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Interactions of adjacent pulsating, erupting and creeping solitons*

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This paper investigates the adjacent interactions of three novel solitons for the quintic complex Ginzburg–Landau equation, which are plain pulsating, erupting and creeping solitons. It is found that different performances are presented for different solitons due to isolated regions of the parameter space where they exist. For example, plain pulsating and erupting solitons exhibit mutual annihilation during collisions with the decrease of total energy, but for creeping soliton, the two adjacent pulses present soliton fusion without any loss of energy. Otherwise, the method for restraining the interactions is also found and it can suppress interactions between these two adjacent pulses effectively.

Keywords: pulsating soliton, erupting soliton, creeping soliton, adjacent interaction
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It is well known that the complex Ginzburg–Landau (CGL) equation is a generic model with both dissipation and dispersion, which can describe a variety of phenomena, such as open flow motions, travelling waves in binary fluid mixtures, and spatially extended nonequilibrium systems.[1−4] In optics, the CGL equation and its modifications are often used to model the propagation of optical pulse in optical transmission systems and laser system with the various nonlinear gains, and have been extensively studied during past years.[5−12]

Recently, three novel solutions, being pulsating, erupting and creeping solitons, have been reported by Soto-Crespo *et al.*[13,14] Based on the three new types of solitons, the effects of nonlinear gradient terms and third-order dispersion on these solitons have also been analysed.[15−17] The results show that even small nonlinear gradient terms or third-order dispersion could dramatically change the behaviour of these solitons.

Also, because the bit rate of communication system depends mostly on the repetitive frequency of the signal pulse, and the separation between two adjacent pulses decreases with the increase of the pulse repetitive frequency, therefore small separation between two adjacent pulses leads to the strong interaction,[18−21] and causes the pulse distortion, which is disadvantageous to the information undistorted transmission. So the interactions between two adjacent solitons should not be ignored in the communication systems with high bit rates.

In this paper, we study the interactions between adjacent plain pulsating, erupting or creeping solitons, which are three novel solutions of the quintic CGL equation. We find that plain pulsating and erupting solitons exhibit mutual annihilation during collisions with the decrease of total energy, but for creeping soliton, the two adjacent pulses present soliton fusion[22,23] without any loss of energy. Otherwise, the method for restraining the interactions is also found and it can suppress interactions between these adjacent pulses effectively. For the plain pulsating soliton, the pulses keep the former characteristics after the adjacent interaction is suppressed. But for erupting and creeping solitons, the pulses do not resume the original figures after the adjacent interaction is suppressed and they present different performances respectively.

The quintic CGL equation can be written as follows:

$$U_z = A_0 U + A_1 U_{tt} + A_2 |U|^2 U + A_3 |U|^4 U, \qquad (1)$$

where U is a slowly varying complex amplitude and A_j (j = 0,1,2,3) are the system parameters of the equation, in which A_0 is real, A_1, A_2 and A_3 are complex constants, i.e., $A_j = A_{jr} + \mathrm{i}A_{ji}$ (j= 1,2,3). The physical meaning of each variable depends on the particular problem. In optics, t is the retarded time, z is

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the propagation distance. The parameter A_0 represents the linear gain (or loss). The real part $A_{1\mathrm{r}}$ of A_1 describes the effect of spectral limitation due to bandwidth-limited amplification, and the imaginary part $A_{1\mathrm{i}}$ of A_1 is the group-delay dispersion coefficient and $A_{1\mathrm{i}} > 0$ corresponds to anomalous dispersion. The real part $A_{2\mathrm{r}}$ of A_2 accounts for nonlinear gain/absorption processes, and the imaginary part $A_{2\mathrm{i}}$ of A_2 is equal to one. The real part $A_{3\mathrm{r}}$ of A_3 represents a higher-order correction to the nonlinear amplification/absorption, namely, the saturation to the nonlinear gain or loss, and the imaginary part $A_{3\mathrm{i}}$ of A_3 is a parabolic correction term to the nonlinear refractive index.

In the following, we will investigate numerically the interactions between adjacent pulsating, erupting and creeping solitons respectively. The initial condition is given by

$$U(t,0) = r\mathrm{sech}(t-q_0)\exp(\mathrm{i}\Omega_1 t) + \mathrm{sech}(t+q_0)\exp(-\mathrm{i}\Omega_2 t), \quad (2)$$

where the r and $2q_0$ account for the ratio of amplitude and the initial separation between adjacent solitons respectively. Ω_1, Ω_2 are the initial phase of the adjacent pulses respectively.

At first we consider the interaction between adjacent pulsating solitons. To understand the interaction between the adjacent pulses, we take $r = 1$ and $\Omega_1 = \Omega_2 = 0$ in Eq.(2), while the parameters of Eq.(1) are fixed as those in Refs.[13,14], $A_0 = -0.1$, $A_{1\mathrm{r}} = 0.08$, $A_{1\mathrm{i}} = 0.5$, $A_{2\mathrm{r}} = 0.66$, $A_{3\mathrm{r}} = -0.1$, and $A_{3\mathrm{i}} = -0.1$. From Fig.1(a), we can see that there is strong interplay between the adjacent pulses when the distance of them is equal to 9. As mentioned in Refs.[24,25], mutual annihilation is exhibited during the collision happened between them. A lot of numerical simulations show that there exits strong attraction between adjacent pulsating solitons and exhibit mutual annihilation during the collision[24,25] when the initial separation is equal to or less than 9. Figures 1(b) and 1(c) present the evolution of the total energy and the pulse centre of different values of q_0. The pulsating solitons resume the originally periodicity and the stationary centre position after the annihilation occurred, which can be seen clearly from Figs.1(b) and 1(c).

Because of the deformation of the pulse caused by adjacent interactions, serious false codes or the missing of the information will occur during the communications. So the interactions are not favourable factors for the transmission of the pulses. Therefore it is important to search for methods to maintain the soliton separation and lack of distortion. To this end, three methods have been proposed:[24] (i) phase alteration between neighbouring solitons, (ii) unequal amplitudes between neighbouring solitons and (iii) inclusion of third-order dispersion. Of these, the second method seems to offer more flexibility and is easier to implement than the other two. But it does not work here, and we choose the first method presented above, which can suppress the interaction of two adjacent breathing solitons effectively.[25] For the pulsating soliton, when the initial separation of the two solitons is 9 and the initial phases are $\Omega_1 = \Omega_2 = 0.018$, the interactions between neighbouring solitons are suppressed effectively, which can be seen clearly from Fig.1(d). But there exists obvious repulsion between them at the first several periods and then keep stable without any interaction. A lot of numerical simulations show that the mutual annihilation can be suppressed effectively for $q_0 = 4.5$ when the initial phase is larger than 0.017, as shown in Fig.1(e) the pulse centre for different values of Ω_1 and Ω_2, which will offer us a larger choosing range of parameters in experiments to control the initial condition.

Next, we consider the interaction between adjacent erupting solitons. The initial pulse is still given by Eq.(2) with $r = 1$, $\Omega_1 = \Omega_2 = 0$, while other parameters adopted are as follows: $A_0 = -0.1$, $A_{1\mathrm{r}} = 0.125$, $A_{1\mathrm{i}} = 0.5$, $A_{2\mathrm{r}} = 1.0$, $A_{3\mathrm{r}} = -0.1$, and $A_{3\mathrm{i}} = -0.6$. Due to the randomness of the erupting soliton itself,[13,14] the interaction is more complicated than that of the pulsating soliton. A lot of numerical calculations show that obvious centre excursion occurred when the transmission distance is longer than 100, whether the step is 0.01 or less, as shown in Fig.2(a). Therefore, for simplifying the calculation and showing the interaction between two adjacent erupting solitons clearly, we take the transmission distance as 100 and the step as 0.001 for all the calculation. Figures 2(b) and 2(c) show the 3D evolution graph and the corresponding centre position for $q_0 = 10$. From Figs.2(b) and 2(c), we can see that mutual annihilation occurred when the transmission distance is larger than 22 and then resume the single pulse figure without the increase of energy. This is similar to the interactions of the pulsating soliton. So we adopt the same method as the pulsating soliton to suppress the interaction between two adjacent erupt-

ing solitons and the results are shown in Figs.2(d) and 2(e) for $q_0 = 10$ and $\Omega_1 = \Omega_2 = 2.2$. The interaction is suppressed effectively, which can be seen clearly from graphs and weak repulsion between adjacent erupting solitons occurred due to the randomness of the erupting soliton itself.

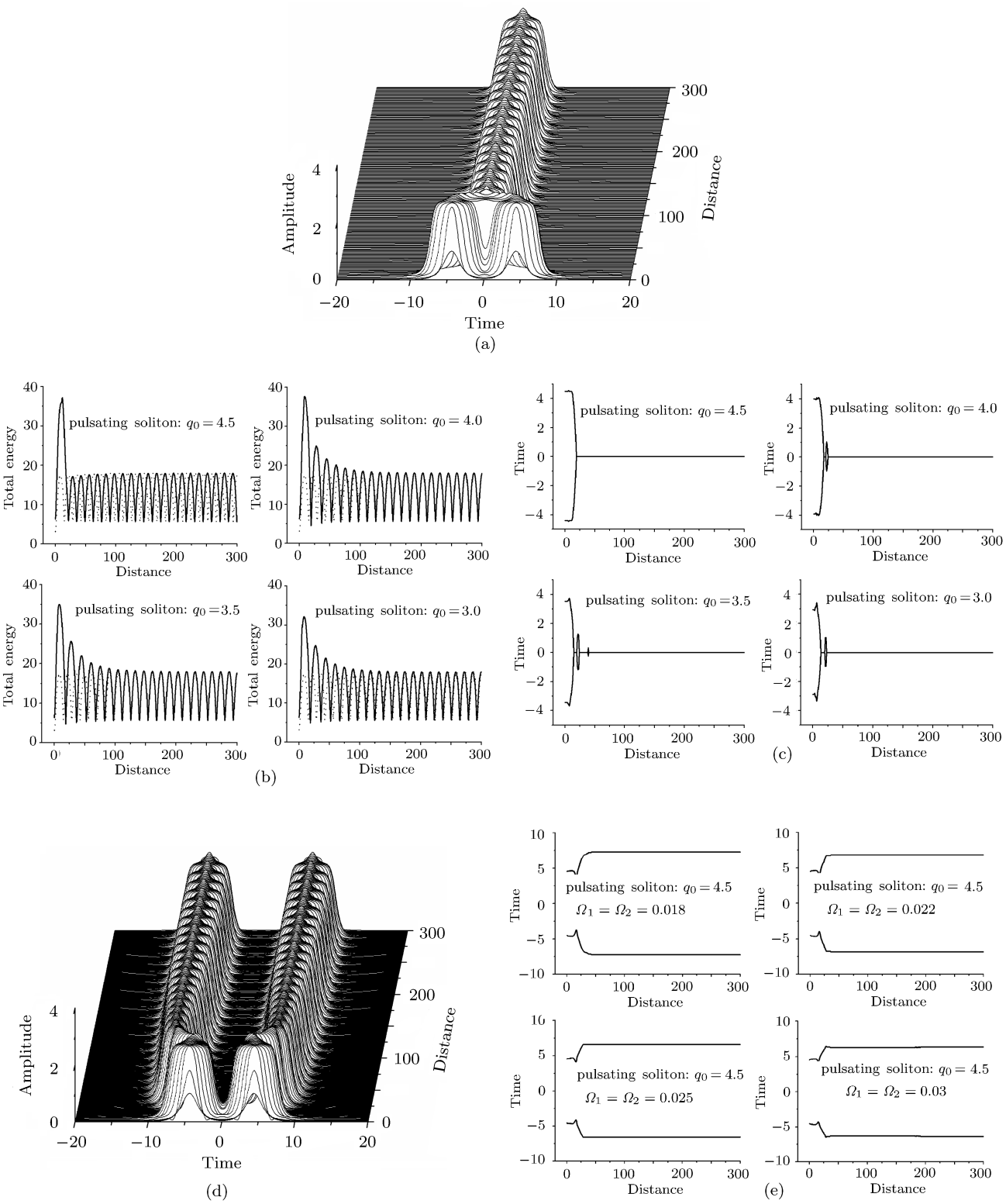


Fig.1. The interaction between the adjacent plain pulsating solitons: (a) the space–time plot when $q_0 = 4.5$, $\Omega_1 = \Omega_2 = 0$; (b) the pulse energy for $q_0 = 4.5, 4.0, 3.5, 3.0$ respectively; (c) the corresponding centre position of (b); (d) the space–time plot when $q_0 = 4.5$, $\Omega_1 = \Omega_2 = 0.018$; and (e) the space–time plot when $q_0 = 4.5$, $\Omega_1 = \Omega_2 = 0.018, 0.022, 0.025, 0.03$ respectively.

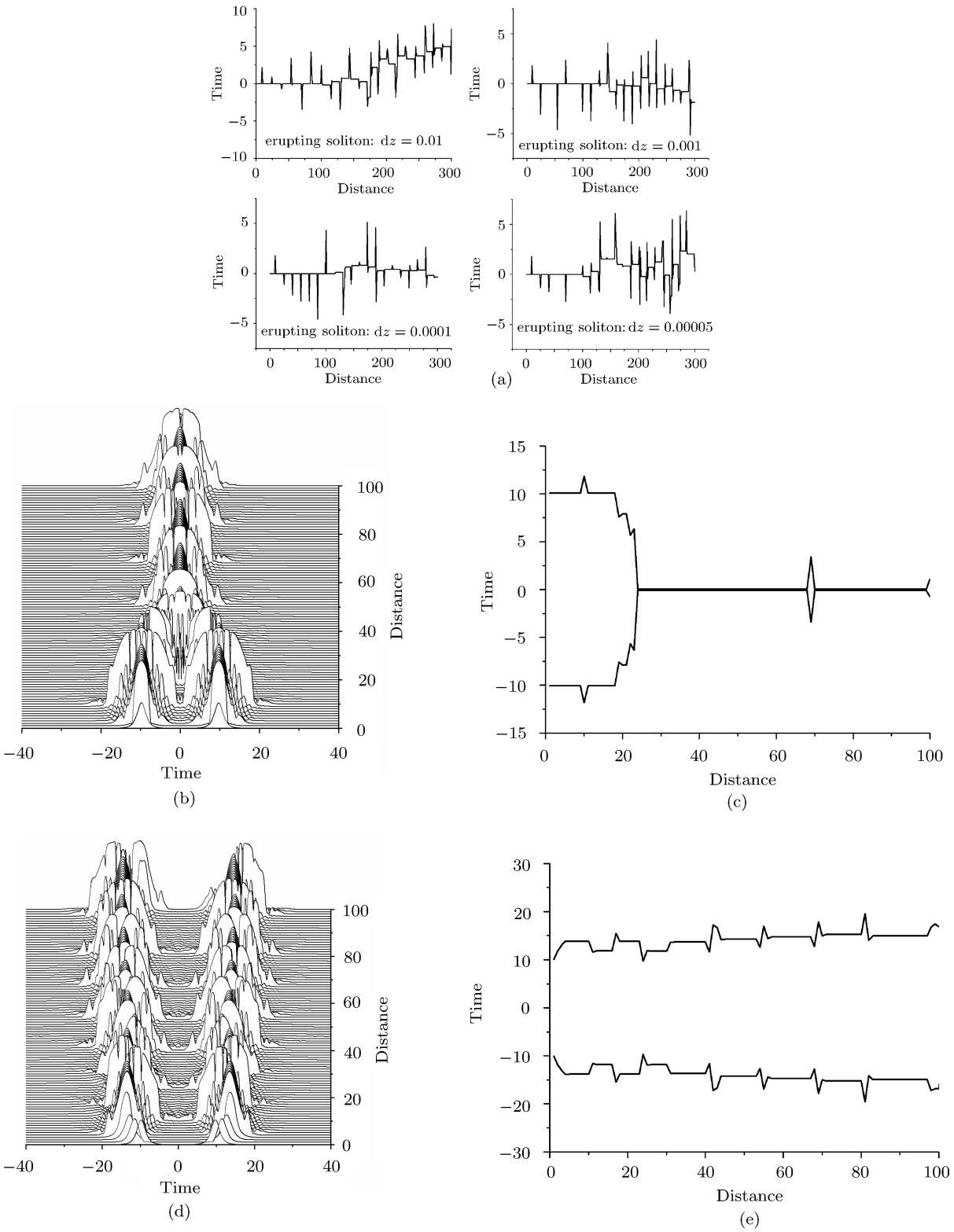


Fig.2. The interaction between two adjacent erupting solitons: (a) the centre position for $\mathrm{d}z = 0.01, 0.001, 0.0001, 0.00005$ respectively; (b) the space–time plot when $q_0 = 10$, $\Omega_1 = \Omega_2 = 0$; (c) the corresponding centre position of (b); (d) the space–time plot when $q_0 = 10$, $\Omega_1 = \Omega_2 = 2.2$; (e) the corresponding centre position of (d).

Finally, we study the interaction of two adjacent creeping solitons. The same initial pulses as Eq.(2) are introduced, in which $r = 1$, $\Omega_1 = \Omega_2 = 0$, and other parameters adopted are $A_0 = -0.1$, $A_{1\mathrm{r}} = 0.101$, $A_{1\mathrm{i}} = 0.5$, $A_{2\mathrm{r}} = 1.3$, $A_{3\mathrm{r}} = -0.3$, and $A_{3\mathrm{i}} = -0.101$. A lot of calculations show that the adjacent creeping solitons present strong repulsion when the initial separation is equal to or larger than 26, as shown in Fig.3(a), the top right corner inset for $q_0 = 13$ the centre position. When the initial separation is smaller than 26, the two adjacent pulses present soliton fusion after a distance of transmission, as shown in Fig.3(a) the top left corner inset and Fig.3(b) for $q_0 = 12$. From the graphs, we can see that the interaction of creeping solitons is different from that of the above two solitons, for which the mutual annihilation is happened. The total energy is not increased after the pulses interaction is come into being. But it is also harmful to the information transmission since the pulses' shape and size are not preserved during collisions, just like solitons. The pulse width after the interaction happened is obviously wider than that of a single pulse before interaction. So the interaction of two adjacent creeping solitons happens with the increase of total energy. To suppress the interaction, the same method is adopted. The results show that, when $q_0 = 12$ and $\Omega_1 = \Omega_2 = 0.023$, the interaction can be suppressed effectively, as shown in Fig.3(a) the bottom left corner inset and Fig.3(c). But the pulse periods are very different from that before the interaction is suppressed. And the pulse width remains almost stable during the transmission. This may be useful in optical communication or other application. But the two pulses present repulsion after the interaction is suppressed for a litter larger values of Ω_1, Ω_2, as shown in Fig.3(a) bottom right corner inset for $\Omega_1 = \Omega_2 = 0.024$.

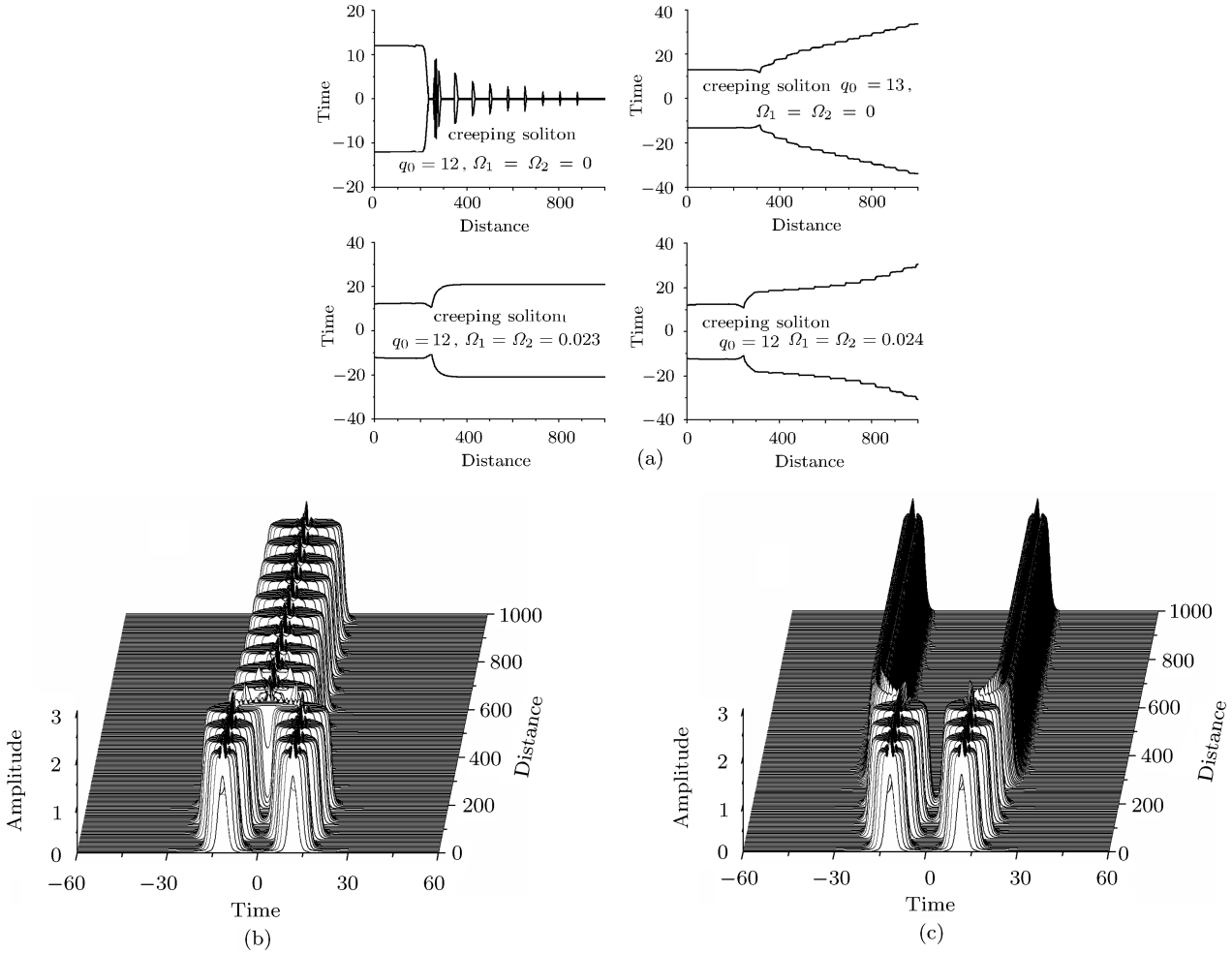


Fig.3. The interaction between two adjacent creeping solitons: (a) the centre position for different q_0 and Ω_1, Ω_2; (b) the space–time plot when $q_0 = 12$, $\Omega_1 = \Omega_2 = 0$; (c) the space–time plot when $q_0 = 12$, $\Omega_1 = \Omega_2 = 0.023$.

In conclusion, we have studied the adjacent interactions and their suppressions of plain pulsating soliton, erupting soliton and creeping soliton, which are the three novel solutions of the quintic CGL equation. The simulated calculated results showed that different behaviours are presented for different solitons due to the relative independent parameters conditions for them respectively. So the suppression effects are also different despite the same method is adopted. For the pulsating and erupting solitons, the pulses can keep the initial properties after the interactions are suppressed. But the repulsion is presented during the transmission for the erupting soliton. For the creeping soliton, the interaction regularity is more complex than the other two, and so the pulses width almost keeps stable after the interaction being suppressed. This may be useful in optical communication or other applications. The results showed above are obtained under the corresponding fixed parameter conditions for different solitons respectively. For the cases of other parameters within the solution existing ranges and whether there is any regularities for the whole ranges are under investigation.

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