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Grey spatial solitons due to two-photon photorefractive effect*

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This paper predicts that grey spatial solitons can exist in two-photon photorefractive materials. In steady state and under appropriate external bias conditions, it obtains the grey spatial soliton solutions of the optical wave evolution equation. The intensity profile, phase distribution, and transverse velocity of these grey solitons are discussed.

Keywords: photorefractive effect, spatial soliton, photorefractive material

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1. Introduction

During the last decade, the optical spatial solitons based on photorefractive effect have attracted much special interest,[1,2] because these photorefractive spatial solitons can be formed at low light intensity and are potentially useful for all-optical switching, beam steering, and optical interconnects. So far, several different types of photorefractive spatial solitons have been investigated: i.e., quasi-steady-state solitons,[3−6] screening solitons,[7−10] photovoltaic solitons,[11−16] screening-photovoltaic solitons,[17−21] self-trapping of incoherent beams[22,23] or white light beams,[24] and holographic solitons.[25−27] In addition, it has also been predicted that polymeric solitons may exist in biased organic photorefractive polymer.[28−30]

All of the above mentioned solitons are resulted from single-photon photorefractive effect. Recently, Castro-Camus and Magana[31] provided a new model of the two-photon photorefractive effect. The Castro-Camus model includes a valence band, a conduction band, and an intermediate allowed level. The intermediate allowed level is used to maintain a quantity of excited electrons from the valence band by the gating beam. These electrons are then excited again to the conduction band by the signal beam. The pattern of the signal beam can induce a spatially dependent charge distribution which gives rise to non-linear changes of refractive index in the medium. Recently, we had just presented a theory based on the Castro-Camus model which gives rise to the evolution equation of one-dimensional spatial solitons in two-photon photorefractive media.[32] In the steady state and under appropriate external bias conditions, our solutions predict not only dark but also bright soliton states. In this paper, we will show that grey spatial solitons can also be supported in two-photon photorefractive media. In the steady state and under appropriate external bias conditions, we obtain the grey spatial soliton solutions of the optical wave evolution equation, and discuss the intensity profile, phase distribution, and transverse velocity of these grey solitons.

2. Evolution equation

To study the grey spatial solitons due to two-photon photorefractive effect, we consider an optical beam which propagates in a photorefractive medium with two-photon photorefractive effect along the z axis and is permitted to diffract only along the x direction. The photorefractive medium is put with its optical c axis oriented along x coordinate and is illuminated by the gating beam. Moreover, let us assume that the polarization of the incident optical beam is parallel to the c axis, and an external bias electric field is applied in the same direction. As usual, we express the optical field of the incident beam in terms of slowly varying envelope ϕ, i.e. $E = \hat{x}\phi(x,z)\exp(\mathrm{i}kz)$, where $k = k_0 n_\mathrm{e} = (2\pi/\lambda_0)n_\mathrm{e}$, n_e is the unperturbed extraor-

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dinary index of refraction, and λ_0 is the free-space wavelength. Under these conditions the optical beam satisfies the following envelope evolution equation:[9]

$$\mathrm{i}\phi_z + \frac{1}{2k}\phi_{xx} - \frac{k_0 n_\mathrm{e}^3 r_{33} E_\mathrm{sc}}{2}\phi = 0, \tag{1}$$

where $\phi_z = \partial\phi/\partial z, \phi_{xx} = \partial^2\phi/\partial x^2, r_{33}$ is the electro-optic coefficient, and $\boldsymbol{E}_\mathrm{sc} = E_\mathrm{sc}\hat{x}$ is the space charge field in the medium.

The space charge field in Eq.(1) can be obtained by[32]

$$E_\mathrm{sc} = E_0\frac{(I_{2\infty} + I_{2\mathrm{d}})(I_2 + I_{2\mathrm{d}} + \gamma_1 N_\mathrm{A}/s_2)}{(I_{2\infty} + I_{2\mathrm{d}} + \gamma_1 N_\mathrm{A}/s_2)(I_2 + I_{2\mathrm{d}})}, \tag{2}$$

where γ_1 is the recombination factor of the IL–VB transition, $I_{2\mathrm{d}} = \beta_2/s_2, s_2$ is photoexcitation cross, β_2 is the thermoionization probability constants for the transition of IL–CB, $I_{2\infty} = I_2(x \to \pm\infty, z)$, I_2 is the intensity of the soliton beam. According to Poynting's theorem, I_2 can be expressed in terms of the envelope ϕ, that is, $I_2 = (n_\mathrm{e}/2\eta_0)|\phi|^2$, where $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$.

Substituting Eq.(2) into Eq.(1), we can establish the envelope evolution equation of the soliton. For convenience, the following dimensionless coordinates and variables are adopted, i.e., $s = x/x_0$, $\xi = z/(kx_0^2)$, $\phi = (2\eta_0 I_\mathrm{d}/n_\mathrm{e})^{1/2}U$, where x_0 is an arbitrary spatial width for scaling. Under these conditions, the following dynamical evolution equation can be obtained:[32]

$$\mathrm{i}U_\xi + \frac{1}{2}U_{\mathrm{ss}} - \frac{\beta(\rho+1)}{\rho+1+\sigma}\left(1 + \frac{\sigma}{|U|^2+1}\right)U = 0, \tag{3}$$

where $\rho = I_{2\infty}/I_{2\mathrm{d}}$, $\beta = (k_0x_0)^2(n_\mathrm{e}^4 r_{33}/2)E_0$, $\sigma = \gamma_1 N_\mathrm{A}/s_2 I_{2\mathrm{d}} = \gamma_1 N_\mathrm{A}/\beta_2$.

We can directly show that Eq.(3) can be derived from the following Lagrangian density

$$L = \frac{\mathrm{i}}{2}(UU_\xi^* - U^*U_\xi) + \frac{1}{2}U_\mathrm{s}U_\mathrm{s}^* + \frac{\beta(\rho+1)}{\rho+1+\sigma}\left[|U|^2 - \rho + \sigma\ln\left(\frac{1+|U|^2}{1+\rho}\right)\right], \tag{4}$$

or Hamiltonian density

$$\hat{H} = \frac{1}{2}|U_\mathrm{s}|^2 + \frac{\beta(\rho+1)}{\rho+1+\sigma}\left[|U|^2 - \rho + \sigma\ln\left(\frac{1+|U|^2}{1+\rho}\right)\right]. \tag{5}$$

Similar to Refs.[33–35], It can also be verified that Eq.(3) has the following invariants, namely, the complementary power

$$N = \int_{-\infty}^{\infty}(|U|^2 - \rho)\mathrm{d}s, \tag{6}$$

the renormalized momentum

$$P = \frac{\mathrm{i}}{2}\int(UU_\mathrm{s}^* - U^*U_\mathrm{s})\left(1 - \frac{\rho}{|U|^2}\right)\mathrm{d}s = \frac{\mathrm{i}}{2}\int(UU_\mathrm{s}^* - U^*U_\mathrm{s})\mathrm{d}s - \rho\mathrm{Arg}U\Big|_{-\infty}^{\infty} \tag{7}$$

and the renormalized Hamiltonian

$$H = \int_{-\infty}^{\infty}\left\{\frac{1}{2}|U_\mathrm{s}|^2 + \frac{\beta(\rho+1)}{\rho+1+\sigma}\left[|U|^2 - \rho + \sigma\ln\left(\frac{1+|\mathrm{U}|^2}{1+\rho}\right)\right]\right\}\mathrm{d}s. \tag{8}$$

3. Characteristics of the grey soliton

Introducing the moving coordinates $\eta = s - \nu\xi$, $\zeta = \xi$, and substituting the transformation

$$U(s,\xi) = A(\eta,\zeta)\exp(\mathrm{i}v\eta)\exp(\mathrm{i}v^2\zeta/2) \tag{9}$$

into Eq.(3), we can show that the new envelope $A(\eta,\zeta)$ satisfies the same evolution equation of Eq.(3), i.e.,

$$\mathrm{i}A_\zeta + \frac{1}{2}A_{\eta\eta} - \frac{\beta(\rho+1)}{\rho+1+\sigma}\left(1 + \frac{\sigma}{|A|^2+1}\right)A = 0, \tag{10}$$

where $A_\zeta = \partial A/\partial\zeta, A_{\eta\eta} = \partial^2 A/\partial\eta^2$. In the new moving-coordinate system v represents the normalized

transverse soliton velocity. From Eqs.(3) and (10), we can find that the solution of Eq.(10) automatically satisfies Eq.(3) and vice versa. According to Refs.[10] and [30] one can know that the grey soliton solutions of Eq.(10) can be expressed as

$$A = \rho^{1/2} y(\eta) \exp\left[\mathrm{i}\mu\xi - \mathrm{i}J\int_0^\eta \frac{\mathrm{d}\tilde{\eta}}{y^2(\tilde{\eta})} + \mathrm{i}\Phi_0\right], \quad (11)$$

where J is a real constant to be determined, $y(\eta)$ is a normalized real function ($|y(\eta)| \leq 1$), and Φ_0 is an arbitrary phase. The normalized real function satisfies the boundary conditions: $y^2(0) = \delta$ ($0 < \delta < 1$, i.e. the intensity is finite at the origin), $\dot{y}(0) = 0, y(\eta \to \pm\infty) = 1$, and all the derivatives of $y(\eta)$ are also zero at infinity. The δ parameter describes the grayness, i.e., the intensity at the beam centre is written as $I(0) = \delta I_0$. From Eqs.(9) and (11) we have

$$U(s,\xi) = \rho^{1/2} y(\eta) \exp\left[\mathrm{i}\left(\mu + \frac{v^2}{2}\right)\xi + \mathrm{i}v\eta - \mathrm{i}J\int_0^\eta \frac{\mathrm{d}\tilde{\eta}}{y^2(\tilde{\eta})} + \mathrm{i}\Phi_0\right]. \quad (12)$$

If we set $J = v$, the solution for the U envelope can be written as

$$U(s,\xi) = \rho^{1/2} y(\eta) \exp\left\{\mathrm{i}\left(\mu + \frac{v^2}{2}\right)\xi + \mathrm{i}\nu\left[\eta - \int_0^\eta \frac{\mathrm{d}\tilde{\eta}}{y^2(\tilde{\eta})}\right] + \mathrm{i}\Phi_0\right\}. \quad (13)$$

Substituting Eq.(13) into Eq.(3) we get the following ordinary differential equation:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\eta^2} - 2\mu y - \frac{v^2}{y^3} - \frac{2\beta(\rho+1)}{\rho+1+\sigma}\left(1 + \frac{\sigma}{1+\rho y^2}\right) y = 0. \quad (14)$$

According to the boundary conditions of $y(\eta)$ at infinity, we obtain

$$v = [-2(\mu+\beta)]^{1/2}. \quad (15)$$

Integrating Eq.(14) once, we find

$$\left(\frac{\mathrm{d}y}{\mathrm{d}\eta}\right)^2 = 2\left[\mu + \frac{\beta(\rho+1)}{\rho+1+\sigma}\right](y^2-1) + v^2\left(1 - \frac{1}{y^2}\right) + \frac{2\beta(\rho+1)\sigma}{\rho(\rho+1+\sigma)}\ln\left(\frac{1+\rho y^2}{1+\rho}\right). \quad (16)$$

Using the boundary conditions of $y(\eta)$ at zero, one get

$$\mu = -\frac{\beta}{\rho+1+\sigma}\left[(\rho+1) + \frac{\sigma}{1-\delta} + \frac{\delta}{(1-\delta)^2}\frac{\sigma(\rho+1)}{\rho}\ln\left(\frac{1+\rho\delta}{1+\rho}\right)\right]. \quad (17)$$

From Eqs.(15) and (17) we can know

$$\frac{2\delta\beta\sigma}{(1-\delta)(\rho+1+\sigma)}\left[1 + \frac{1}{1-\delta}\frac{\rho+1}{\rho}\ln\left(\frac{1+\rho\delta}{1+\rho}\right)\right] > 0. \quad (18)$$

Expanding $\ln\left(\dfrac{1+\rho\delta}{1+\rho}\right) = \ln\left[1 - \dfrac{\rho(1-\delta)}{1+\rho}\right]$ into series, we can prove that the quantity in the square bracket of inequality (18) is negative, so we can deduce that

$$\beta < 0. \quad (19)$$

Expression (19) indicates that the grey spatial solitons can be formed in two-photon photorefractive medium only when the external bias field is applied in the opposite direction with respect to the optical c axis of the medium (that is, $E_0 < 0$). By further integrating Eq.(16), we arrive at

$$\eta = \pm\left[\frac{2(\rho+1+\sigma)}{-\beta\sigma}\right]^{1/2} \int_{\sqrt{\delta}}^{y} \left[w\left(\tilde{y}^2 + \frac{1}{\tilde{y}^2} - 2\right) + \left(1 - \frac{1}{\tilde{y}^2}\right) - \frac{\rho+1}{\rho}\ln\left(\frac{1+\rho\tilde{y}^2}{1+\rho}\right)\right]^{-1/2} \mathrm{d}\tilde{y}, \quad (20)$$

where

$$w = \frac{1}{1-\delta}\left[1 + \frac{\delta}{1-\delta}\frac{\rho+1}{\rho}\ln\left(\frac{1+\rho\delta}{1+\rho}\right)\right]. \tag{21}$$

From Eq.(20) it can be shown that the intensity of full width at half maximum (FWHM) of the grey spatial soliton in two-photon photorefractive materials can be expressed as

$$W = \left[\frac{2(\rho+1+\sigma)}{-\beta\sigma}\right]^{1/2} \int_{\sqrt{\delta}}^{\sqrt{(1+\delta)/2}} \left[w\left(\tilde{y}^2 + \frac{1}{\tilde{y}^2} - 2\right) + \left(1 - \frac{1}{\tilde{y}^2}\right) - \frac{\rho+1}{\rho}\ln\left(\frac{1+\rho\tilde{y}^2}{1+\rho}\right)\right]^{-1/2} \mathrm{d}\tilde{y}. \tag{22}$$

To illustrate our results, we take the following parameters[31,32,36] $n_{\mathrm{e}} = 2.2$, $r_{33} = 30 \times 10^{-12}\,\mathrm{mV^{-1}}$, $s_1 = 1.06 \times 10^{-6}\,\mathrm{m^2}$, $s_2 = 1.06 \times 10^{-6}\,\mathrm{m^2}$, $\beta_1 = 0.05$, $\beta_2 = 0.05$, $\gamma_1 = 3.3 \times 10^{-17}\,\mathrm{m^3{\cdot}s^{-1}}$, and $N_{\mathrm{A}} = 10^{22}\,\mathrm{m^{-3}}$. Other parameters are $\lambda_0 = 0.5\,\mu\mathrm{m}$, $x_0 = 10\,\mu\mathrm{m}$, $\rho = 10$, and the value of the electric field adopted here is $E_0 = -2 \times 10^6\,\mathrm{V{\cdot}m^{-1}}$, (i.e. $\beta = -11.13$). The intensity profile and phase distribution of the grey solitons are shown in Fig.1 based on the above mentioned parameters.

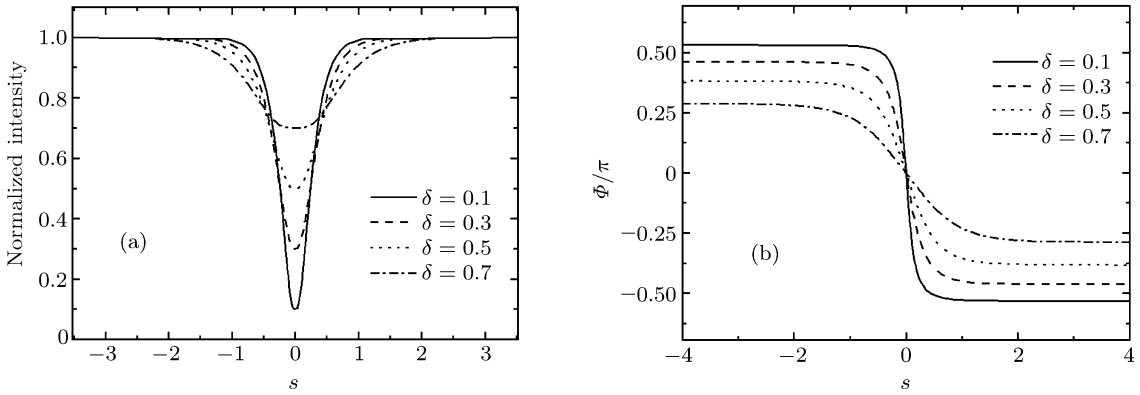


Fig.1. (a) Normalized intensity profiles of grey solitons in two-photon photorefractive medium for various values of the grayness parameter δ when $\beta = -11.13$, $\rho = 10$; (b) corresponding phase profiles of these grey solitons.

According to Eqs.(15) and (17), we can get the dimensionless transverse velocity of the grey soliton

$$v = \left\{\frac{2\delta\beta\sigma}{(1-\delta)(\rho+1+\sigma)}\left[1 + \frac{\rho+1}{(1-\delta)\rho}\ln\left(\frac{1+\rho\delta}{1+\rho}\right)\right]\right\}^{1/2}. \tag{23}$$

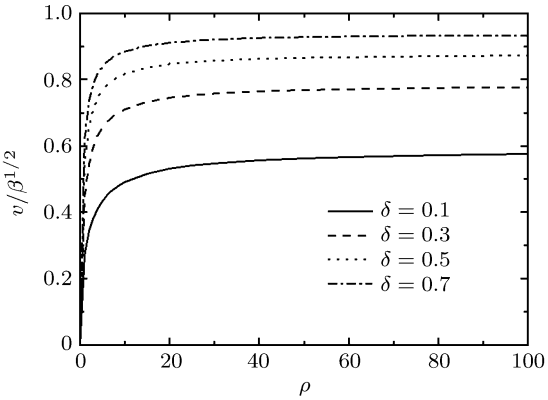


Fig.2. Transverse velocity v of a grey soliton in two-photon photorefractive medium as a function of ρ and δ.

The dependence of the dimensionless transverse velocity versus ρ for various values of the grayness δ is shown in Fig.2.

The angle θ between the propagation direction of the grey spatial soliton in two-photon photorefractive medium and the z-axis can be expressed as

$$\mathrm{tg}\theta = \frac{x}{z} = \frac{v}{kx_0} = \frac{1}{kx_0}\left\{\frac{2\delta\beta\sigma}{(1-\delta)(\rho+1+\sigma)}\left[1+\frac{\rho+1}{(1-\delta)\rho}\ln\left(\frac{1+\rho\delta}{1+\rho}\right)\right]\right\}^{1/2}$$
$$= n_{\mathrm{e}}(r_{33}E_0)^{1/2}\left\{\frac{2\delta\beta\sigma}{(1-\delta)(\rho+1+\sigma)}\left[1+\frac{\rho+1}{(1-\delta)\rho}\ln\left(\frac{1+\rho\delta}{1+\rho}\right)\right]\right\}^{1/2}. \quad (24)$$

4. Conclusions

In conclusion, the evolution equation of one-dimensional spatial solitons in the photorefractive media with two-photon photorefractive effect is presented by using Castro-Camus model, which predicts that grey spatial solitons are possible in two-photon photorefractive media in the steady state and under appropriate external bias conditions. The intensity profile, phase distribution, and transverse velocity of these grey solitons are presented. It is shown that the grey spatial solitons can be formed in two-photon photorefractive medium only when the external bias field is applied in the contrary direction with respect to the optical c axis of the medium (that is, $E_0 < 0$). It is found that the FWHM of these grey spatial solitons is inversely proportional to the square root of the absolute value of the external bias field.

The two-photon photorefractive grey spatial solitons have similar fundamental properties to the previously investigated photorefractive grey spatial solitons. They all require a bias field (or photovoltaic field) in opposition to the optical c axis of the medium, and their FWHM is inversely proportional to the square root of the absolute value of the bias field (or photovoltaic field). The greatest difference between these grey solitons is that the two-photon photorefractive grey solitons rely on two-photon photorefractive effect to set up space charge field. The two-photon photorefractive grey solitons require a gating beam (with a different wave length to the signal beam) to produce a quantity of excited electrons from the valence band of the material. Without the gating beam, the signal beam cannot evolve into a spatial soliton. By adjusting the gating beam, one can control the formation of the two-photon photorefractive grey solitons. Moreover, the waveguides induced by single-photon photorefractive solitons may be partly erased by the reading beams, whereas the two-photon photorefractive solitons have the advantage of inducing permanent waveguide with no erasure problem.

Finally, we think that other aspects of the two-photon photorefractive solitons, which include the stability of these solitons, the coupling of these solitons, etc., also deserve further study. We want to investigate these issues in the future.

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