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Quantum dense coding using a peculiar tripartite entangled state*

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Following a recent proposal (*Phys. Lett.* A **346** (2005) 330) about quantum dense coding using a tripartite entangled GHZ state and W state, this paper proposes an experimentally feasible scheme for dense coding in cavity QED by using another peculiar tripartite entangled state. In the scheme the atoms interact simultaneously with a highly detuned cavity mode with the assistance of a classical field, the successful probability of dense coding scheme with this peculiar tripartite entangled state equals 1.

Keywords: quantum dense coding, tripartite entangled state, cavity QED

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1. Introduction

Quantum entanglement has played a key role in recent years as a potential resource for quantum communication and quantum information processing.[1−5] Also multi-particle entanglement, involving several distant parties, has been explored extensively: for instance, teleportation,[6−13] quantum telecloning,[14,15] quantum secret sharing,[16−18] quantum computation,[19] quantum cryptography[20] and so on.

Another important application of multi-particle entanglement is quantum dense coding which can transmit two bits of classical information by sending only one qubit with the help of entanglement while the sender and the receiver share an entangled quantum state.[21] Two-particle quantum dense coding has been demonstrated by using optical systems and nuclear magnetic resonance techniques.[22−24] Some theoretical schemes of quantum dense coding has been proposed by using bipartite or tripartite entanglement.[25−29] As for two different kinds of genuine tripartite entanglement, the GHZ state and W state are proposed to utilize in the scheme for quantum dense coding also.[30−32] In this paper, the main object is to realize quantum dense coding by using a peculiar tripartite entangled state which is a different GHZ state or W state in cavity QED. In our scheme the atoms of the entangled tripartite are shared by three parties. During the passage of the atoms through the cavity field, a strong classical field is present, so the photo-number dependent parts are cancelled. Thus this scheme is insensitive to both the cavity decay and the thermal field.[30−33] Compared with the scheme of quantum dense coding by using the GHZ state or W state, the scheme using this peculiar tripartite entanglement has some distinctive improvement. The scheme can be realized and is controlled by the measurement basis of the third particle.

2. Quantum dense coding with the peculiar tripartite entangled state

In our previous work, we have prepared a peculiar tripartite entangled state via cavity QED as follows:[34−37]

$$|\psi\rangle = \lambda_1 |eeg\rangle_{123} + \lambda_2 |ege\rangle_{123} + \lambda_3 |gee\rangle_{123} + \lambda_4 |ggg\rangle_{123}\,, \quad (1)$$

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where $\sum_i |\lambda_i|^2 = 1$, and the parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ can be chosen independently according to our requirement. This peculiar tripartite entangled state has features of both the GHZ state (i.e. $\tau_{123} > 0$) and the W state (i.e. the remaining reduced density matrices ρ_{ij} retain entanglement according to the positive partial transposition (PPT) criterion) simultaneously. However, its entanglement properties are not completely identical to either the GHZ state or the W state. When $\lambda_1 = \lambda_2 = \frac{1}{2}, \lambda_3 = \lambda_4 = \frac{1}{2}\mathrm{i}$, Eq.(1) becomes

$$\begin{aligned} |\psi\rangle = \ & \frac{1}{2}|eeg\rangle_{123} + \frac{1}{2}|ege\rangle_{123} + \frac{1}{2}\mathrm{i}|gee\rangle_{123} \\ & + \frac{1}{2}\mathrm{i}|ggg\rangle_{123}, \end{aligned} \tag{2}$$

where the three atoms 1, 2 and 3 are held by Alice, Bob and Charlie respectively.

The purpose of this scheme is that the sender (Alice) sends classical information to the receiver (Bob), while the third party (Charlie) supervises and controls the transmission through his measurement. To realize the quantum dense coding, Alice performs one of the four local operations $I, \sigma^x, \mathrm{i}\sigma^y, \sigma^z$ (I is the identity operator and σ^i are three Pauli operators) on her atom (atom 1). The operation denotes 2 bits of classical information, and will transform the state of three atoms into one of the following four states respectively:

$$\begin{aligned} |\psi^1\rangle_{123} = & \left(\frac{1}{2}|ee\rangle_{12} + \frac{1}{2}\mathrm{i}|gg\rangle_{12}\right)|g\rangle_3 \\ & + \left(\frac{1}{2}|eg\rangle_{12} + \frac{1}{2}\mathrm{i}|ge\rangle_{12}\right)|e\rangle_3, \end{aligned} \tag{3a}$$

$$\begin{aligned} |\psi^2\rangle_{123} = & \left(\frac{1}{2}|ge\rangle_{12} + \frac{1}{2}\mathrm{i}|eg\rangle_{12}\right)|g\rangle_3 \\ & + \left(\frac{1}{2}|gg\rangle_{12} + \frac{1}{2}\mathrm{i}|ee\rangle_{12}\right)|e\rangle_3, \end{aligned} \tag{3b}$$

$$\begin{aligned} |\psi^3\rangle_{123} = & \left(-\frac{1}{2}|ge\rangle_{12} + \frac{1}{2}\mathrm{i}|eg\rangle_{12}\right)|g\rangle_3 \\ & + \left(-\frac{1}{2}|gg\rangle_{12} + \frac{1}{2}\mathrm{i}|ee\rangle_{12}\right)|e\rangle_3, \end{aligned} \tag{3c}$$

$$\begin{aligned} |\psi^4\rangle_{123} = & \left(-\frac{1}{2}|ee\rangle_{12} + \frac{1}{2}\mathrm{i}|gg\rangle_{12}\right)|g\rangle_3 \\ & + \left(-\frac{1}{2}|eg\rangle_{12} + \frac{1}{2}\mathrm{i}|ge\rangle_{12}\right)|e\rangle_3. \end{aligned} \tag{3d}$$

Now the information is encoded into the pure entangled state, which is shared among the three parties (Alice, Bob and Charlie), then Alice sends her atom to one of the receivers (i.e. Bob). With the help of Charlie's computation measurement on his atom 3, if the measurement result is $|g\rangle$, the quantum states of atoms 1 and 2 will collapse into

$$|\psi^1\rangle_{12} = \frac{1}{\sqrt{2}}(|ee\rangle_{12} + \mathrm{i}|gg\rangle_{12}), \tag{4a}$$

$$|\psi^2\rangle_{12} = \frac{1}{\sqrt{2}}(|ge\rangle_{12} + \mathrm{i}|eg\rangle_{12}), \tag{4b}$$

$$|\psi^3\rangle_{12} = \frac{1}{\sqrt{2}}(-|ge\rangle_{12} + \mathrm{i}|eg\rangle_{12}), \tag{4c}$$

$$|\psi^4\rangle_{12} = \frac{1}{\sqrt{2}}(-|ee\rangle_{12} + \mathrm{i}|gg\rangle_{12}); \tag{4d}$$

if the measurement result is $|e\rangle$, the quantum states of atoms 1 and 2 will collapse into

$$|\phi^1\rangle_{12} = \frac{1}{\sqrt{2}}(|eg\rangle_{12} + \mathrm{i}|ge\rangle_{12}), \tag{5a}$$

$$|\phi^2\rangle_{12} = \frac{1}{\sqrt{2}}(|gg\rangle_{12} + \mathrm{i}|ee\rangle_{12}), \tag{5b}$$

$$|\phi^3\rangle_{12} = \frac{1}{\sqrt{2}}(-|gg\rangle_{12} + \mathrm{i}|ee\rangle_{12}), \tag{5c}$$

$$|\phi^4\rangle_{12} = \frac{1}{\sqrt{2}}(-|eg\rangle_{12} + \mathrm{i}|ge\rangle_{12}). \tag{5d}$$

Now Bob let atom 1 and 2 be simultaneously in a single mode cavity and at the same time two atoms are driven by a classical field. The interaction between the single-mode cavity and the two driven atoms can be described, in the rotating-wave approximation as follows:[38,39]

$$\begin{aligned} H = \ & \varpi_0 S_z + \varpi_a a^+ a + \sum_{j=1}^{2} \{g(a^+ S_j^- + a S_j^+) \\ & + \Omega[S_j^+ \exp(-\mathrm{i}\varpi\tau) + S_j^- \exp(\mathrm{i}\varpi\tau)]\}, \end{aligned} \tag{6}$$

where a^+ and a are the creation and annihilation operators for the cavity mode, g is the coupling constant between cavity and particle, Ω is the Rabi frequency, ϖ_0, ϖ_a and ϖ are atomic transition frequency, cavity frequency and the frequency of the driven classical field respectively. While the case of $\varpi_0 = \varpi$, the evolution operator of the system, in the interaction picture, is $U(t) = \exp(-\mathrm{i}H_0 t)\exp(-\mathrm{i}H_\mathrm{e} t)$, where $H_0 = \sum_{j=1}^{2} \Omega(S_j^+ + S_j^-)$, H_e is the effective Hamiltonian. In the strong driving regime $\Omega \gg \delta$ and the case $\delta \gg g$, there is no energy exchange between the atomic system and the cavity, thus the scheme is insensitive to both the cavity decay and the thermal field. Then in the interaction picture, the effective interaction Hamiltonian reads:[38,39]

$$\begin{aligned} H_\mathrm{e} = \ & \frac{\lambda}{2}\left[\sum_{j=1}^{2}(|e\rangle_{jj}\langle e| + |g\rangle_{jj}\langle g|) + \sum_{j=1;i\neq j}^{2}(S_j^+ S_i^+ \right. \\ & \left. + S_j^+ S_i^- + \mathrm{H.c.})\right], \end{aligned} \tag{7}$$

where $\lambda = g/2\delta$. If two atoms are simultaneously sent into the cavity, then using the above cavity, we can verify the following evolution:[30−33]

$$|g\rangle|g\rangle \longrightarrow e^{-i\lambda\tau}[\cos(\lambda\tau)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle) - i\sin(\lambda\tau)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle)], \tag{8a}$$

$$|g\rangle|e\rangle \longrightarrow e^{-i\lambda\tau}[\cos(\lambda\tau)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle) - i\sin(\lambda\tau)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle)], \tag{8b}$$

$$|e\rangle|g\rangle \longrightarrow e^{-i\lambda\tau}[\cos(\lambda\tau)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle) - i\sin(\lambda\tau)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle)], \tag{8c}$$

$$|e\rangle|e\rangle \longrightarrow e^{-i\lambda\tau}[\cos(\lambda\tau)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle)(\cos(\Omega\tau)|e\rangle - i\sin(\Omega\tau)|g\rangle) - i\sin(\lambda\tau)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle)(\cos(\Omega\tau)|g\rangle - i\sin(\Omega\tau)|e\rangle)]. \tag{8d}$$

Choosing to adjust the interaction time $\lambda\tau = \pi/4$ and modulate the driving field $\Omega\tau = \pi$, after interaction we can lead the quantum state of the two atom system in Eq.(4) to

$$|\psi^1\rangle_{12} \longrightarrow |e\rangle_1|e\rangle_2, \tag{9a}$$

$$|\psi^2\rangle_{12} \longrightarrow |g\rangle_1|e\rangle_2, \tag{9b}$$

$$|\psi^3\rangle_{12} \longrightarrow i|e\rangle_1|g\rangle_2, \tag{9c}$$

$$|\psi^4\rangle_{12} \longrightarrow i|g\rangle_1|g\rangle_2; \tag{9d}$$

and similarly the other 4 states in Eq.(5) will evolve to:

$$|\phi^1\rangle_{12} \longrightarrow |e\rangle_1|g\rangle_2, \tag{10a}$$

$$|\phi^2\rangle_{12} \longrightarrow |g\rangle_1|g\rangle_2, \tag{10b}$$

$$|\phi^3\rangle_{12} \longrightarrow i|e\rangle_1|e\rangle_2, \tag{10c}$$

$$|\phi^4\rangle_{12} \longrightarrow i|g\rangle_1|e\rangle_2. \tag{10d}$$

Hence, the Bell-state measurement can be achieved by detecting atoms 1 and 2 separately. With the outcome of the measurement on his two atoms, Bob can distinguish Alice's operation on atom 1. That is to say, Bob receives two bits of classical information from Alice while Alice only sends him one qubit and the whole process is controlled by Charlie in a quantum way. So, the goal of dense coding is achieved, and if they cooperate with each other, they can read original information with probability equal to one. That is to say, the role of Charlie is very important. Charlie can control the transmission by sending his measurement result or not to Bob. Only after Bob receives Charlie's classical information (measurement result) can he know the state of his two atoms. Otherwise, if one of them is dishonest, for example Charlie, Bob still knows which type Alice's operation belongs to, $(I, i\sigma^y)$ or (σ^x, σ_z). But he cannot further discriminate which one Alice's operation is. On the other hand, if Bob is dishonest, Charlie will know nothing about Alice's operation. So the one who received Alice's atom has a higher probability of being a successful cheat compared with the party who has not.

3. Conclusion

In conclusion, we have proposed a simple scheme for quantum dense coding via a peculiar tripartite entanglement using the interaction between atoms and cavity field. Our quantum dense coding scheme is realized between two parties by the control of the third one. If and only if when they cooperate with each other can they read Alice's operational information. Any attempt to get complete information without the cooperation of the third party cannot succeed in a deterministic way. In comparison with previous work,[30] our scheme has some distinctive improvement. The successful probability of the dense coding scheme with this peculiar tripartite entangled state equals 1 in comparison with the scheme using W state which is a probabilistic one. On the other hand, in comparison with the scheme using the GHZ state, in our scheme, Charlie does not need a Hadamard operation before he performs the computation measurement on his atom. In the experiment, the Rydberg atoms with principal quantum numbers are 49, 50 and 51, so the radiative time is about $T_r = 3 \times 10^{-2}$s, and the coupling constant is $g = 2\pi \times 24$kHz.[40] The required atom–cavity–field interaction time has the order of $T \approx 10^{-4}$ s. Then the time needed to complete the whole scheme is much shorter than T_r. That is to say, all the operation are within current techniques, so this scheme based on cavity QED techniques might be realizable at present.

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