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Nonclassical properties in the resonant interaction of a three level ∧-type atom with two-mode field in coherent state

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In this paper, we study the nonclassical properties of the electromagnetic field resulting from the interaction of a three-level ∧-type atom with a two-mode field initially in the coherent state, such as squeezing properties and sub-Poisson statistics. We show that the squeezing can be enhanced by selective atomic measurement.

Keywords: resonant interaction, selective atomic measurement, two-mode squeezing

PACC: 4250

In the past few years, nonclassical properties of quantum states have attracted much attention. It has been shown that light squeezing is possible in the two-level or three-level Jaynes–Cummings models with a coherent field.[1−9] Later, Gerry and Ghosh[10] showed that the squeezing may be greatly enhanced in two-level Jaynes–Cummings model, even for low photon number, by selective atomic measurements. Lately, squeezing enhancement has been found in other models, such as the two-photon Jaynes–Cummings model.[11] In this paper, we show that the squeezing exists in the resonant interaction of a three-level ∧-type atom with a two-mode field initially in the coherent state, and is enhanced after selective atomic measurement. We also study the possible nonclassical properties such as sub-Poisson statistics, violations of the Cauchy–Schwarz inequality before and after selective atomic measurement.

First, we introduce the criterion of judging the existence of nonclassical properties. The quadrature operators for the two modes are defined as

$$d_1 = \frac{1}{2^{\frac{3}{2}}}(a^+ + a + b^+ + b), d_2 = \frac{1}{2^{\frac{3}{2}}\mathrm{i}}(a - a^+ + b - b^+), \tag{1}$$

the variances satisfy the uncertainty relation

$$\left\langle (\triangle d_1)^2 \right\rangle \left\langle (\triangle d_2)^2 \right\rangle \geq \frac{1}{16}. \tag{2}$$

When $\left\langle (\Delta d_i)^2 \right\rangle < \frac{1}{4}$, $i = 1$ or 2, the ith quadrature component is squeezed. We characterize squeezing by the parameter[10]

$$q_i = \frac{\left\langle (\triangle d_i)^2 \right\rangle - 0.25}{0.25}. \tag{3}$$

If there is squeezing in the ith quadrature, $-1 \leq q_i < 0$.

We calculate the Mandel Q-parameters defined for modes a and b as

$$Q_a = \frac{\langle a^+ a a^+ a\rangle - \langle a^+ a\rangle^2 - \langle a^+ a\rangle}{\langle a^+ a\rangle}, \tag{4}$$

$$Q_b = \frac{\langle b^+ b b^+ b\rangle - \langle b^+ b\rangle^2 - \langle b^+ b\rangle}{\langle b^+ b\rangle}. \tag{5}$$

When $-1 \leq Q_i < 0$ $(i = a, b)$, sub-Poisson statistics exists in the ith mode.

We also consider the second-order cross-correlation function defined as

$$Q_{ab} = \frac{\langle a^+ a b^+ b\rangle}{\langle a^+ a\rangle \langle b^+ b\rangle} - 1. \tag{6}$$

The function is zero for uncorrelated states, positive for correlated states, and negative for anticorrelated states. As a test for violations of Cauchy–Schwarz inequality,[12] we follow Agarwal[13] and consider the quantity

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$$I_0 = (\frac{\langle a^+a^+aa\rangle\langle b^+b^+bb\rangle}{\langle a^+ab^+b\rangle^2})^{1/2} - 1, \tag{7}$$

when $I_0 < 0$, the inequality is violated.

We consider a three-level ∧-type atom interacting resonantly with a two-mode cavity field initially in the two-mode coherent state. Let the upper atomic level be denoted by $\mid i\rangle$, and the two lower levels by $\mid g\rangle$ and $\mid e\rangle$, respectively. In the interaction picture, the corresponding Hamiltonian in rotating-wave approximation is[14]

$$H_I = \lambda_1(a \mid i\rangle\langle e \mid + a^+ \mid e\rangle\langle i \mid) + \lambda_2(b \mid i\rangle\langle g \mid + b^+ \mid g\rangle\langle i \mid), \tag{8}$$

where $a^+(a)$ and $b^+(b)$ are the creation (annihilation) operators for the two cavity mode, and λ_1 and λ_2 are the atom-field coupling constants. In what follows, we set $\lambda_1 = \lambda_2 = \lambda$. We assume that the atom is initially on the upper level and the cavity field is initially in a two-mode coherent state, i.e.

$$\mid \psi_{\rm f}(0)\rangle = \sum_{m,n=0}^{\infty} C_{m,n} \mid m,n\rangle, \tag{9}$$

$$C_{m,n} = {\rm e}^{(-\bar{m}-\bar{n})/2}\frac{\alpha^m\beta^n}{\sqrt{m!n!}},$$

where $\bar{m} =\mid \alpha^2 \mid$ is the average photon number in a-mode, and $\bar{n} =\mid \beta^2 \mid$ is the average photon number in b-mode. In this paper, we assume that α (β) is a real value for simplicity. Then the initial state of the system is given by

$$\mid \psi(0)\rangle = \sum_{m,n=0}^{\infty} C_{m,n} \mid i,m,n\rangle. \tag{10}$$

After an interaction time t, the system evolves to the state

$$\mid \psi(t)\rangle = \sum_{m,n=0}^{\infty} C_{m,n}(\cos(\Omega t) \mid i,m,n\rangle - {\rm i}\sin\varphi\sin(\Omega t) \mid e,m+1,n\rangle - {\rm i}\cos\varphi\sin(\Omega t) \mid g,m,n+1\rangle), \tag{11}$$

where $\Omega = \lambda\sqrt{m+n+2}$, $\sin\varphi = \lambda\sqrt{m+1}/\Omega$, $\cos\varphi = \lambda\sqrt{n+1}/\Omega$.

We now consider the second case that the atom is selectively measured. If the atom is found in state $\mid e\rangle$, the field state collapses to

$$\mid \psi_{\rm f}(t)\rangle = \frac{1}{N}\sum_{m,n=0}^{\infty} C_{m,n}\sin\varphi\sin(\Omega t) \mid m+1,n\rangle, \tag{12}$$

where N is a normalization factor.

We then consider the third case that the atom is subjected to a classical resonant microwave field after leaving the cavity, then the atom undergoes the transformations as follows:[15]

$$\mid g\rangle \to \cos\left(\frac{\theta}{2}\right) \mid g\rangle + {\rm i}{\rm e}^{-{\rm i}\Phi}\sin\left(\frac{\theta}{2}\right) \mid e\rangle, \tag{13}$$

$$\mid e\rangle \to \cos\left(\frac{\theta}{2}\right) \mid e\rangle + {\rm i}{\rm e}^{{\rm i}\Phi}\sin\left(\frac{\theta}{2}\right) \mid g\rangle, \tag{14}$$

where θ and Φ can be controlled by modulating amplitude and phase of classical field. Here, we make $\theta = \frac{\pi}{2}$, $\Phi = -\frac{\pi}{2}$. Then the system state is given by

$$\begin{aligned}\mid \psi(t)\rangle = \sum_{m,n=0}^{\infty} C_{m,n}[&\cos(\Omega t) \mid {\rm i},m,n\rangle - \frac{\rm i}{\sqrt{2}}\sin\varphi\sin(\Omega t)(\mid e,m+1,n\rangle \\ &+ \mid g,m+1,n\rangle) - \frac{\rm i}{\sqrt{2}}\cos\varphi\sin(\Omega t)(\mid g,m,n+1\rangle - \mid e,m,n+1\rangle)].\end{aligned} \tag{15}$$

Now the atom is also selectively measured. If the atom is found in state $\mid e\rangle$, the field state collapses to

$$\mid \psi_f(t)\rangle = \frac{1}{N'} \sum_{m,n=0}^{\infty} C_{m,n}(\cos\varphi \sin(\Omega t) \mid m, n+1\rangle - \sin\varphi \sin(\Omega t) \mid m+1, n\rangle), \tag{16}$$

where N' is a normalization factor.

Now we start to discuss the nonclassical properties of the cavity field. Here we consider the fact that the two cavity mode cannot always include the same number of photons in experiment. We plot q_1 versus the scaled interaction time $T = \lambda t$ (q_2 never shows squeezing) for the three states in Eqs.(11), (12) and (16), which are shown in Figs.1–6. We also make a comparison between the cases of less average photon number and larger average photon number. For the same state, when $T \leq 5$, the squeezing for the case $\bar{n} = 5,\ \bar{m} = 8$ is even slightly greater than that for the case $\bar{n} = 20,\ \bar{m} = 23$. But after a longer interaction time, the squeezing for the case $\bar{n} = 5,\ \bar{m} = 8$ (recurs at round $T \sim 18$) is greatly less than that for the case $\bar{n} = 20,\ \bar{m} = 23$ (recurs at round $T \sim 32$). Then for $\bar{n} = 5$, $\bar{m} = 8$, when no state selective measurement is made on the atom, the squeezing can only reach 35%, but when the direct measurement is performed and if the atom is detected in the state $\mid e\rangle$, the maximum squeezing will rise to nearly 55%, being greater than that in the case without measurement. When the microwave field is applied and if the atom is also detected in the $\mid e\rangle$, the maximum squeezing is also nearly 55%, which is not enhanced compared with direct measurement. The similar behaviour occurs for $\bar{n} = 20$, $\bar{m} = 23$, the difference is that the maximum squeezing can reach nearly 75% after selective atomic measurement.

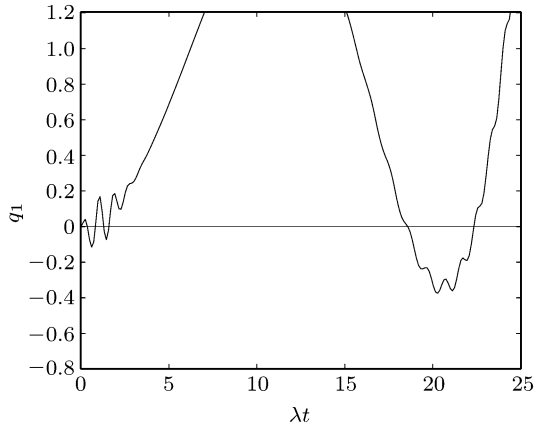


Fig.1. Squeezing for less average photons for the two cavity mode ($\alpha^2 = 5$, $\beta^2 = 8$) when no selective atomic measurement is made.

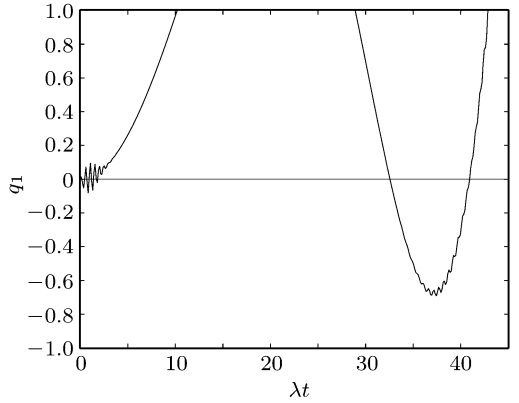


Fig.2. Squeezing for larger average photons for the two cavity mode ($\alpha^2 = 20$, $\beta^2 = 23$) when no selective atomic measurement is made.

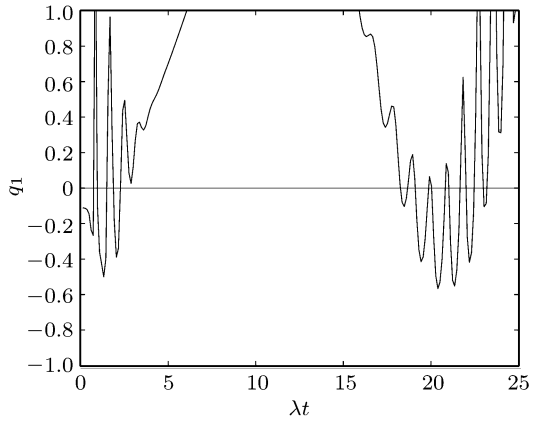


Fig.3. Squeezing for $\alpha^2 = 5$, $\beta^2 = 8$ when direct selective measurement is performed. The atom is detected in state $\mid e\rangle$.

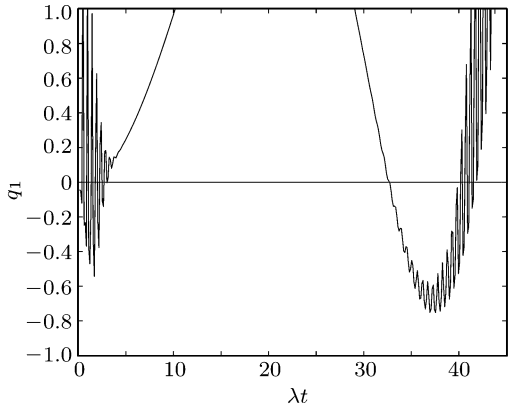


Fig.4. Squeezing for $\alpha^2 = 20$, $\beta^2 = 23$ when direct selective measurement is performed. The atom is detected in state $\mid e\rangle$.

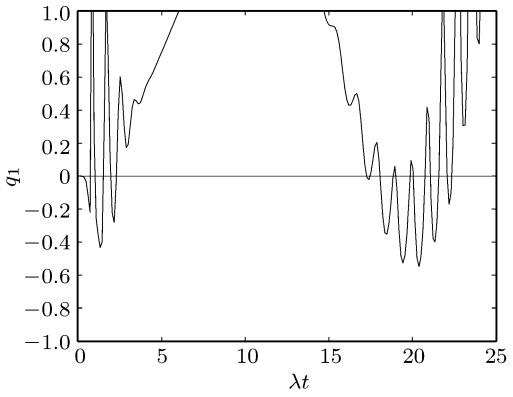


Fig.5. Squeezing for $\alpha^2 = 5$, $\beta^2 = 8$ with the classical field is applied with parameters $\theta = \dfrac{\pi}{2}$ and $\Phi = -\dfrac{\pi}{2}$. The atom is detected in state $\mid e\rangle$.

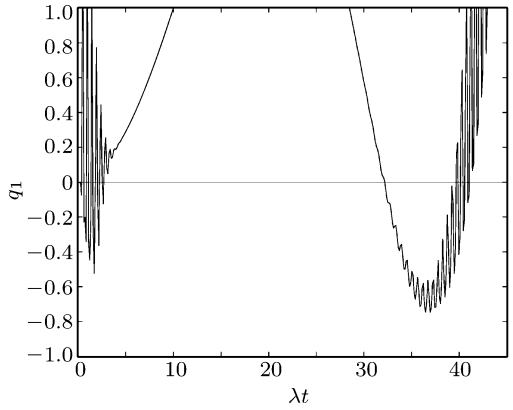


Fig.6. Squeezing for $\alpha^2 = 20$, $\beta^2 = 23$ with the classical field is applied with parameters $\theta = \dfrac{\pi}{2}$ and $\Phi = -\dfrac{\pi}{2}$. The atom is detected in state $\mid e\rangle$.

Figures 7–9 show $Q_{\rm a}(Q_{\rm b})$ versus the scaled interaction time $T = \lambda t$ (here we consider the condition that $0 \leq T \leq 60$). We observe negative values of $Q_{\rm a}$ and $Q_{\rm b}$. Figure 7 shows that $Q_{\rm a}(Q_{\rm b})$ oscillates fast in most of the interaction time in the case that no selective measurement is performed. The oscillations show the features of sub-Poisson statistics. It can be seen that the amplitude of the oscillations decreases to negative in discrete timeslices. Thus in this case sub-Poisson statistics only exists in these timeslices. Figure 8 indicates that $Q_{\rm a} < 0$ in almost all the time, and $Q_{\rm b} < 0$ at some discrete timeslices. In other words, in the case that direct selective measurement is made, sub-Poissonian statistics exists in a-mode nearly all the time, but exists in b-mode for a shorter time. If the classical field is applied to the atom and selective measurement is also made, sub-Poisson statistics never exists, as shown in Fig.9.

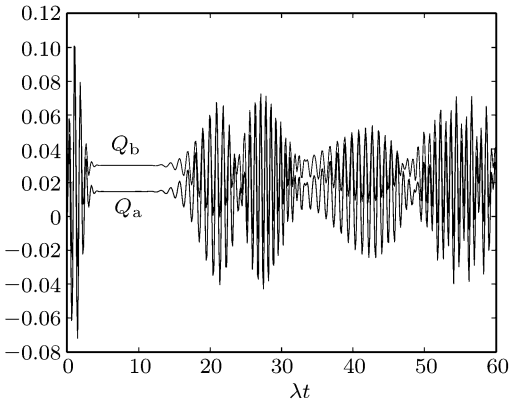


Fig.7. Mandel's Q-parameters for the two cavity mode ($\alpha^2 = 5$, $\beta^2 = 8$), respectively. No selective atomic measurement is made.

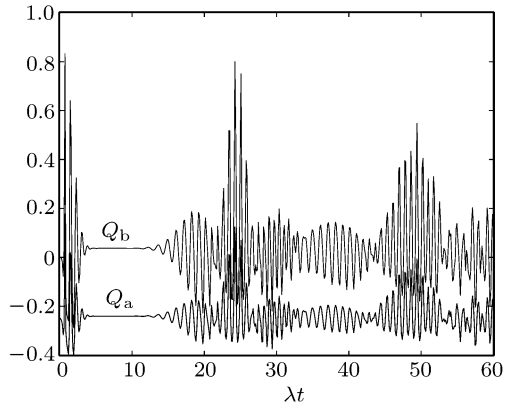


Fig.8. Same as Fig.7, but direct selective atomic measurement is performed. The atom is detected in state $\mid e\rangle$.

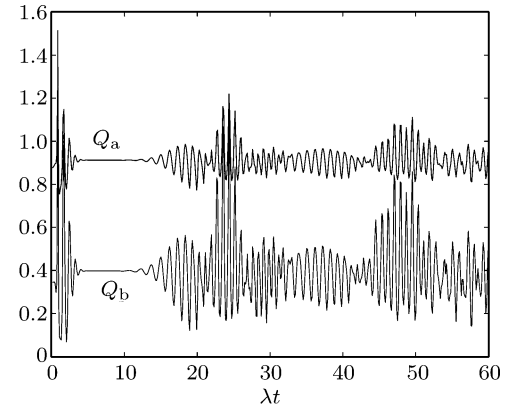


Fig.9. As Fig.7, but the classical field is applied with parameters $\theta = \dfrac{\pi}{2}$ and $\Phi = -\dfrac{\pi}{2}$. The atom is detected in state $\mid e\rangle$.

We also plot $Q_{\rm ab}$ and I_0 versus the interaction time $T = \lambda t$ (here we consider the condition that $0 \leq T \leq 60$). Figures 10 and 12 show $I_0 > 0$ for all t in the cases that no measurement is made and

selective atomic measurement is performed after applying the classical field, it indicates that both the cases do not violate the Cauchy–Schwarz inequality. In Fig.12, $Q_{\rm ab} < 0$ nearly all the time, indicating that the two-mode field always anticorrelates.

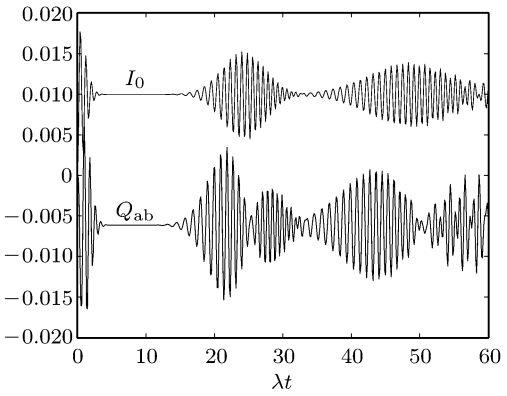


Fig.10. The second-order cross-correlation behaviour and test of violations of the Cauchy–Schwarz inequality for the two cavity mode ($\alpha^2 = 5$, $\beta^2 = 8$) when no selective atomic measurement is made.

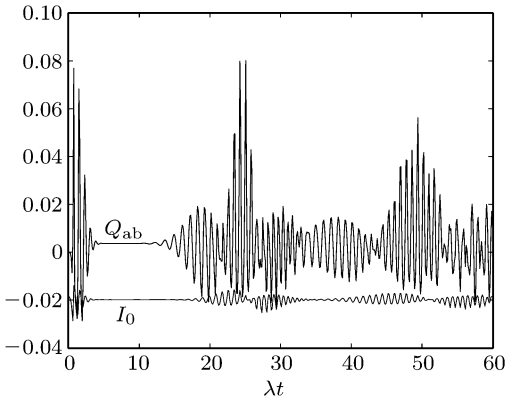


Fig.11. Same as Fig.10, but direct selective atomic measurement is performed. The atom is detected in state $\mid e\rangle$.

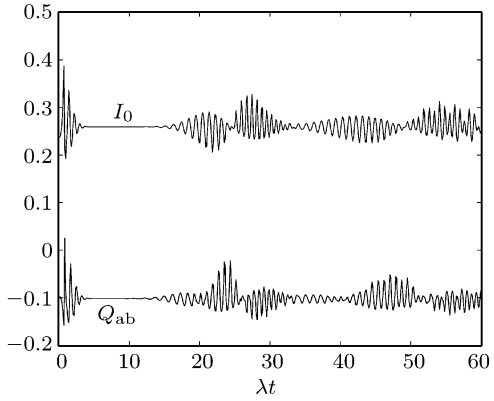


Fig.12. As Fig.10, but the classical field is applied with parameters $\theta = \dfrac{\pi}{2}$ and $\Phi = -\dfrac{\pi}{2}$. The atom is detected in state $\mid e\rangle$.

In Fig.10, $Q_{\rm ab} > 0$ is possible, it indicates that the two-mode cavity field is correlated in some time. In the case that direct measurement is made, as showed in Fig.11, $I_0 < 0$ all the time, and $Q_{\rm ab} > 0$ at many discrete timeslices. It indicates that the cavity field violates the Cauchy–Schwarz inequality, and the possibility of being correlated is greater than that of being anticorrelated.

In summary, we have studied the nonclassical properties existing in the two-mode coherent cavity field resonantly interacting with three-level $\wedge$-type atom. By comparing the different cases of the system state, we find that the squeezing exists in the three field states of Eqs.(11), (12) and (16), respectively. In the case that the atom is directly measured (Eq.(12)) or measured after resonantly interacting with the microwave field (Eq.(16)) the squeezing is greatly enhanced. We also find that for the field after direct atomic measurement the sub-Poisson statistics and violation of the Cauchy–Schwarz inequality are stronger than for the other cases.

References

[1] Meystre P A and Zubairy M S 1982 *Phys. Lett.* A **89** 390

[2] Compagno G, Peng J S and Persico F 1986 *Opt. Commun.* **57** 415

[3] Knight P L 1986 *Phys. Scr.* T **12** 51

[4] Shumovsky A S, Kien F L and Aliskenderov E I 1987 *Phys. Lett.* A **124** 351

[5] Zhu S Y, Lin Z D and Li X S 1988 *Phys. Lett.* A **182** 89

[6] Mahran M H, El-Samman A R A and Obada A S F 1989 *J. Mod. Opt.* **36** 53

[7] Li X S, Lin D L, George T F and Liu Z D 1989 *Phys. Rev.* A **40** 228

[8] Liu Z D, Li X and Lin D L 1991 *Phys. Rev.* A **44** 6144

[9] Zhu A D, Zhang S and Zhao Y F 2005 *Chin. Phys.* **14** 1152

[10] Gerry C C and Ghosh H 1997 *Phys. Lett.* A **229** 17

[11] Ye S Y 2006 *Commun. Theor. Phys.* **45** 147

[12] Lu H and Guo G C 1999 *Acta Phys. Sin.* (Overseas Edition) **8** 577

[13] Agarwal G S 1988 *J. Opt. Soc. Am.* B **5** 1940

[14] Yang Z B 2006 *J. Phys.* B **39** 603

[15] Guo G C and Zheng S B 1996 *Phys. Lett.* A **223** 332