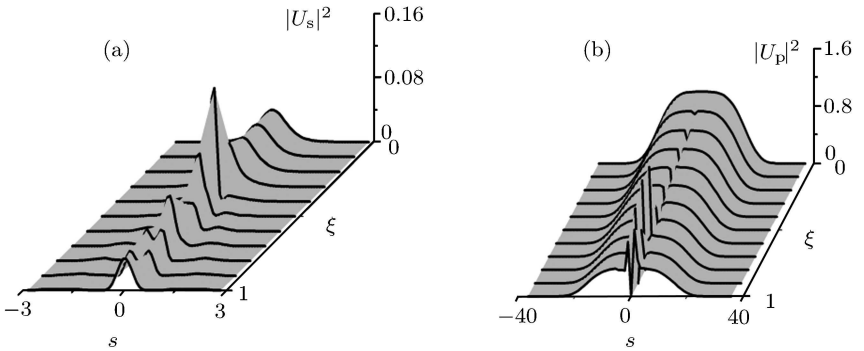



**Fig.4.** The simulations of the evolution of the signal and pump beams in PR two-wave mixing system with a quite weak pump beam and the MI process occurring for the pump beam. (a) The signal beam; (b) the pump beam. Initial condition: $I_{\rm s} = 0.04$, $I_{\rm p} = 1$, $W_{\rm s} = 0.54$, and $W_{\rm p} = 40$. System parameters: $\alpha = 0.47$, $g = 0.483$, and $g_0 = -86.2$.

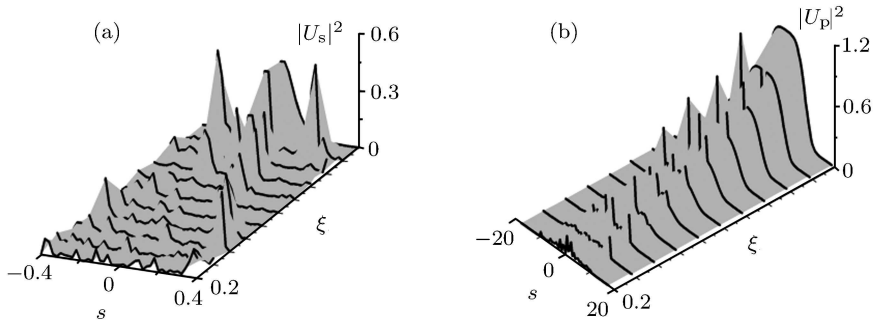



**Fig.5.** The simulations of the evolution of the signal and pump beams in PR two-wave mixing system with a weak pump $I_p = 1$ (a) the signal beam and (b) the pump beam. The other parameters are the same as that in Fig.2(b).

### 3.3. Simulation for a possible experimental system

As we know, the larger the ratio, the longer the regime in which the quasi solitons exist. Because a real PR crystal usually has a size of the order of one centimetre, such a small size would need but a small value of $R$ to ensure the regime length to be much longer than the crystal size. In order to check this, we perform the simulations close to the situation of a possible experimental system including a SNB:60 crystal with $n_e = 2.35$, $n_0 = 2.37$, $\gamma_{13}$ =47 pm/V, $\gamma_{33} = 235$ pm/V, and $\varepsilon = 880$ at $\lambda_0 = 0.5\,\mu$m and $T = 300$ K.[38] To do so, we take the dimensionless system parameters (i.e., $g = z_0\Gamma/2$ and $g_0 = z_0\Gamma_0$) in Eq.(2) with a background according to above material parameters and the bias field. From the PR theory,[39] the intensity and phase coupling coefficients (i.e., $\Gamma$ and $\Gamma_0$) satisfy $\Gamma/2 + \mathrm{i}\Gamma_0 = \delta\tilde{E}E_s/(\tilde{E}+E_s)$, where $\tilde{E} = E_d + \mathrm{i}E_0$ with $E_0$ being the bias field and $E_d = 4\pi k_B T \sin\theta/(\lambda_0 e)$ being the diffusion field (where $k_B$ is the Boltzmann constant, $e$ is the electron charge, and $T$ is the absolute temperature); $E_s = eN_A\lambda_0/(4\pi\varepsilon\varepsilon_0 \sin\theta)$ is the saturation field with $N_A$ being the acceptor density and $\varepsilon$ and $\varepsilon_0$ being the effective dielectric constant and the permittivity of vacuum, respectively; and $\delta = \pi n_e^3 \gamma_{\mathrm{eff}}/\lambda_0$ with $\gamma_{\mathrm{eff}}$ being the effective electro-optic coefficient. If $E_0 > 0$ (or $< 0$) means that the bias field is applied along the $c$-axis direction (or the reverse), assuming $\Gamma_0 > 0$ when $E_0 > 0$, then for this configuration, $\gamma_{\mathrm{eff}}$ can be expressed as $\gamma_{\mathrm{eff}} = \bar{n}^3\gamma_{33}\cos^2\theta - \bar{n}^{-1}\gamma_{13}\sin^2\theta$, where $\bar{n} = n_e/n_o$ and $n_e$ is the extraordinary index of refraction.[40] For a given value of $E_0$, we can simultaneously get the values of $\Gamma$ and $\Gamma_0$, and then get the values of $g$ and $g_0$.

In a possible experiment, we may start with injecting a pair of signal and pump beams into a PR crystal, and then adjust the bias field $E_0$ gradually until the signal beam forms a quasistable solitary state. Our simulations are taken close to this process. Considering that the laser beam used in experiments has the usual Gaussian shape instead the sech-type given by Eq.(5), the initial signal beam is assumed as $U_s(s, \xi = 0) = I_s^{1/2}\exp(-2s^2/\sigma_s^2)$, where $\sigma_s$ is the width of the Gaussian beam, measured from the peak intensity to where it decreases to 1/e of its peak value. The corresponding actual width is then $\sigma_{as} = \sigma_s x_0$.

Figure 6 shows three examples with $\theta = 0.1^\circ$ and $\lambda = 500$ nm in a SBN:60 crystal.

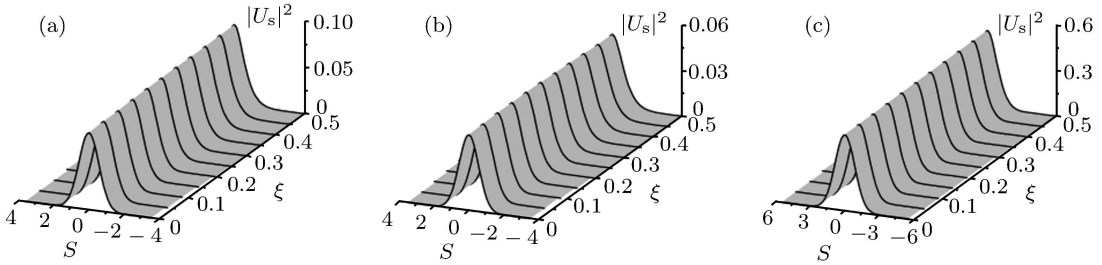



**Fig.6.** Simulations of the propagation for the input Gaussian signal beam in a PR two-wave mixing system with a background according to a possible experiment (a) initial beam parameters: $I_s = 0.1$, $I_p = 10$, $\sigma_s = 1.5$, and $W_p = 5$; system parameters: $\alpha = 0.465$, $g = 0.523$, and $g_0 = -104$; (b) initial beam parameters: $I_s = 0.05$, $I_p = 4$, $\sigma_s = 1.36$, and $W_p = 5$; system parameters: $\alpha = 0.4$, $g = 0.483$, and $g_0 = -86.2$; (c) initial beam parameters: $I_s = 0.5$, $I_p = 20$, $\sigma_s = 2.2$, and $W_p = 5$; system parameters: $\alpha = 0.35$, $g = 0.396$, and $g_0 = -20.7$.

For these values, if $x_0$ and $I_0$ are taken as $x_0 = 25\,\mu$m and $I_0 = 1\,\mathrm{mW/cm^2}$, the actual length of the crystal is about 0.93 cm for $\xi = 0.5$, which is a common case for a PR sample. As one can see, the signal beam can nearly keep its shape unchanged in the sample, just like that behaved by a real soliton. This is conceivable because the quasistable solitary state performs its oscillation in intensity and width with a very long quasi period over propagation distance of tens of centimetres, compared with the usual length of a PR crystal. The evolution of the corresponding pump beams does not show here, and these beams exhibit a common decaying process without MI. For the three examples, the values of those dimensionless system parameters and initial beam parameters are listed in the caption of Fig.6 and the values of corresponding actual parameters are listed in Table 1. Obviously, these actual parameter values are taken in a reasonable and realizable range for a possible experiment. Emphatically, although the ratio of the pump to signal beam to be large enough is very important for the quasi soliton formation, it is not necessary to take the pump beam so extremely strong that is not realistic. Instead, a pump beam 40 to 100 times stronger than the signal beam would be enough to ensure the signal to form a quasi-soliton in a PR crystal whose length is about one centimetre.

**Table 1.** The actual parameter values corresponding to those dimensionless parameters used in the simulations shown in Fig.6.

| Parameters and unit | (a) | (b) | (c) |
|---|---|---|---|
| peak intensity of signal beam/(mW/cm$^2$) | 0.1 | 0.05 | 0.5 |
| width of signal beam/$\mu$m | 37.5 | 34 | 55 |
| peak intensity of pump beam/(mW/cm$^2$) | 10 | 4 | 20 |
| width of pump beam/$\mu$m | 105 | 105 | 105 |
| intensity ratio of the pump to signal beam, $R$ | 100 | 80 | 40 |
| crystal loss $\alpha_0$/cm$^{-1}$ | 0.50 | 0.43 | 0.376 |
| bias field $E_0$/(V/cm) | 3000 | 2500 | 600 |
| intensity coupling coefficients $\Gamma$/cm$^{-1}$ | 0.562 | 0.519 | 0.426 |
| phase coupling coefficients $\Gamma_0$/cm$^{-1}$ | $-55.9$ | $-46.3$ | $-11.1$ |

# 4. Conclusion

In this work, a model has been developed that can describe the transversal variation of both signal and pump beams simultaneously in a PR two-wave mixing system. Based on this model, direct simulations of the dynamical evolution of the beams have been performed. The results demonstrate that, when the intensity ratio of the pump to signal beam is large enough, the pump beam exhibits a common decaying process without MI, while the signal beam has a very robust spatially localized structure within a regime in which the pump beam is depleted slightly. The generic behaviour of such a robust structure is quasiperiodic oscillation in width and intensity, which persists over tens of diffraction lengths, signifying that the signal beam cloud evolve into a quasistable spatial soliton over very long propagation distance. When the propagation distance overruns the regime so that the pump beam is depleted evidently, however, the signal beam cannot remain in its quasistable solitonic structure and breaks up gradually. If the ratio is not large enough, the pump beam is apt to show MI, making the signal beam to oscillate strongly in intensity and width or completely break up.

The most essential prediction for actual experiments is that observation of an effectively stable soliton is quite possible in a PR two-wave mixing system and the conditions (such as the bias field and the intensity ratio) needed to perform such experiments are readily realizable.

# References

[1] Liu J S 2003 *Opt. Lett.* **28** 2237
[2] Pereira N R and Stenflo L 1977 *Phys. Fluids* **20** 1733
[3] Bekki N and Nozaki K 1984 *J. Phys. Soc. Jan.* **53** 1581
[4] Akhmediev N N, Afanasjev V V and Soto-Crespo J M

1996 *Phys. Rev.* E **53** 1190
[5] Akhmediev N N and Afanasjev V V 1995 *Phys. Rev. Lett.* **75** 2320
[6] Soto-Crespo J M, Akhmediev N N and Afanasjev V V 1996 *J. Opt. Soc. Am.* B **13** 1439
[7] Akhmediev N N and Ankiewizc A 1997 *Solitons: Nonlinear Pulses and Beams* (London: Chapman and Hall) Chap 13 p276
[8] Akhmediev N N and Ankiewizc A in 2001 *Spatial Solitons,* edited by Trillo S and Torruellas W (Berlin: Springer) p311
[9] Liu J S, Zhang Y Y and Eichler H J 2005 *J. Mod. Opt.* **52** 563
[10] Liu J S, Zhang H L, Zhang G Y and Wang C 2006 *Chin. Phys.* **15** 394
[11] Cohen O, Carmon T, Segev M and Odoulov S 2002 *Opt. Lett.* **27** 2031
[12] Liu J S and Lu K Q 1998 *Acta Phys. Sin.* **47** 1509 (in Chinese)
[13] Hou C F, Li S Q, Li B and Sun X D 2001 *Acta Phys. Sin.* **50** 73 (in Chinese)
[14] Liu J S and Zhang D Y 2000 *Chin. Phys.* **9** 667
[15] Liu J S 2001 *Chin. Phys.* **10** 1037
[16] Liu J S and Zhang D Y 2001 *Acta Phys. Sin.* **50** 880 (in Chinese)
[17] Hao Z H and Liu J S 2002 *Acta Phys. Sin.* **51** 818 (in Chinese)
[18] Hao Z H and Liu J S 2002 *Acta Phys. Sin.* **51** 2772 (in Chinese)
[19] Liu J S and Hao Z H 2002 *Chin. Phys.* **11** 254
[20] He G G, Wang X S and She W L 2002 *Acta Phys. Sin.* **51** 2270 (in Chinese)
[21] Liu J S and Hao Z H 2003 *Chin. Phys.* **12** 1124
[22] Liu J S 2004 *Acta Phys. Sin.* **53** 3014 (in Chinese)
[23] Liu J S and Hao Z H 2004 *Chin. Phys.* **13** 704
[24] Wang X H and Gou Q 2005 *Acta Phys. Sin.* **54** 3183 (in Chinese)
[25] Liu J S and Du Z M 2005 *Acta Phys. Sin.* **54** 2739 (in Chinese)
[26] Hou C F, Pei Y B, Zhou V X and Sun X D 2005 *Chin. Phys.* **14** 349
[27] Bosshard C, Mamyshev P V and Stegeman G I 1994 *Opt. Lett.* **19** 90
[28] Mamyshev P V, Bosshard C and Stegeman G I 1994 *J. Opt. Soc. Am.* B **11** 1254
[29] Macuil R D and Castillo M D I 2003 *J. Opt.* A: *Pure Appl. Opt.* **5** S493
[30] Malomed B A, Gölles M, Uzunov I M and Lederer F 1997 *Physica Scripta* **5** 73
[31] Afanasjev V V 1995 *Opt. Lett.* **20** 704
[32] Mihalache D, Mazilu D, Crasovan L C, Malomed B A, Lederer F and Torner L 2003 *Phys. Rev.* E **68** 046612
[33] Leblond H, Malomed B A and Mihalache D 2005 *Phys. Rev.* E **71** 036608
[34] Desyatnikov A S and Kivshar Y S 2001 *Phys. Rev. Lett.* **87** 033901
[35] Beckwitt K, Chen Y F, Wise F W and Malomed B A 2003 *Phys. Rev.* E **68** 057601
[36] Towers I N, Malomed B A and Wise F W 2003 *Phys. Rev. Lett.* **90** 123902
[37] Luther-Davies B and Stegeman G I in 2001 *Spatial Solitons* **e**dited by Trillo S and Torruellas W (Berlin: Springer) p22
[38] Vazqez R A, Ewbank M D and Neurgaonkar R R 1991 *Opt. Commun.* **80** 235
[39] Yeh P 1989 *IEEE J. Quant. Electron.* **25** 484
[40] Oda I, Otani Y, Liu L and Yoshizawa T 1998 *Opt. Commun.* **148** 95