Vol 16 No 1, January 2007
1009-1963/2007/16(01)/0137-06

Chinese Physics

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Focusing properties of Gaussian Schell-model beams by an astigmatic aperture lens*

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(Received 23 March 2006; revised manuscript received 24 June 2006)

This paper studies the focusing properties of Gaussian Schell-model (GSM) beams by an astigmatic aperture lens. It is shown that the axial irradiance distribution, the maximum axial irradiance and its position of focused GSM beams by an astigmatic aperture lens depend upon the astigmatism of the lens, the coherence of partially coherent light, the truncation parameter of the aperture and Fresnel number. The numerical calculation results are given to illustrate how these parameters affect the focusing property.

Keywords: Gaussian Schell-model beam, focusing properties, astigmatism, aperture lens

PACC: 4255, 4280, 4260K

1. Introduction

Laser beam focusing is encountered in many applications, and has been widely investigated.[1] In the past several years, focusing properties of partially coherent beams have attracted much attention because of unique characteristics.[2−12] However, most of the research works have been performed on the assumption that the focusing lens has no aberration; few works have been concerned with the influence of spherical aberration on the focusing properties.[6,7] It has been demonstrated that the spherical aberration of the lens influences the focusing properties of partially coherent beams: i.e., the focusing properties of partially coherent beams may take place appreciable changes comparing with aberration-free case. In a practical case, a simple lens has aberration, especially phase aberrations. This paper is aimed at studying the focusing properties of Gaussian Schell-model (GSM) beams by an astigmatic aperture lens. The influence of coherence of partially coherent light, truncation parameter of the aperture, astigmatism of the lens and Fresnel number on the axial irradiance distribution of the focused field is studied. In Section 2, based on the propagation law of partially coherent beams, a theoretical model for GSM beams passing through the aperture lens is presented, where the effect of the astigmatism of the lens is considered. Detailed numerical calculations and analyses are given in Section 3. Finally, a brief summary of the main results concludes this paper.

2. Theoretical model

Consider GSM beams focused by an astigmatic aperture lens shown in Fig.1, which is located at the $z = 0$ plane.



Fig.1. Schematic illustration of GSM beams focused by an astigmatic aperture lens.

The cross-spectral density function of GSM beams in front of the aperture lens is expressed as [13]

*Project supported by the Natural Science Foundation of Henan Province, China (Grant No 0611054000).
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$$W(x_1', y_1', x_2', y_2', 0) = I_0 \exp\left[-\left(\frac{x_1'^2 + x_2'^2}{w_0^2} + \frac{y_1'^2 + y_2'^2}{w_0^2}\right)\right] \exp\left\{-\left[\frac{(x_1' - x_2')^2}{2\sigma_0^2} + \frac{(y_1' - y_2')^2}{2\sigma_0^2}\right]\right\}, \tag{1}$$

where w_0 and σ_0 denote the waist width and spatial correlation length of GSM beams respectively, I_0 is the irradiance constant,$(x_1', y_1'), (x_2', y_2')$ are transversal coordinates of two points at the $z = 0$ plane.

According to the propagation law of partially coherent light,[13,14] the irradiance of GSM beams through a paraxial optical $ABCD$ system is given by

$$\begin{aligned} I(x, y, z) &= \left(\frac{k}{2\pi B}\right)^2 \int_{-a}^{a}\int_{-a}^{a}\int_{-a}^{a}\int_{-a}^{a} W(x_1', y_1', x_2', y_2', 0) \exp\{\mathrm{i}kC_6[(x_1'^2 - x_2'^2) - (y_1'^2 - y_2'^2)]\} \\ &\quad \times \exp\left\{-\frac{\mathrm{i}k}{2B}[A((x_1'^2 + y_1'^2) - (x_2'^2 + y_2'^2)) - 2x(x_1' - x_2') - 2y(y_1' - y_2')]\right\} \mathrm{d}x_1'\mathrm{d}y_1'\mathrm{d}x_2'\mathrm{d}y_2', \end{aligned} \tag{2}$$

where k is the wave number, (x, y) denote transversal coordinates of a point at the z plane, and the transfer $ABCD$ matrix is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - z/f & z \\ -1/f & 1 \end{pmatrix}, \tag{3}$$

where f is the focal length of the lens. In Eq.(2) the astigmatism of the lens is considered and is characterized by a term $\exp[-\mathrm{i}kC_6(x'^2 - y'^2)]$,[15] where C_6 is the astigmatic coefficient, but other phase aberration can be neglected. The aperture is a square with full width $2a$ and the integral is performed with respect to the aperture. In the following, only the axial $(x = y = 0)$ irradiance is considered.

Substituting Eqs.(1) and (3) into (2), we get the axial irradiance

$$I(0, \Delta z) = I_0 \left(\frac{N}{1 + \Delta z}\right)^2 M_1(0, \Delta z) M_2(0, \Delta z), \tag{4}$$

where

$$M_1(0, \Delta z) = \frac{1}{2}\sqrt{\frac{\pi}{Q_2}} \int_{-\delta}^{\delta} \exp\left(Q_1 u^2 - \frac{u^2}{4Q_2\alpha^4}\right) \left(\mathrm{erf}\left[\mathrm{i}\frac{u - 2\delta Q_2\alpha^2}{2\sqrt{Q_2}\alpha^2}\right] - \mathrm{erf}\left[\mathrm{i}\frac{u + 2\delta Q_2\alpha^2}{2\sqrt{Q_2}\alpha^2}\right]\right) \mathrm{d}u, \tag{5}$$

$$M_2(0, \Delta z) = \frac{1}{2}\sqrt{\frac{\pi}{Q_4}} \int_{-\delta}^{\delta} \exp\left(Q_3 \upsilon^2 - \frac{\upsilon^2}{4Q_4\alpha^4}\right) \left(-\mathrm{erf}\left[\mathrm{i}\frac{\upsilon - 2\delta Q_4\alpha^2}{2\sqrt{Q_4}\alpha^2}\right] + \mathrm{erf}\left[\mathrm{i}\frac{\upsilon + 2\delta Q_4\alpha^2}{2\sqrt{Q_4}\alpha^2}\right]\right) \mathrm{d}\upsilon, \tag{6}$$

$$N = \frac{w_0^2}{\lambda f} \text{(Fresnel number)}, \tag{7}$$

$$\delta = \frac{a}{w_0} \text{(truncation parameter)}, \tag{8}$$

$$\alpha = \frac{\sigma_0}{w_0} \text{(relative coherent length)}, \tag{9}$$

$$u = \frac{x'}{w_0} \text{(relative transversal coordinate in the } x \text{ direction)}, \tag{10}$$

$$\nu = \frac{y'}{w_0} \text{(relative transversal coordinate in the } y \text{ direction)}, \tag{11}$$

$$\Delta z = \frac{(z - f)}{f} \text{(relative propagation distance)}, \tag{12}$$

$$Q_1 = -1 - \frac{1}{2\alpha^2} + \mathrm{i}\left(2\pi N f C_6 + \frac{\pi N \Delta z}{1+\Delta z}\right), \quad (13a)$$

$$Q_2 = -1 - \frac{1}{2\alpha^2} + \mathrm{i}\left(-2\pi N f C_6 - \frac{\pi N \Delta z}{1+\Delta z}\right), \quad (13b)$$

$$Q_3 = -1 - \frac{1}{2\alpha^2} + \mathrm{i}\left(-2\pi N f C_6 + \frac{\pi N \Delta z}{1+\Delta z}\right), \quad (13c)$$

$$Q_4 = -1 - \frac{1}{2\alpha^2} + \mathrm{i}\left(2\pi N f C_6 - \frac{\pi N \Delta z}{1+\Delta z}\right), \quad (13d)$$

erf is the error function. Equation (4) indicates that the axial irradiance depends on the spatial correlation parameter α, astigmatism coefficient C_6, truncation parameter δ, Fresnel number N, and relative propagation distance Δz. Equation (4) does not have an analytical solution. To show how the coherence of partially coherent light, the astigmatism of the lens, truncation parameter of the aperture and Fresnel number N affect the axial irradiance in the focused field, we perform the numerical calculations of Eq.(4).

3. Numerical results and discussions

Numerical calculations were performed by using Eq.(4) to illustrate focusing properties of GSM beams focused by an astigmatic aperture lens. Some typical examples are shown in Figs.2–6. Figure 2 gives the axial irradiance distribution $I(0, \Delta z)$ of GSM beams focused by an astigmatic aperture lens for different values of Fig.2(a) $\alpha = 0.5$ and $\delta = 0.6$; Fig.2(b) $\delta = 0.6$ and $C_6 = 0.5 \times 10^{-3}\,\mathrm{mm}^{-1}$; Fig.2(c) $\alpha = 0.5$ and $C_6 = 0.5 \times 10^{-3}\,\mathrm{mm}^{-1}$. The other calculation parameters are chosen as $N = 2$ and $f = 300\,\mathrm{mm}$. As can be seen, the axial irradiance distribution of focused GSM beams show unsymmetrical behaviour with respect to the geometrical focal plane, the maximum axial irradiance $I_{\max}$ changes and the position of maximum axial irradiance $\Delta z_{\max}$ moves toward the lens, which depends on C_6, α and δ.

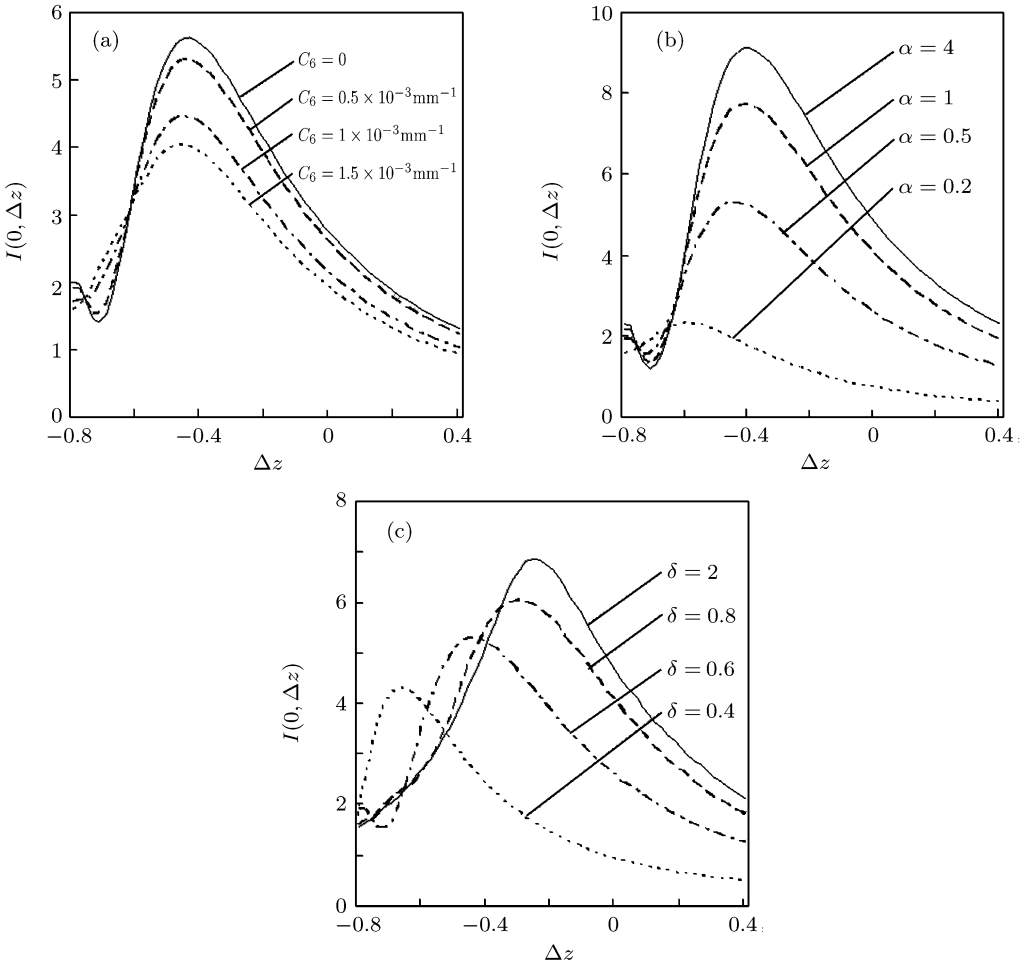


Fig.2. The axial irradiance distribution $I(0, \Delta z)$ of GSM beams focused by an astigmatic aperture lens: (a) $\alpha = 0.5$ and $\delta = 0.6$; (b) $\delta = 0.6$ and $C_6 = 0.5 \times 10^{-3}\mathrm{mm}^{-1}$; (c) $\alpha = 0.5$ and $C_6 = 0.5 \times 10^{-3}\mathrm{mm}^{-1}$. The irradiance $I(0, \Delta z)$ is normalized to I_0.

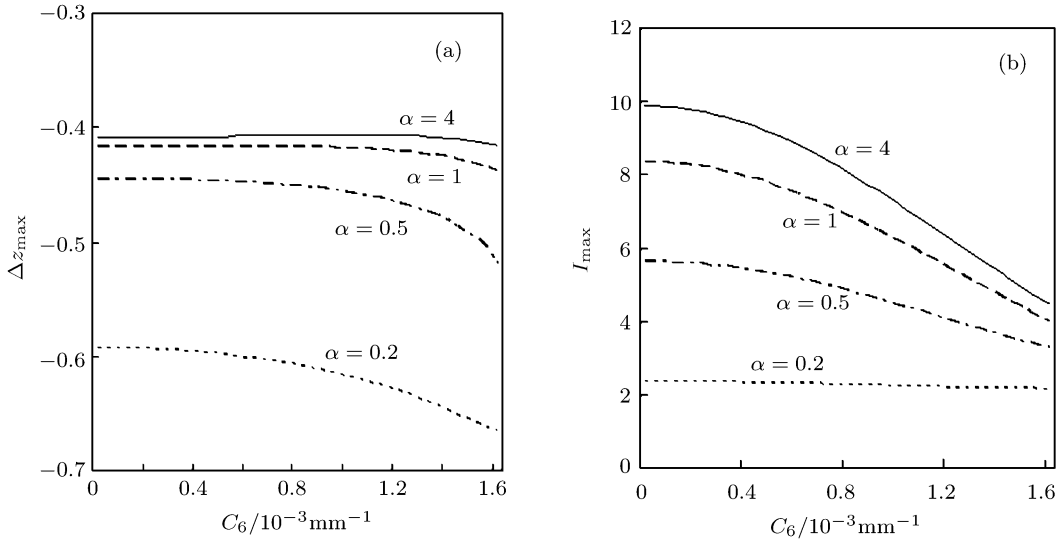


Fig.3. (a) The position of maximum axial irradiance $\Delta z_{\max}$ and (b) the maximum axial irradiance $I_{\max}$ of GSM beams as a function of the astigmatism coefficient C_6. The irradiance is normalized to I_0.

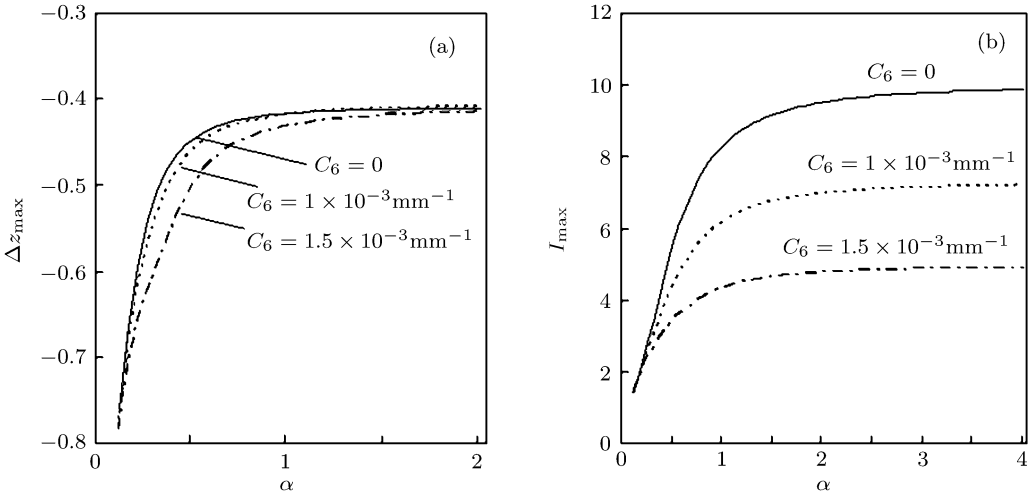


Fig.4. (a) The position of maximum axial irradiance $\Delta z_{\max}$ and (b) the maximum axial irradiance $I_{\max}$ of GSM beams as a function of the coherence parameter α. The irradiance is normalized to I_0.

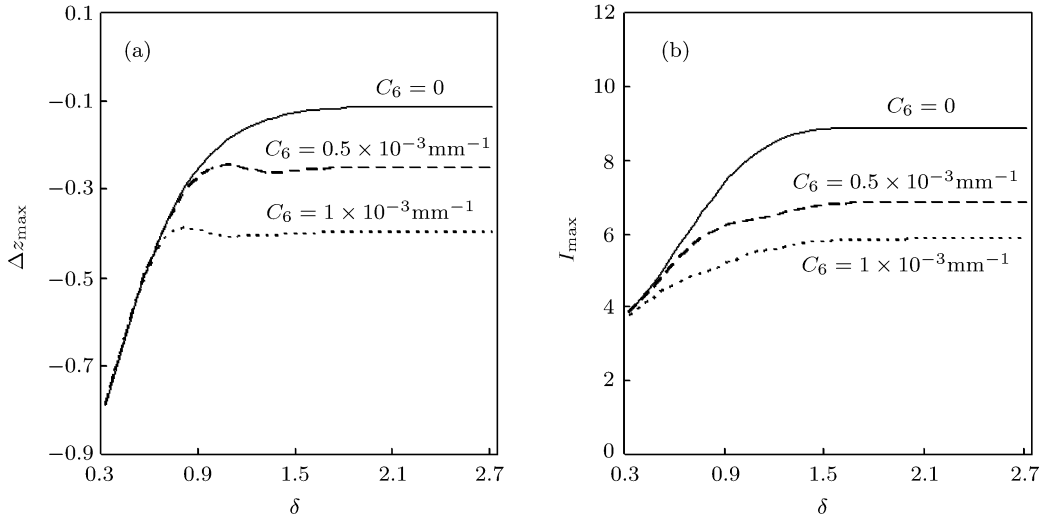


Fig.5. (a) The position of maximum axial irradiance $\Delta z_{\max}$ and (b) the maximum axial irradiance $I_{\max}$ of GSM beams as a function of the truncation parameter δ. The irradiance is normalized to I_0.

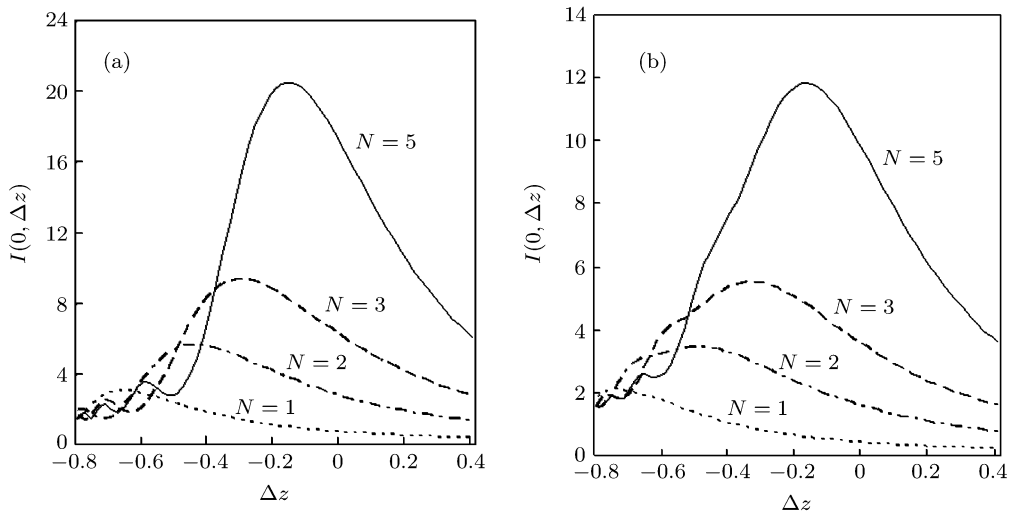


Fig.6. The axial irradiance distribution $I(0, \Delta z)$ of GSM beams focused by an astigmatic aperture lens: (a) $C_6 = 0$; (b) $C_6 = 1.5 \times 10^{-3}\text{mm}^{-1}$. The irradiance $I(0, \Delta z)$ is normalized to I_0.

A qualitative description of the dependence of the position of maximum axial irradiance $\Delta z_{\max}$ and maximum axial irradiance $I_{\max}$ is compiled in Figs.3–5. In Fig.3, we plot the position of maximum axial irradiance $\Delta z_{\max}$ and maximum axial irradiance $I_{\max}$ as a function of the astigmatism coefficient C_6 for different values of the spatial correlation parameter $\alpha = 0.2, 0.5, 1$ and 4. The other calculation parameters are chosen as $N = 2, f = 300\,\text{mm}$ and $\delta = 0.6$. From Fig.3(a), it is shown that as C_6 increases, $\Delta z_{\max}$ moves toward the lens. From $C_6 = 0$ to $0.8 \times 10^{-3}\,\text{mm}^{-1}$, $\Delta z_{\max}$ moves toward the lens gradually, then as C_6 continues to increase, $\Delta z_{\max}$ moves toward the lens noticeably. From Fig.3(b), we find that when C_6 varies, for lower coherence case $\alpha = 0.2$, the smaller variation of the maximum axial irradiance $I_{\max}$ with C_6 is found. However, for the large coherence case $\alpha = 0.5, 1$ and 4, the change of C_6 results in the remarkable change of the maximum axial irradiance $I_{\max}$.

In Fig.4, we give the position of maximum axial irradiance $\Delta z_{\max}$ and maximum axial irradiance $I_{\max}$ as a function of the coherence parameter α for different values of the astigmatism coefficient $C_6 = 0, 1.0 \times 10^{-3}\,\text{mm}^{-1}$ and $1.5 \times 10^{-3}\,\text{mm}^{-1}$. The other calculation parameters are the same as those in Fig.3. It is found from Fig.4(a) that as the coherence decreases, $\Delta z_{\max}$ moves toward the lens. However, when the coherence increases to $\alpha = 1$ and continues to increase, $\Delta z_{\max}$ changes gradually. For the case of the spatially full coherence, the position of maximum axial irradiance $\Delta z_{\max}$ approaches to a certain value -0.41 with different $C_6 = 0, 1.0 \times 10^{-3}\,\text{mm}^{-1}, 1.5 \times 10^{-3}\,\text{mm}^{-1}$. From Fig.4(b), we find that the maximum axial irradiance $I_{\max}$ increases with increasing coherence α.

In Fig.5, we plot the position of maximum axial irradiance $\Delta z_{\max}$ and maximum axial irradiance $I_{\max}$ as functions of the truncation parameter δ for different values of the astigmatism coefficient $C_6 = 0, 0.5 \times 10^{-3}\,\text{mm}^{-1}$, $1.0 \times 10^{-3}\,\text{mm}^{-1}$. The other calculation parameters are chosen as $N = 2, f = 300\,\text{mm}$ and $\alpha = 0.5$. It is shown that, as α decreases, $\Delta z_{\max}$ moves toward the lens and the maximum axial irradiance $I_{\max}$ decreases. Moreover, our calculations show that as δ increases, and the aperture effect can be neglected, $\Delta z_{\max}$ and $I_{\max}$ both become nearly uniform. For example, for the case of $C_6 = 0, 0.5 \times 10^{-3}\,\text{mm}^{-1}, 1.0 \times 10^{-3}\,\text{mm}^{-1}$ in Fig.5(a), $\Delta z_{\max} = -0.11, -0.25, -0.40$, respectively.

The axial irradiance distribution of GSM beams for different values of the Fresnel number $N = 1 (f = 600\,\text{mm})$, $2 (f = 300\,\text{mm})$, $3 (f = 200\,\text{mm})$ and $5 (f = 120\,\text{mm})$ is plotted in Fig.6(a) $C_6 = 0$ and Fig.6(b) $C_6 = 1.5 \times 10^{-3}\,\text{mm}^{-1}$, the other calculation parameters are $\delta = 0.6$ and $\alpha = 0.5$. From Fig.6 we see that the position of maximum axial irradiance $\Delta z_{\max}$ moves toward the lens as the Fresnel number N decreases, and the maximum axial irradiance $I_{\max}$ increase with increasing Fresnel number N. It is also found that when the lens is aberration-free $C_6 = 0$, the position also moves toward the lens (see Fig.6(a)), which was found in Refs.[2,7].

4. Conclusions

In this paper the focusing properties of GSM beams by an astigmatic aperture lens have been studied. It has been shown that the axial irradiance distribution becomes unsymmetrical with respect to the geometrical focal plane, so that there exists focal shift for the astigmatic aperture lens system in general. The position of maximum axial irradiance $\Delta z_{\max}$ moves toward the lens with increasing astigmatism coefficient C_6, and with decreasing the coherence α, truncation parameter δ, and Fresnel number N. The maximum axial irradiance $I_{\max}$ increases with increasing the coherence α, truncation parameter δ, and Fresnel number N, and with decreasing the astigmatism coefficient C_6. Focusing properties of GSM beams through an aperture lens without aberration can be treated as special cases of $C_6 = 0$, which show similar behaviour as those in Refs.[2,7]. Our study discusses that the position of maximum axial irradiance towards the lens in the astigmatism case is always closer than that for the aberration-free case, the maximum axial irradiance in the astigmatism case is always smaller than that for the aberration-free case. The results obtained in this paper would be useful for the practical design and use of the aperture lens in focusing and collimating GSM and astigmatic GSM beams.

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