Vol 15 No 9, September 2006
1009-1963/2006/15(09)/2185-08

Chinese Physics

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Charge coupled devices (CCD) photometry and long-term optical variability of gamma-ray-loud BL Lacertae object Mkn 421*

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(Received 12 January 2006; revised manuscript received 15 May 2006)

Evidence for the long-term optical curve variability and colour behaviour of the BL Lac object Mkn 421 is presented. Our results show that the amplitude of the optical variations of Mkn 421 is only about ΔB=4.7 magnitude in the B-band for its photometric history from 1899 to 2002. The results of optical photometric monitoring of the Mkn 421 from April 2000 to Jan 2002 are provided. During our observation, Mkn 421 shows significant rapid variations and exhibits short time variability of 2.42 hours in the B band. A strong correlation between the $B-V$ colour index and the magnitude in the B band is found. Our charge coupled device (CCD) photometry of Mkn 421 shows that the measured results are in good agreement with the predicted optical variability period of about 23 years.

Keywords: BL Lac object Mkn 421, photometry, mass of central black hole, colour index
PACC: 9870Q, 9760L

1. Introduction

A variable γ-ray source, Mkn 421 at redshift Z=0.0308, is well known to be of the blazar class of active galactic nuclei with the x-ray selected BL Lac object. Mkn 421 is very active at high energies. In 1991, γ-ray emissions from Mkn 421 was detected by using the energetic gamma ray experiment telescope (EGRET), which was the first detection of a BL Lac-type γ-ray-loud source. Its integrated photon flux above 100 MeV was 1.4±0.3 $\times 10^{-9}\mathrm{m}^{-2}\cdot\mathrm{s}^{-1}$, and its differential photon energy spectrum could be represented by a power law with an exponent of –1.96±0.14.[1] Subsequently, Mkn 421 was detected once again as an extragalactic source of TeV γ-ray by the Whipple observatory γ-ray telescope,[2] and has been confirmed to be a TeV γ-ray source by the high energy gamma ray astronomy (HEGRA) collaboration.[3] The spectral energy distribution (SED) shows a double-peaked feature with two maxima.[4] The Whipple observed significant variabilities on a few-hour timescale on 14 and 15 May 1994, during which the average source flux about 250 GeV increased by a factor of 10.[5] An extremely rapid burst of TeV photons from Mkn 421 was obtained, with the flux increasing by a factor of 20–25 on a timescale of about 50±5 min.[6] A similar dramatic outburst in the optical band was obtained by Xie *et al.*[7] According to Xie *et al*,[7] a variability timescale of 2.5 hours can be inferred (on 13 January, 1986 a 1.4-mag increase in the B band was recorded), from which the authors estimated the mass of the Mkn 421 black hole to be about 1.8 $\times 10^7$ $M_\odot$, where $1M_\odot$=1.9889 $\times 10^{30}$ kg. Many observations to search for its variability in optical waveband have been performed. Miller[8] reported the light curve in the B band from 1899 to 1975 for Mkn 421. After examining the data from 1974 to 1982, Gagen–Torn *et al*[9] concluded that Mkn 421 varied in B band by an amplitude of 1.5 mag on characteristic timescales of days to years. Liu *et al*[10] collected all available observed data in B from 22 publications and gave the light curve in the B band from 1899

*Project supported by an CRCG grant of the University of Hong Kong, Yunnan Provincial Foundation of Science of China (Grant No 2004A0027M) and the National Natural Science Foundation of China(Grant No 10363002), and the Natural Science Foundation of China Education Department(Grant No 02142).
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to 1991 for Mkn 421. The light curve is very complicated and consists of a set of outbursts with a very large duration. The brightness of Mkn 421 varies from B=11.6 mag to more than 16 mag. Using the Jurkevich method of computing periods of cyclic phenomena, Liu *et al*[10] revealed from the light curve a possible period of 23.1±1.1 years, from which the authors estimated the mass of the Mkn 421 black hole to be about $2\times10^6 M_\odot$.

Mkn 421 is the brightest BL Lac object at UV, x-ray and TeV γ-ray wavelengths, which are the final tails of the two components in the SED.[11,12] Optical observation is generally used only for a comparison with the rapid high-energy flares,[7] and there is a lack of analysis of its variability on a larger temporal scales.[13] We collected many optical observations during the past few years, including the results from charge coupled devices (CCD) photometry in April 2000-to-Jan 2002, in order to prove that there exists a period of 23.1±1.1 years and the variation in the period under the thermal instability of a slim accretion dick around a massive black hole with mass $2\times10^6 M_\odot$.

In this paper we present the long-term light curve in the B band from 1899 to 2002 and colour behaviour of Mkn 421, based on the published blue (B), visual (V) and red (R) data. We present the results of optical photometric (B, V, R, I) monitoring of the Mkn 421 from April 2000 to Jan 2002. Here I stands for infrared. The results show significant rapid variation (2.5 hours) in the B band. The period obtained from our CCD photometry of Mkn 421 is in good agreement with the period of about 23 years between outbursts found previously by Liu *et al*.[10]

2. Observation and data reduction

From April 2000 to Jan 2002, Mkn 421 was observed with a 102-cm RCC telescope at Yunnan Observatory equipped with a direct CCD (1024×1024 pixels) camera. The scale of the CCD was 0.38 per pixel. The telescope was used at the $f/13.3$ Cassergrain focus. We measured the fluxes through standard Johnson broadband filters relative to stars included in the frames. The filters were as follows:

B–GC385(2mm)+BG12(1mm)+BG18(2mm)

V–GG495(2mm)+BG18(2mm)

R–RG610(3mm)+66.2500(1mm)

I–RG716(3mm)+60.5050(1mm)

The standard stars in the fields of Mkn 421 were taken from Ref.[14]. The integration time was 4–7 min. The magnitude of Mkn 421 was derived, using the standard stars within the same CCD field and all observed data were processed using the photometric tool, APPHOT, in the IRAF software package. The flat field images were taken at dusk and dawn when possible. The bias (taken with at least 15 images each night) was taken at the beginning of the observation and the end, separately. The observations are summarized in Table 1. The first column gives the date, the second the Julian date, the third the observed magnitudes, the fourth the observational uncertainties at 1σ level, and the last column the magnitude system. The observational uncertainties are standard errors derived from two comparison stars in the same CCD field.

Table 1. The observational data.

UT Date (1)	JD (2)	Mag (3)	σ (4)	Band (5)
4 April 2000	2451639.050	13.35	0.01	V
	.053	13.80	0.01	B
	.057	13.81	0.01	B
	.060	13.35	0.01	V
	.063	13.32	0.01	V
	.067	13.83	0.01	B
	.070	13.83	0.01	B
	.072	13.35	0.01	V
	.075	13.18	0.01	R
	.077	13.18	0.01	R
	.079	13.18	0.01	R
	.081	13.19	0.01	R
	.083	13.35	0.01	V
	.085	13.35	0.01	V
	.088	13.33	0.01	V
	.090	13.34	0.01	V
	.093	13.84	0.01	B
4 April 2000	2451639.096	13.86	0.01	B
	.098	13.30	0.02	V
	.100	13.32	0.02	V
	.103	13.88	0.01	B
	.107	13.90	0.01	B
	.110	13.89	0.01	B
	.112	13.31	0.01	V
	.114	13.32	0.02	V
	.117	13.92	0.01	B
	.120	13.90	0.01	B
	.122	13.31	0.01	V
	.125	13.30	0.01	V
	.126	12.22	0.01	I
	.127	13.22	0.01	R
	.129	13.20	0.01	R
	.131	13.30	0.01	V
	.132	12.31	0.01	I
	.133	13.25	0.01	R
	.136	13.30	0.01	V
	.139	13.92	0.01	B
	.141	13.32	0.01	V

Table. 1.(*Continued*)

UT Date (1)	JD (2)	Mag (3)	σ (4)	Band (5)
	.143	13.31	0.01	V
	.146	13.92	0.01	B
	.148	13.33	0.01	V
	.150	13.32	0.01	V
	.153	13.94	0.01	B
	.156	13.93	0.01	B
	.157	13.22	0.01	R
	.158	13.23	0.02	R
	.160	13.36	0.02	V
	.163	13.95	0.01	B
	.166	13.96	0.01	B
	.168	13.36	0.01	V
	.169	13.20	0.01	R
	.172	13.94	0.01	B
	.175	13.94	0.01	B
	.176	12.22	0.02	I
	.177	12.25	0.02	I
	.180	13.90	0.02	B
	.183	13.32	0.02	V
	.185	13.91	0.02	B
	.187	13.30	0.02	V
5 April 2000	2451640.032	13.80	0.01	B
	.035	13.50	0.01	V
	.037	13.82	0.01	B
	.039	13.84	0.01	B
	.041	13.52	0.01	V
	.043	13.58	0.01	V
	.046	13.90	0.01	B
	.049	13.91	0.01	B
	.052	13.60	0.01	V
	.054	13.54	0.01	V
	.057	13.94	0.01	B
	.060	13.93	0.01	B
	.062	13.50	0.01	V
	.064	13.52	0.01	V
	.068	13.90	0.01	B
	.071	13.87	0.01	B
	.073	13.50	0.01	V
	.075	13.46	0.01	V
	.078	13.85	0.02	B
	.081	13.84	0.02	B
	.084	13.44	0.01	V
	.086	13.40	0.01	V
	.089	13.80	0.01	B
	.092	13.83	0.01	B
	.095	13.82	0.02	B
	.097	13.40	0.01	V
	.100	13.38	0.01	V
	.103	13.75	0.01	B
	.106	13.70	0.01	B
	.108	13.33	0.01	V
	.110	13.30	0.01	V
	.113	13.60	0.01	B
	.116	13.62	0.01	B
	.118	13.31	0.01	V
	.120	13.29	0.01	V
	.123	13.57	0.01	B
	.126	13.54	0.01	B
	.128	13.27	0.01	V

Table. 1.(*Continued*)

UT Date (1)	JD (2)	Mag (3)	σ (4)	Band (5)
	.130	13.26	0.01	V
	.133	13.51	0.01	B
	.135	12.85	0.01	R
	.136	12.85	0.01	R
	.137	12.85	0.01	R
	.139	13.20	0.01	V
	.142	13.21	0.01	V
	.145	13.45	0.02	B
	.148	13.40	0.02	V
	.150	13.18	0.01	V
	.152	13.19	0.01	V
	.155	13.38	0.01	B
	.158	13.40	0.01	B
	.161	13.40	0.01	B
	.163	13.20	0.01	V
	.165	13.23	0.02	V
	.167	13.21	0.02	V
	.170	13.50	0.02	B
	.173	13.53	0.02	B
	.175	13.30	0.02	V
	.177	13.60	0.02	B
	.179	13.64	0.02	B
	.182	13.24	0.02	V
	.185	13.66	0.02	B
15 Feb 2001	2451956.230	13.50	0.02	B
	.233	13.26	0.02	V
	.237	13.30	0.01	V
	.240	13.52	0.01	B
	.243	13.51	0.01	B
	.245	13.25	0.01	V
	.247	13.24	0.02	V
	.250	13.53	0.02	B
	.253	13.49	0.01	B
	.256	13.25	0.01	V
15 Feb 2001	2451956.259	13.26	0.01	V
	.262	13.50	0.01	B
	.265	13.53	0.01	B
	.268	13.25	0.01	V
	.271	13.27	0.01	V
	.274	13.52	0.01	B
	.277	13.55	0.01	B
	.280	13.27	0.01	V
	.284	13.28	0.01	V
16 Feb 2001	2451957.207	13.30	0.02	V
	.287	13.29	0.01	V
	.290	13.28	0.01	V
	.293	13.56	0.01	B
	.296	13.57	0.01	B
	.299	13.56	0.02	B
	.310	13.57	0.03	B
	.313	13.55	0.01	B
	.316	13.53	0.02	B
	.319	13.53	0.02	B
	.321	13.26	0.02	V
	.324	13.24	0.02	V
	.328	13.53	0.01	B
	.331	13.52	0.01	B
	.334	13.23	0.01	V
	.337	13.23	0.02	V

Table. 1.(*Continued*)

UT Date (1)	JD (2)	Mag (3)	σ (4)	Band (5)
	.340	13.50	0.02	*B*
	.343	13.50	0.02	*B*
	.346	13.49	0.02	*B*
	.350	13.24	0.02	*V*
	.352	13.23	0.02	*V*
	.354	13.49	0.02	*B*
	.357	13.48	0.02	*B*
	.360	13.22	0.02	*V*
	.363	13.21	0.02	*V*
	.366	13.45	0.02	*B*
	.369	13.45	0.02	*B*
	.373	13.24	0.01	*V*
	.377	13.42	0.01	*B*
	.380	13.43	0.02	*B*
	.382	13.23	0.02	*V*
	.384	13.21	0.02	*V*
16 Feb 2001	2451957.387	13.42	0.02	*B*
	.390	13.40	0.02	*B*
	.393	13.41	0.02	*B*
	.397	13.27	0.02	*V*
	.210	13.60	0.02	*B*
	.213	13.61	0.02	*B*
	.214	13.60	0.02	*B*
	.217	13.29	0.01	*V*
	.220	13.30	0.01	*V*
16 Feb 2001	2451957.224	13.58	0.01	*B*
	.227	13.58	0.01	*B*
	.229	13.28	0.01	*V*
	.233	13.26	0.01	*V*
	.237	13.55	0.01	*B*
	.240	13.57	0.01	*B*
	.244	13.57	0.01	*B*
	.247	13.28	0.01	*V*
	.250	13.25	0.01	*V*
	.253	13.25	0.01	*V*
	.257	13.54	0.01	*B*
	.260	13.53	0.01	*B*
	.263	13.50	0.01	*B*
	.267	13.24	0.01	*V*
	.270	13.23	0.01	*V*
	.273	13.48	0.01	*B*
	.276	13.47	0.02	*B*
	.279	13.49	0.02	*B*
	.282	13.20	0.02	*V*
	.285	13.20	0.02	*V*
	.290	13.19	0.02	*V*
	.293	13.20	0.02	*V*
	.296	13.45	0.02	*B*
	.298	13.47	0.02	*B*
	.300	13.44	0.02	*B*
	.303	13.43	0.02	*B*
	.306	13.45	0.02	*B*
	.309	13.44	0.02	*B*
	.313	13.23	0.01	*V*
	.317	13.22	0.01	*V*
	.320	13.42	0.01	*B*
	.323	13.41	0.01	*B*
	.327	13.40	0.01	*B*
	.330	13.22	0.01	*V*

Table. 1.(*Continued*)

UT Date (1)	JD (2)	Mag (3)	σ (4)	Band (5)
	.333	13.23	0.01	*V*
	.337	13.22	0.01	*V*
	.339	13.38	0.01	*B*
	.343	13.39	0.01	*B*
	.346	13.37	0.01	*B*
	.349	13.21	0.01	*V*
	.353	13.22	0.02	*V*
	.357	12.75	0.02	*R*
	.360	12.74	0.02	*R*
	.361	12.73	0.02	*R*
	.364	13.35	0.02	*B*
	.369	13.34	0.02	*B*
	.372	13.34	0.02	*B*
	.375	13.20	0.01	*V*
	.378	13.19	0.01	*V*
	.381	13.19	0.01	*V*
	.384	13.35	0.01	*B*
	.387	13.40	0.01	*B*
	.390	13.21	0.01	*V*
8 Jan 2002	2452282.370	13.90	0.04	*B*
	.372	13.89	0.04	*B*
	.374	13.89	0.04	*B*
	.376	13.90	0.04	*B*
	.379	13.16	0.04	*R*
	.383	13.18	0.04	*R*
	.386	13.15	0.04	*R*
	.389	13.09	0.04	*R*
	.391	13.10	0.04	*R*
	.392	13.11	0.04	*R*
	.394	13.10	0.04	*R*
	.396	13.09	0.04	*R*
	.398	13.12	0.04	*R*
	.400	13.10	0.04	*R*
	.401	13.10	0.04	*R*
	.403	13.11	0.04	*R*
	.405	13.12	0.04	*R*
	.407	13.12	0.04	*R*

We compile all available B-band optical data on Mkn 421 from Refs.[7–9], [13] and [15–35]. The B band data for Mkn 421 consist of 565 observations dating from 1900 to 1991[10] and 238 observations dating from 1994 to 2001. Since we search for the long time variability, we increase data points by 238 in Liu *et al*'s[10] figures. The B-band observations are used in this paper because there are more data available in the B-band than in other bands. In order to probe the long-term behaviour of the variations and to counteract the heavy weighting of data and to reduce small amplitude intra-month fluctuations, we have calculated 100-day means of the light curve. This interval is long enough to smooth the light curve considerably. Because of the different qualities of our data in different epochs, the impact of flickering on recent data is washed out, but still significant in the early data. As

the Mkn 421 varies in intensity by about 1.4 mag on a timescale of 52 min in the B band,[7] the largest difference between early epoch data point and the mean value can be estimated only within an uncertainty of 1.4 mag. Our purpose is to investigate the long-term (about 100 years) light curve and large-amplitude variations. We do not show individual error-bars. This average light curve consists of 140 data points, dating back to 1899.

3. Results

3.1. Long-term optical variability

An historical mean light curve for Mkn 421 is shown in Fig.1, where it can be seen that Mkn 421 is a very active object, with a range of variation of ΔB=4.7 mag. The source reached a maximum brightness of 11.60 mag on 19 January, 1934 and was brighter than 12.75 mag of a 100-day mean on four occasions in 1901, 1916, 1936[8] and 1982.[10] Later on it reached B=12.83 mag of a 100-day mean in 1997.[13] Before 1991, the resulting 100-d mean of the light curve is similar to that presented by Miller.[8] And Liu *et al*[10] reported a historical light curve of about 90 years. The peaks in 1916, 1936 and 1982 remained unchanged. Regular optical photometric monitoring of Mkn 421 has been carried out since 1916, showing five long-term phases of activity (on a timescale from one year to a few years) until 1982. Since 1991 the optical flux has increased (flares have been detected) and reached a brighter B=12.83 mag in 1997. Tosti *et al*[13] presented the results of optical photometric (UBVRI) monitoring of Mkn 421 from November 1994 to June 1997. They found during 1995 and the first half of 1996 (from JD 2449676 to 2450233) that the Mkn 421 showed small amplitude variations and a modulation on a some-month timescale. In winters 1996–97 a large outburst occurred, and a maximum of about B=12.83 mag of 100-day mean, one of the highest values of brightness reported in the historical light curve, was observed. Our light curves for the B, V and R bands observed from April 2000 to Jan 2002 are shown in Fig.2 and Table 1. They indicate that the source varies slowly during our monitoring period. The amplitudes of the observed variability are ΔB=0.56 mag, ΔV=0.41 mag, ΔR=0.43 mag. Also, the R, V and B bands light curves are well correlated with each other. The results given by Xie *et al* (2002) and our observations show that Mkn 421 was in faint state, and since 1997 a large outburst has occurred and the faint state of B has reached about 13.90 mag. This value is in good agreement with B=13.95 mag in 1978.[20] Our recent CCD photometry of Mkn 421 from April 2000 to Jan 2002 shows that it was the state with the same brightness as that on 23 May 1977 to 21 June 1979.[19]

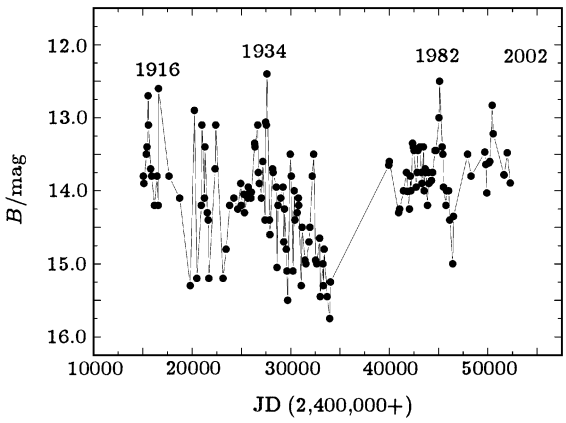


Fig.1. The mean long-term light curve of Mkn 421 over a 100-day mean in the B band from 1899 to 2002.

3.2. Short variability timescales

The optical variability of Mkn 421 shows two modes: a longer mode on a time scale from several years to ten years and a short mode on a timescale from several d to a few months. On the short variability timescales, a rapid decline of 1.6 mag in brightness in 16 days was found in January 1942.[8] Xie *et al*[36] observed a 1.4-mag increase in the B band in about 2 days. The brighter magnitude variation detected by Villata *et al*[33] was about 0.4 mag in both R and B bands in about 60 days. The steepest variation soared to 0.19-mag in one days in the R band.[33] Hurst *et al*[37] observed a 0.9 mag increase in the V band in about 40 days.

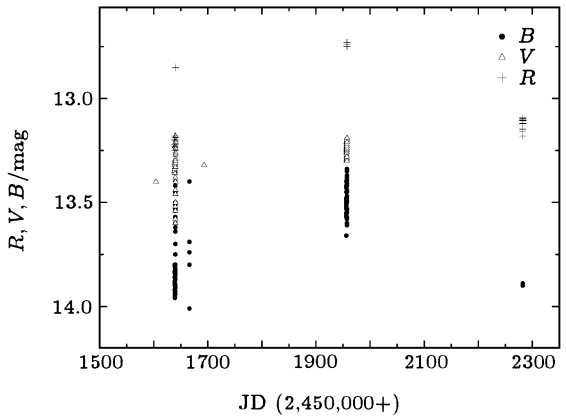


Fig.2. The light curve of Mkn 421 in the B, V and R bands from April 2000 to January 2002.

In this paper, we define the observed value of Δt_{min} as a minimum variability timescale for a given object. This timescale corresponds to a nominal change of 50% or more in the emitted intensity in the B, V, and R wave bands.[38−41] In addition, the amplitude of optical variability on this timescale must be more than 5δ, where δ is the maximum total observational root-mean-square error.

Figure 3 shows the new observations obtained on 5 April 2000. Our current data of the light curve show that a short timescale variability was observed to be on 2.42 hours in the B band. The rapidly optical variation to a amplitude of $\Delta B = 0.54$ mag and a confidence level larger than 4δ occurred between JD 2451640.057 and JD 2451640.158 and is shown in Table 1 and Fig.3. In Fig.3, it is shown that the source brightens from B=13.94 mag to B=13.40 mag in

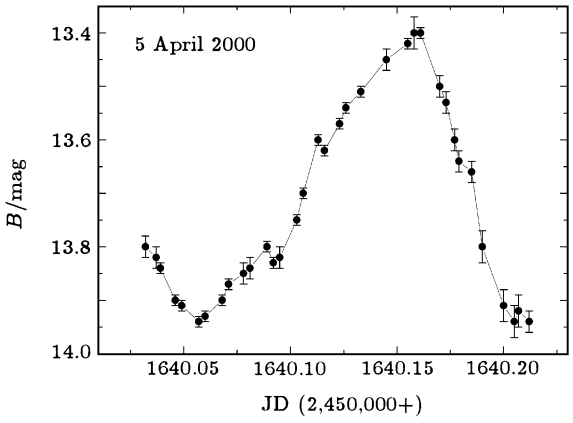


Fig.3. B band light curve of Mkn 421 on 5 April 2000.

145 minutes. We also examined the CCD brightness measurements of one of our field standards. The maximal deviation of the field standard star for one night is 0.06 mag. From these data, we infer that the time evolution of the standard stars should not cause more than 6% error. This result shows a typical rapid timescale of variability for Mkn 421 to be $\Delta t = 2.5$h, which is similar to that given by Xie *et al.*[36] For the emission from a spherical blob with an observed time variability Δt, relativistic causality requires the size of the emission region, R, in the jet to satisfy the inequality

$$R \leq \frac{c\Delta t\delta}{(1+z)} \quad \text{cm}, \tag{1}$$

where c is the light speed, δ is the Doppler factor, z is the cosmological redshift: z=0.0308 for Mkn 421. According to Fabian and Rees[42] accretion theory, we can use an average optical index $\alpha_{\rm opt} = 1.5$ and average value in the V band to estimate the Doppler factor $\delta = 1.4$ for Mkn 421. Using the inequality and the variability timescale for optical emission $\Delta t = 145$ min, the size of the optical emission region is estimated to be R=3.5×10^{12}m.

3.3. Colour indices

As the Mkn 421 object is violent variable, its colour indices should ideally be derived from simultaneous observations in the various colours. However, if the interval between the B and V observations is sufficiently small, these observations could be regarded as quasi-simultaneous. In our monitoring programmed from 4 April 2000 to 8 Jan 2002, the quasi-simultaneous observations show that the intra-day variabilities are ΔB=0.56 mag, ΔV=0.41 mag, ΔR=0.43 mag. The results are shown in Table 1. We notice that the maximal deviation of the differential magnitude between stars 1 and 2 is too large, so the confidence level on this variability timescale is low. Data of the interval between the B, V and R observations from Villata *et al*[33] and Table 1 could be regarded as quasi-simultaneous. For studying variations in the colour, we use the data from Villata *et al*[33] and Table 1. Some data in B and R immediately before or after V measuring, the time interval for completing one set of observations was generally 5–30 min (27 data points). In our observations, the time interval was generally 5–10 min (114 data points). We select those observations in Table 1 when the B, V and R photometry is obtained within less than 10 min, which is one order of magnitude smaller than the variability timescale. We have calculated the $B-R$ and $B-V$ colour indices for Mkn 421. They are displayed in Fig.4 and Fig.5. In Fig.4 shown $B-R$ is against R magnitude, and in Fig.5 $B-V$ against B magnitude. Errors on the averages could be quite large, due to the variability of Mkn 421. By linear regression, we obtain the anti-correlation

$$B-R = (0.1298 \pm 0.0045)R - (1.032 \pm 0.061), \tag{2}$$

with a correlation coefficient r=0.8603 (27 data points from Villata *et al*[33] and 17 data points from Table 1), and the chance probability P=10^{-5}, the standard deviation S$_{(B-R)}$=0.26;

$$B-V = (0.7257 \pm 0.0056)B - (9.054 \pm 0.076), \tag{3}$$

with a correlation coefficient r=0.97706 (114 data points in Table 1), and the chance probability

P=10^{-7}, the standard deviation $S_{(B-V)} = 0.27$. It is evident that the colour index is also variable, and it is greater when the source is fainter (see Fig.4 and Fig.5). We find the strong correlation between the $B-V$ colour index and B magnitude is in good agreement with that recently reported by Xie *et al*,[35] fitting with a regression line $B - V = 0.71B - 9.34$ (5 data points) for Mkn 421. For PKS 0735+178, OI 090.4, OJ-131 and ON+231, Xie *et al*[39,43] have also observed similar correlations between the $B-V$ colour index and B magnitude. Tosti *et al*[13] have observed similar correlations between the $U - I$ colour index and V magnitude for Mkn 421 (39 data points).

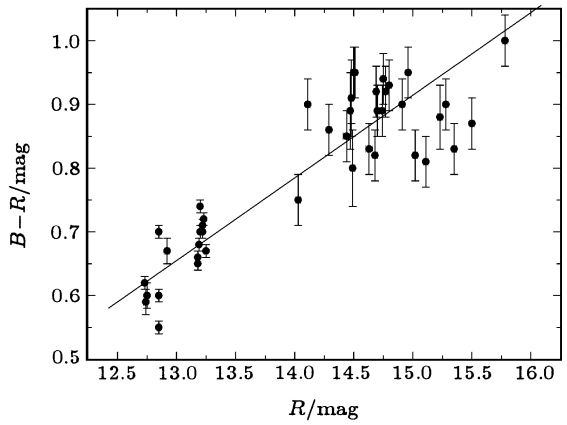


Fig.4. The $(B - R)$ Colour indices against the R magnitude in the R band. The solid line is the regression line.

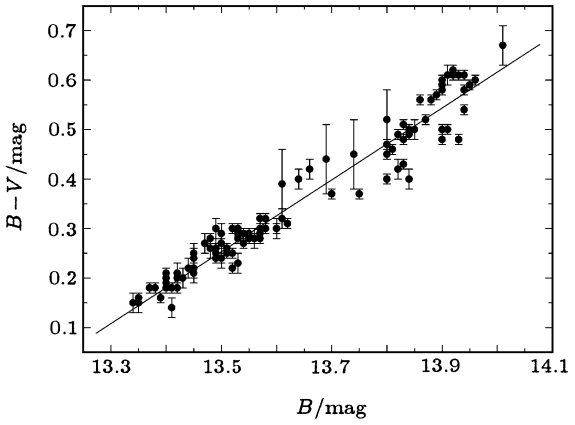


Fig.5. The $(B - V)$ Colour indices against the B magnitude in the B band. The solid line is the regression line.

In Fig.4 and Fig.5 we present the results of colour index $(B - R)$ as a function of the R magnitude and $(B - V)$ as a function of the V magnitude. From the figures we can see that there appears the reddening of the colour indices when the source becomes fainter. Tosti *et al*[13] related this effect to the host galaxy contribution to the total light. Nevertheless, it could be an intrinsic characteristic of the emission mechanism. The reddening is a typical feature discovered in other BL Lacs that have an evident host galaxy, therefore this phenomenon is generally interpreted as the consequence of the thermal emission of the galaxy: when the synchrotron continuum of the AGN becomes less luminous then the spectrum of the galaxy is well observable and the colour index is redder.[13,44,45] We should check the correlation and analysis of the data of photometric monitoring in the future.

4. Discussion

We have constructed the historical light curve of Mkn 421 and found that it is very complicated and consists of many outbursts. Possible long-period variations in Mkn 421 and possible period of 23.1±1.1 years in the mean level of light curve are reported to be consistent with recent behaviours. Our result from CCD photometry is in agreement with the prediction of the optical variability period of about 23.1 years between outbursts found previously by Liu *et al.*[10] The period of 23 years is about half the time interval between the well observed outbursts in 1936 and 1982. If the period is correct, outbursts probably occurred between 1954 and 1963, unfortunately, no published data are available. Based on this period we can predict that the next outburst should be at its maximum around 2005.

Using the historical light curve of Mkn 421 and the theory of the slim accretion disc in AGNs, Liu *et al*[10] have obtained the mass of central black hole to be about 2 $\times 10^6$ $M_\odot$. Besides, in OJ 287,[44] ON 231,[46] 2200+230[47] and 0735+178,[48] several working groups have also obtained similar longer period optical variations on historical light curve and the mass of central black hole. On the basis of the accretion disc theory[27,49] and observation, the short time variability is 2.42 hours, We have also obtained a similar result for the mass of central black hole about M=10^6 $M_\odot$. Obviously, the results are very interesting. For the mass of central black hole 2 $\times 10^6$ $M_\odot$, the Schwarzschild radius is $R_{\rm g}$ =2GM/ c^2 = 0.6 $\times 10^{10}$ m. Therefore, we can obtain the relation between the size of the emission region R and the Schwarzschild radius of the central black hole, $R \simeq 205\ R_{\rm g}$. This is a very important result. It is well known that in the accretion disc theory R is a very important parameter.

Our observations show a large amplitude

variation (Δm=0.54 mag) on a short timescale (Δt=145 min) in Mkn 421. The results have important implications for theoretical models to explain optical and gamma-ray variability of BL Lac objects. The TeV gamma-ray minimum variability timescale, which is smaller than the GeV minimum variability timescale, is similar to the optical minimum variability timescale for Mkn 421[8,50] and provides a new clue to the nature of the gamma-ray emission.[51] Perhaps TeV gamma-ray and GeV gamma-ray come from two different emission regions.

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