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Stochastic resonance in a gain–noise model of a single-mode laser driven by pump noise and quantum noise with cross-correlation between real and imaginary parts under direct signal modulation*

Chen Li-Mei(陈黎梅)[a)b)†], Cao Li(曹 力)[b)d)], and Wu Da-Jin(吴大进)[c)d)]

[a)] *Department of Physics, Shenzhen University, Shenzhen* 518057, *China*
[b)] *State Key Laboratory of Laser Technology, Huazhong University of Science and Technology, Wuhan* 430074, *China*
[c)] *Department of Physics, Huazhong University of Science and Technology, Wuhan* 430074, *China*
[d)] *China Centre of Advanced Science and Technology (World Laboratory), Beijing* 100080, *China*

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Stochastic resonance (SR) is studied in a gain–noise model of a single-mode laser driven by a coloured pump noise and a quantum noise with cross-correlation between real and imaginary parts under a direct signal modulation. By using a linear approximation method, we find that the SR appears during the variation of signal-to-noise ratio (SNR) separately with the pump noise self-correlation time τ, the noise correlation coefficient between the real part and the imaginary part of the quantum noise $\lambda_{\rm q}$, the attenuation coefficient γ and the deterministic steady-state intensity I_0. In addition, it is found that the SR can be characterized not only by the dependence of SNR on the noise variables of τ and $\lambda_{\rm q}$, but also by the dependence of SNR on the laser system variables of γ and I_0. Thus our investigation extends the characteristic quantity of SR proposed before.

Keywords: stochastic resonance, direct modulation signal, quantum noise with cross-correlation between real and imaginary part, linear approximation method
PACC: 4250, 0540, 4260K

1. Introduction

Since Benzi *et al*[1] first discovered the phenomenon of stochastic resonance (SR) in 1981, it has been widely investigated and confirmed experimentally.[2−6] Traditionally, the dependence of signal-to-noise ratio (SNR) on noise intensity is used to quantify the characteristic quantity of SR.[2,3,5] In 1998, Barzykin*et al*[7] proposed that, in a broad sense, the SR phenomenon could also be described by the dependence of SNR on noise correlation time. However, the noise and its correlation form can greatly influence the SR in a dynamic system. Since Fulinski *et al*[8,9] proposed that two different noises could be cross-correlated under a certain condition, the statistic properties and SR of a single-mode laser system driven by a correlated quantum noise and pump noise have been extensively investigated.[10−12] For a laser system, considering that the real part and the imaginary part of a quantum noise[13,14] originate from a common bath, the two parts may be cross-correlated in some situations. Therefore, Zhou *et al*[15] assumed that there existed cross-correlation between the two parts of the quantum noise. Then Ke *et al*[16] and Zhang *et al*[17] studied the influences on the statistic properties of a single-mode laser driven by the cross-correlation between the two parts of the quantum noise, and found that the transient properties that they obtained and the experimental results were in good agreement with each other. In the process of optical communication, a signal inevitably modulates noises in laser output intensity, which is the so-called modulated noises. In 1992, Dykman *et al*[18] studied the phenomenon of stochastic resonance appearing in an asymmetric bistable model driven by modulated noises. They found that the modulated noises could better describe and reveal stochastic res-

*Project supported by the National Natural Science Foundation of China (Grant No 10275025).
†E-mail: chenlm@szu.edu.cn

onance characteristics, and their results were in good agreement with experimental results. Because of its significance in both theory and practice, the dynamic system driven by modulated noises has attracted much attention. [19−22]

In this paper, we focus on the study of the SR phenomenon in a gain–noise model of a single-mode laser driven by a coloured pump noise and a quantum noise with cross-correlation between its real and imaginary parts under a direct signal modulation. By using a linear approximation method, we find that the SR phenomenon appears during the variation of SNR with both the pump noise self-correlation time τ and the noise correlation coefficient $\lambda_{\rm q}$. This implies that the SR phenomenon can be described not only by the dependence of SNR upon the pump noise correlation time τ as proposed by Barzykin *et al* in Ref.[7], but also by the dependence of SNR upon the noise correlation coefficient $\lambda_{\rm q}$. A deepgoing study shows that the SR also appears in the curves of SNR versus the attenuation coefficient γ and the deterministic steady-state intensity I_0. This reveals that the SR can be characterized not only by the dependence of SNR on the noise variables of τ and $\lambda_{\rm q}$ but also by the dependence of SNR on the laser system variables of γ and I_0. In this way, our investigation extends the characteristic quantity of SR as proposed in Ref.[7].

2. SNR in a gain–noise model of a single-mode laser

The intensity equation in a gain–noise model of a single-mode laser with a direct signal modulation is given by[17,20]

$$\frac{{\rm d}I}{{\rm d}t} = 2\Big(\frac{A_{\rm g}}{1+I} - a_l\Big)I + P(1-|\lambda_{\rm q}|) + \frac{2I}{1+I}P_{\rm R}(t)B\cos(\Omega t) + 2\sqrt{I}\varepsilon_{\rm r}(t), \quad (1)$$

where noises $P_{\rm R}(t)$ and $\varepsilon_{\rm r}(t)$ are assumed as follows:

$$\begin{aligned}
&\langle P_{\rm R}(t)\rangle = \langle \varepsilon_{\rm r}(t)\rangle = 0,\\
&\langle P_{\rm R}(t)P_{\rm R}(s)\rangle = \frac{Q}{2\tau}\exp\Big(-\frac{|\,t-s|}{\tau}\Big),\\
&\langle \varepsilon_{\rm r}(t)\varepsilon_{\rm r}(s)\rangle = P(1+|\lambda_{\rm q}|)\delta(t-s),\ \ (-1\le\lambda_{\rm q}\le 1),\\
&\langle P_{\rm R}(t)\varepsilon_{\rm r}(s)\rangle = \langle \varepsilon_{\rm r}(t)P_{\rm R}(s)\rangle = 0. \quad (2)
\end{aligned}$$

In Eqs.(1) and (2), I is the laser intensity; $A_{\rm g}$ and $a_{\rm l}$ represent the gain and loss coefficients respectively; B and Ω are the amplitude and frequency of the periodic signal respectively; $P_{\rm R}(t)$ is the real part of the pump noise, and $\varepsilon_{\rm r}(t)$ is the quantum noise of phase locking; Q and P are the intensities of the pump noise and quantum noise respectively; τ is the pump noise self-correlation time; $\lambda_{\rm q}$ is the cross-correlation coefficient between the real part and the imaginary part of the quantum noise. The term $P_{\rm R}(t)\ B\cos(\Omega t)$ on the right-hand side in Eq.(1) implies that $P_{\rm R}(t)$ is modulated by the periodic signal directly.

Let

$$I = I_0 + \delta(t), \quad (3)$$

where $I_0 = (A_{\rm g} - a_{\rm l})/a_{\rm l}$ is the deterministic steady-state intensity in which no noise is taken into account, and $\delta(t)$ is a perturbation term. Linearizing Eq.(1) around the deterministic steady-state intensity I_0 yields the linear equation of the laser intensity as follows:

$$\frac{{\rm d}\delta(t)}{{\rm d}t} = -\gamma\delta(t) + P(1-|\lambda_{\rm q}|) + \frac{2I_0}{1+I_0}P_{\rm R}(t)B\cos(\Omega t) + 2\sqrt{I_0}\varepsilon_{\rm r}(t), \quad (4)$$

where the attenuation coefficient $\gamma = 2a_{\rm l}(A_{\rm g}-a_{\rm l})/A_{\rm g}$. Integrating Eq.(4) leads to

$$\begin{aligned}
\delta(t) = {}& \exp(-\gamma t)\frac{P(1-|\lambda_{\rm q}|)}{\gamma}[\exp(\gamma t)-1] + \exp(-\gamma t)\frac{2I_0}{1+I_0}\int_0^t B\cos(\Omega\, u)P_{\rm R}(u)\exp(\gamma\, u){\rm d}u\\
&+ \exp(-\gamma t)2\sqrt{I_0}\int_0^t \varepsilon_{\rm r}(u)\exp(\gamma u){\rm d}u. \quad (5)
\end{aligned}$$

The steady-state mean intensity correlation function is given by[2,23]

$$C(t) = \lim_{t'\to\infty}\overline{\langle I(t'+t)I(t')\rangle}\,, \quad (6)$$

where

$$\overline{\langle I(t'+t)I(t')\rangle} = \frac{\Omega}{2\pi}\int_{t'}^{t'+2\pi/\Omega}\langle I(t'+t)I(t')\rangle{\rm d}t'$$

and

$$\langle I(t'+t)I(t')\rangle = I_0^2 + I_0\langle\delta(t')\rangle + I_0\langle\delta(t'+t)\rangle + \langle\delta(t'+t)\delta(t')\rangle. \tag{7}$$

Combining Eq.(4) with Eq.(5) leads to the intensity correlation function, which holds only in the case of $\gamma \neq 1/\tau$, as follows:

$$\begin{aligned} C(t) = I_0^2 &+ \frac{2\,I_0P(1-|\lambda_{\rm q}|)}{\gamma} + \frac{P^2(1-|\lambda_{\rm q}|)^2}{\gamma^2} \\ &+ \Big[\frac{I_0^2B^2Q}{(1+I_0)^2\,\tau}\Big(\frac{\gamma_1}{(\Omega^2+\gamma_1^2)\gamma} - \frac{\Omega^2+\gamma_1\gamma_1'}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}\Big) - \frac{2\,I_0P(1+|\lambda_{\rm q}|)}{\gamma}\Big]{\rm e}^{-\gamma|t|} \\ &+ \frac{I_0^2B^2Q}{(1+I_0)^2\tau}\frac{1}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}[(\Omega^2+\gamma_1\gamma]_1)\cos(\Omega\,t) + \frac{2\Omega}{\tau}\sin(\Omega\,t)]{\rm e}^{-\tau^{-1}|t|}, \end{aligned} \tag{8}$$

where $\gamma_1 = \gamma + \tau^{-1}$, and $\gamma_1' = \gamma - \tau^{-1}$.

Convert Eq.(8) into the power spectrum $S(\omega)$ by Fourier transform, and we will have

$$S(\omega) = S_1(\omega) + S_2(\omega), \tag{9}$$

and

$$\begin{aligned} S_1(\omega) &= \frac{I_0^2B^2Q}{(1+I_0)^2\tau^2}\frac{\Omega^2+\gamma_1\gamma_1'}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}[\frac{1}{\tau^{-2}+(\Omega+\omega)^2} + \frac{1}{\tau^{-2}+(\Omega-\omega)^2}], \\ S_2(\omega) &= \frac{2I_0^2B^2\gamma}{(1+I_0)^2\,\tau}(\frac{\gamma_1}{(\Omega^2+\gamma_1^2)\gamma} - \frac{\Omega^2+\gamma_1\gamma_1'}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)})\frac{1}{(\omega^2+\gamma^2)} - \frac{4\,I_0P(1+|\lambda_{\rm q}|)}{(\omega^2+\gamma^2)}, \end{aligned}$$

where $S_1(\omega)$ and $S_2(\omega)$ are the power output spectra of signal and noise, respectively.

The SNR is defined as the ratio of the power output of the signal $P_{\rm s}$ to the power output of the broadband noise at $\omega = \Omega$ (only for the spectrum when $\omega > 0$).

Representing SNR with variable R, we have

$$R = \frac{P_{\rm s}}{S_2(\omega=\Omega)}. \tag{10}$$

Inserting $P_{\rm s} = \int_0^\infty S_1(\omega){\rm d}\omega$ and $S_2(\omega=\Omega)$ into Eq.(10), the SNR is obtained as follows:

$$R = \frac{\dfrac{I_0^2B^2\pi}{(1+I_0)^2\tau}\dfrac{\Omega^2+\gamma_1\gamma_1'}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}}{\dfrac{2I_0^2B^2\gamma}{(1+I_0)^2\,\tau}\Big(\dfrac{\gamma_1}{(\Omega^2+\gamma_1^2)\gamma} - \dfrac{\Omega^2+\gamma_1\gamma_1'}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}\Big)\dfrac{1}{(\Omega^2+\gamma^2)} - \dfrac{4\,I_0P(1+|\lambda_{\rm q}|)}{(\Omega^2+\gamma^2)}}. \tag{11}$$

3. Discussion

According to the expression of SNR (11), we discuss SR from two aspects: one is SR during the variation of SNR with the noise variables of τ and $\lambda_{\rm q}$. And the other is SR during the variation of SNR with the system variables of γ and I_0.

3.1. SR in the $R-\tau$ curve and the $R-\lambda_{\rm q}$ curve

According to the expression (11), with the cross-correlation coefficient between the real part and the imaginary part of the quantum noise $\lambda_{\rm q}$, the quantum noise intensity P, and the pump noise intensity Q taken as the parameters, we depict the $R-\tau$ curves in Fig.1. It can be seen obviously from Fig.1 that a peak exists in each of the $R-\tau$ curves, indicating that the SR appears during the variation of SNR with the pump noise self-correlation time τ. Meanwhile, Figs.1(a) and 1(b) also show that with increasing values of $|\lambda_{\rm q}|$ and P, the resonance peaks become lower and flatter, indicating that the smaller the values of $|\lambda_{\rm q}|$ and P are, the stronger the SR is in the $R-\tau$ curves. From Fig.1(c) we find that with increasing Q,

the peaks become higher and sharper, showing that the bigger the value of Q is, the stronger the SR is in the $R-\tau$ curves.

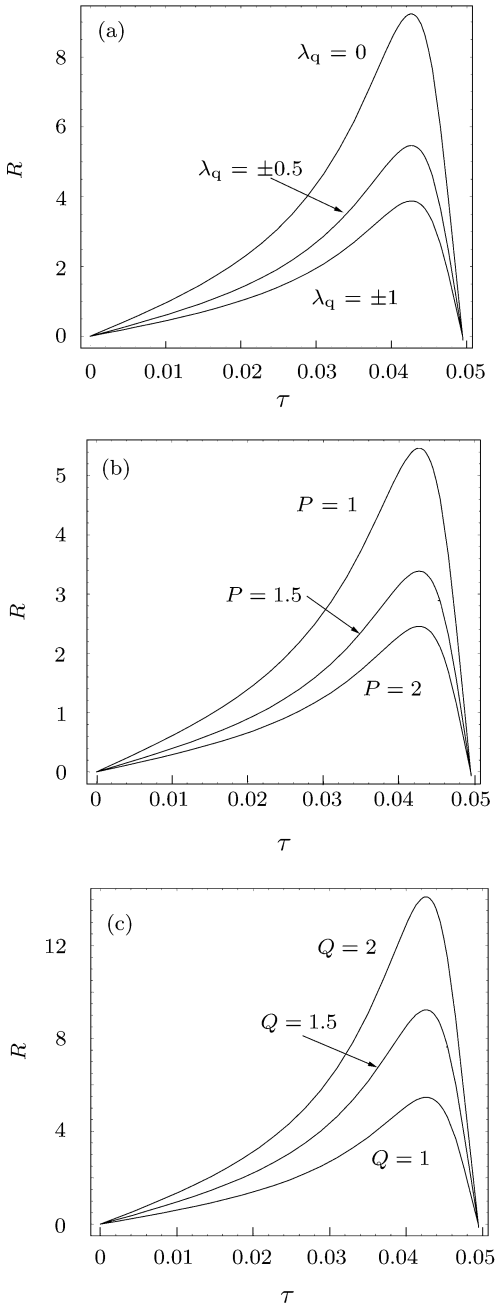


Fig.1. $R-\tau$ curves, with λ_q, P and Q taken as the parameters ($I_0 = 1, \gamma = 20$, $\Omega = 3$, $B = 1$; (a) $P = 1$, $Q = 1$; (b) $\lambda_q = 0.5$, $Q = 1$; (c) $P = 1$, $\lambda_q = 0.5$).

With the quantum noise intensity P, the pump noise intensity Q and the pump noise self-correlation time τ taken as the parameters, the $R-\lambda_q$ curves are plotted from the expression (11) in Fig.2. Figure 2 shows that each of the $R-\lambda_q$ curves exhibits a peak at $\lambda_q = 0$, indicating that the SR also appears during the variation of SNR with the cross-correlation coefficient λ_q. And from Fig.2(a) we see that the peaks become lower and flatter with increasing P, revealing that the smaller the value of P is, the stronger the SR is in the $R-\lambda_q$ curves. And Figs.2(b) and 2(c) show that with increasing Q and τ, the peaks become higher and sharper, displaying that the bigger the values of Q and τ are, the stronger the SR is in the $R-\lambda_q$ curves.

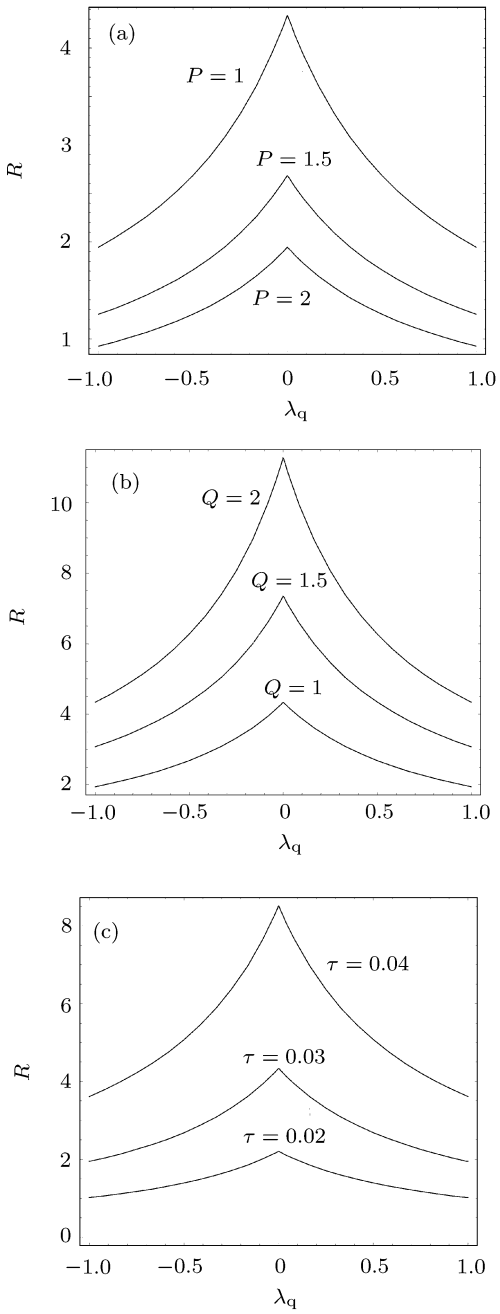


Fig.2. $R-\lambda_q$ curves, with P, Q and τ taken as the parameters ($I_0 = 1$, $\gamma = 20$, $\Omega = 3$, $B = 1$; (a) $\tau = 0.03$, $Q = 1$; (b) $\tau = 0.03$, $P = 1$; (c) $P = 1$, $Q = 1$).

From the above analyses, it is found that the SR phenomenon appears in both the $R-\tau$ curve and the $R-\lambda_{\rm q}$ curve. Thus we can conclude that the SR can be described not only by the dependence of SNR on the pump noise correlation time τ as proposed by Barzykin *et al* in Ref.[7], but also by the dependence of SNR on the noise correlation coefficient $\lambda_{\rm q}$. Hence, in this way, we have extended the characteristic quantity of SR proposed in Ref.[7]. Furthermore, with increasing Q and decreasing P, the SR phenomenon becomes stronger in the $R-\tau$ and $R-\lambda_{\rm q}$ curves.

3.2. SR in the $R-\gamma$ curve and the $R-I_0$ curve

According to expression (11), with the cross-correlation coefficient between the real part and the imaginary part of the quantum noise $\lambda_{\rm q}$, the quantum noise intensity P, the pump noise intensity Q, and the pump noise self-correlation time τ taken as the parameters, we depict the $R-\gamma$ curves in Fig.3 and the $R-I_0$ curves in Fig.4.

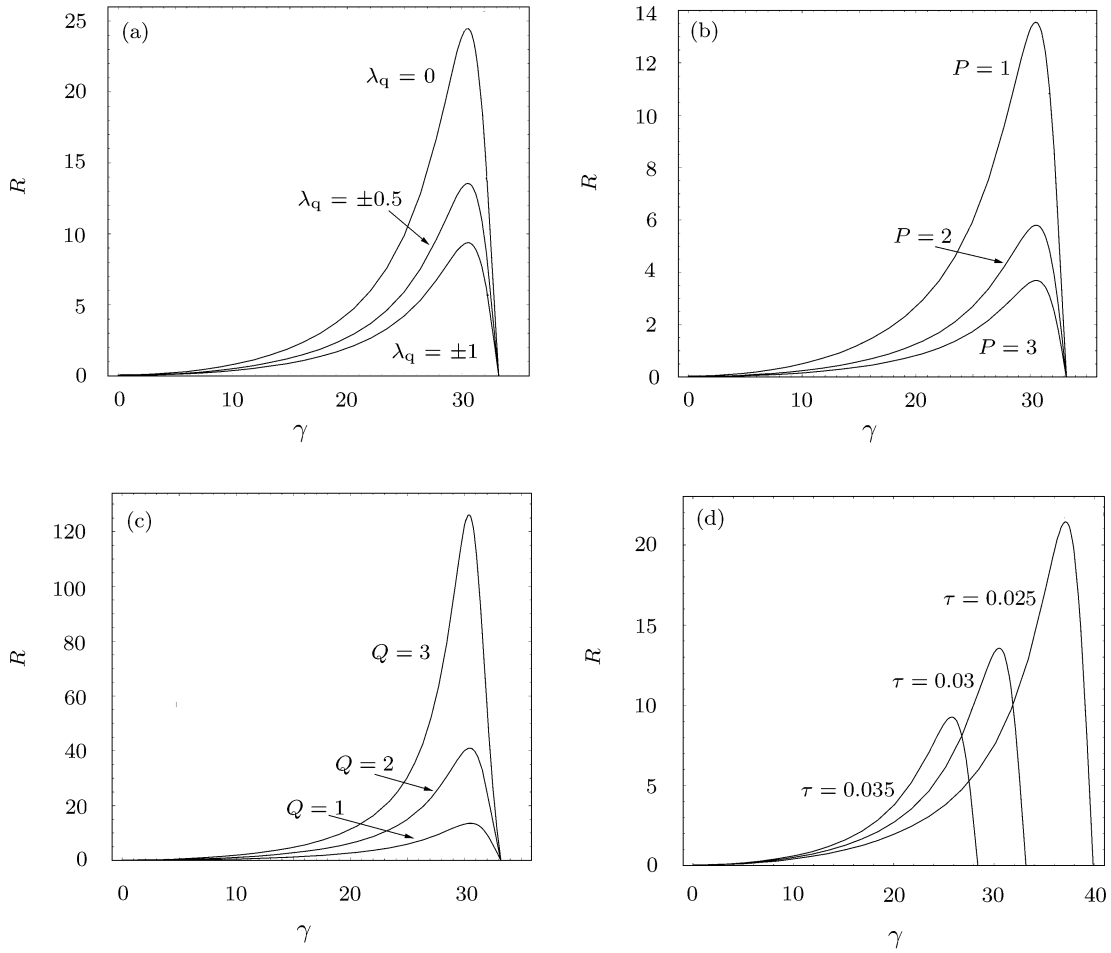


Fig.3. $R-\gamma$ curves with $\lambda_{\rm q}$, P , Q and τ taken as the parameters ($I_0=1$, $\Omega=3$, $B=1$; (a) $\tau=0.03$, $P=1$, Q = 1; (b) $\lambda_{\rm q}=0.5$, $\tau=0.03$, $Q=1$; (c) $\lambda_{\rm q}=0.5$, $\tau=0.03$, $P=1$; (d) $\lambda_{\rm q}=0.5$, $P=1$, $Q=1$).

As shown in Figs.3 and 4, there also exists a peak in each of the $R-\gamma$ and $R-I_0$ curves respectively, indicating that the SR also appears during the variation of SNR with both the attenuation coefficient γ and the deterministic steady-state intensity I_0. From Figs.3(a), 3(b), 4(a) and 4(b) we see obviously that with increasing values of $|\lambda_{\rm q}|$ and P, the resonance peaks become lower and flatter in the $R-\gamma$ and $R-I_0$ curves, showing that the smaller the values of $|\lambda_{\rm q}|$ and P are, the stronger the SR is in these two curves. Fig-

ures 3(c) and 4(c) show that with increasing Q, the peaks become higher and sharper, displaying that the bigger the value of Q is, the stronger the SR is in the $R-\gamma$ and $R-I_0$ curves. And from Figs.3(d) and 4(d) we see that with increasing τ the peaks become lower and flatter and the peak positions move towards the left in the $R-\gamma$ curves, but the peaks become higher and sharper and peak positions do not change in the $R-I_0$ curves, revealing that the bigger the value of τ is, the weaker the SR is in the $R-\gamma$ curves but the stronger the SR is in the $R-I_0$ curves.

From the above analyses, we can conclude that the SR phenomenon appears in both the $R-\gamma$ curve and the $R-I_0$ curve. It means that the SR can be characterized not only by the dependence of SNR on the noise variables of τ and $\lambda_{\rm q}$ but also by the dependence of SNR on laser system variables of γ and I_0. In this way, we have extended the characteristic quantity of SR proposed in Ref.[7] once more. And furthermore, with increasing Q and decreasing $|\lambda_{\rm q}|$ and P, the SR phenomenon becomes stronger in the $R-\gamma$ and $R-I_0$ curves.

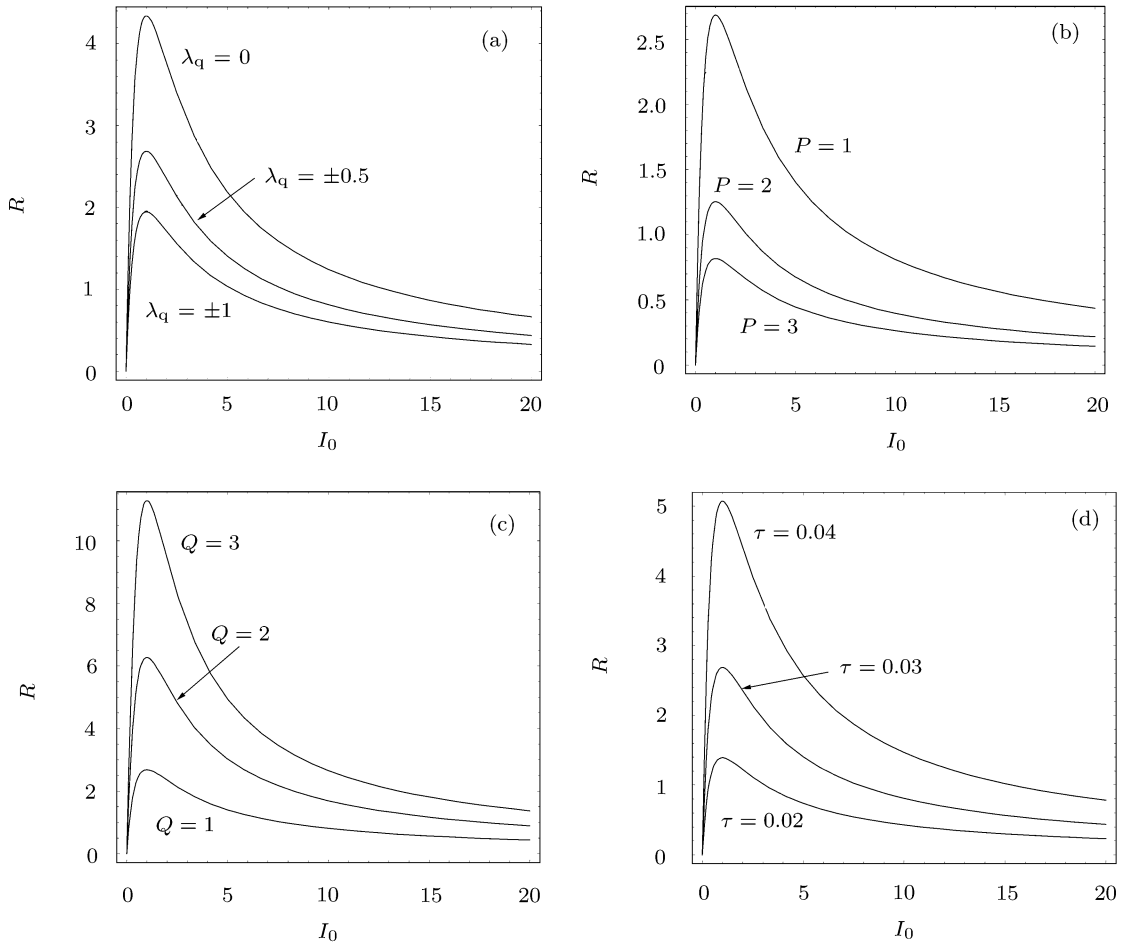


Fig.4. $R-I_0$ curves, with $\lambda_{\rm q}$, P , Q and τ taken as the parameters ($I_0=1$, $\Omega=3$, $B=1$; (a) $\tau=0.03$, $P=1$, $Q=1$; (b) $\lambda_{\rm q}=0.5$, $\tau=0.03$, $Q=1$; (c) $\lambda_{\rm q}=0.5$, $\tau=0.03$, $P=1$; (d) $\lambda_{\rm q}=0.5$, $P=1$, $Q=1$).

4. Conclusions

By introducing the cross-correlation between the real part and the imaginary part of a quantum noise and a direct signal modulation into a gain–noise model of a single-mode laser system, we find that the SR appears in the $R-\tau$, $R-\lambda_{\rm q}$, $R-\gamma$ and $R-I_0$ curves. Therefore it is concluded that the SR can be described not only by the dependence of SNR on the self-correlation time τ as proposed in Ref.[7], but also by the dependence of SNR on the cross-correlation

coefficient λ_q, the attenuation coefficient γ and the deterministic steady-state intensity I_0. And moreover, with increasing Q and decreasing P, the SR phenomenon becomes stronger in the $R-\tau$, $R-\lambda_q$, $R-\gamma$ and $R-I_0$ curves. And with increasing $|\lambda_q|$, the SR becomes weaker in the $R-\tau$, $R-\gamma$ and $R-I_0$ curves. And then with increasing τ, the SR becomes stronger in $R-I_0$ and the $R-\lambda_q$ curves but weaker in $R-\gamma$ curves.

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