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Accurate analysis of arbitrarily-shaped helical groove waveguide*

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This paper presents a theory on accurately analysing the dispersion relation and the interaction impedance of electromagnetic waves propagating through a helical groove waveguide with arbitrary groove shape, in which the complex groove profile is synthesized by a series of rectangular steps. By introducing the influence of high-order evanescent modes on the connection of any two neighbouring steps by an equivalent susceptance under a modified admittance matching condition, the assumption of the neglecting discontinuity capacitance in previously published analysis is avoided, and the accurate dispersion equation is obtained by means of a combination of field-matching method and admittance-matching technique. The validity of this theory is proved by comparison between the measurements and the numerical calculations for two kinds of helical groove waveguides with different groove shapes.

Keywords: helical groove waveguide, dispersion characteristic, interaction impedance, travelling wave tube

PACC: 7170G, 7145I, 7110

1. Introduction

Being a potential beam-wave interaction structure working in both slow wave regimes and fast wave regimes, the helical groove waveguide (HGW)[1−8] has attracted much attention due to its large transverse size, high machining accuracy, super-thermal conductivity and low loss, which make it worth to be considered as a slow wave structure in the millimetre wave band. Recently, there has been increasing interest in the theoretical and experimental study of variable shape HGWs for the use in normal travelling-wave tube (TWT)[1−6] and gyro-TWT.[7] The authors of Ref.[3] have developed a simple approach for the analysis of arbitrary-shaped HGW, in which the groove profile is approximately represented by a series of rectangular steps and each step supports the fundamental TE mode. This analysis can give reasonable predications on dispersion characteristics, and the theoretical results are in agreement with the experimental data. However, there are some discrepancy between them in the high-frequency band, which may be caused by the neglect of the influence of higher-order evanescent modes on groove regions. In this regard, we will develop this approach by introducing the effect of the evanescent modes on the groove regions, so as to give an accurate dispersion equation and interaction impedance. The calculated results by using the present theory are in a better agreement with the experimental data as compared to the prior theory.[3]

2. Theory

The longitudinal section of an arbitrary-shaped helical groove waveguide is shown in Fig.1, where δ denotes the groove width, L denotes the length of the period, and r_a and r_c denote the inner and outer radii of the groove, respectively.

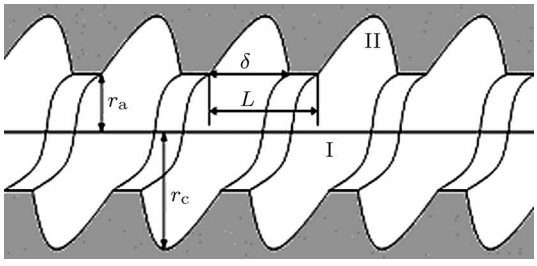


Fig.1. The longitudinal section of the arbitrary helical groove.

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In the following analysis, this structure is divided into two regions: the groove region II and the interaction region I. The fields in each region can be obtained by solving the wave equations, and the dispersion equation can be derived by the matching conditions of the fields at the interfaces.

A. Fields in interaction region I($r \le r_a$)

In the central region, the cylindrical coordinate system (r, φ, z) is employed to solve the Maxwell equations. By Floquet's theorem of periodic structure, the field expressions can be expressed as

$$\left.\begin{aligned}
E_z^{\mathrm{I}} &= \sum_{m=-\infty}^{\infty} A_m^{\mathrm{I}} \gamma_m^2 F_m(\gamma_m r) \mathrm{e}^{\mathrm{j}m\varphi - \mathrm{j}\beta_n z}, \\
H_z^{\mathrm{I}} &= \mathrm{j} \sum_{m=-\infty}^{\infty} B_m^{\mathrm{I}} \gamma_m^2 F(\gamma_m r) \mathrm{e}^{\mathrm{j}m\varphi - \mathrm{j}\beta_n z}, \\
E_\varphi^{\mathrm{I}} &= \mp \sum_{m=-\infty}^{\infty} \left[A_m^{\mathrm{I}} (m\beta_m / r) F_m(\gamma_m r) - B_m^{\mathrm{I}} \omega\mu_0 \gamma_m F_m'(\gamma_m r) \right] \mathrm{e}^{\mathrm{j}m\varphi - \mathrm{j}\beta_n z}, \\
H_\varphi^{\mathrm{I}} &= \pm \mathrm{j} \sum_{m=-\infty}^{\infty} \left[A_m^{\mathrm{I}} (k^2 \gamma_m / \omega\mu_0) F_m'(\gamma_m r) - B_m^{\mathrm{I}} (m\beta_m / r) F_m(\gamma_m r) \right] \mathrm{e}^{\mathrm{j}m\varphi - \mathrm{j}\beta_n z}, \\
E_r^{\mathrm{I}} &= \pm \mathrm{j} \sum_{m=-\infty}^{\infty} \left[A_m^{\mathrm{I}} \beta_m \gamma_m F_m'(\gamma_m r) - B_m^{\mathrm{I}} \omega\mu_0 (m/r) F_m(\gamma_m r) \right] \mathrm{e}^{\mathrm{j}m\varphi - \mathrm{j}\beta_n z}, \\
H_r^{\mathrm{I}} &= \pm \sum_{m=-\infty}^{\infty} \left[A_m^{\mathrm{I}} (k^2 m / \omega\mu_0 r) F_m(\gamma_m r) - B_m^{\mathrm{I}} \beta_m \gamma_m F_m'(\gamma_m r) \right] \mathrm{e}^{\mathrm{j}m\varphi - \mathrm{j}\beta_n z},
\end{aligned}\right\} \tag{1}$$

where $\beta_m = \beta_0 + (2\pi m/L)$ is the axial propagation constant of the mth space harmonic, $\gamma_m = \left|\sqrt{\beta_m^2 - k^2}\right|$ is the radial propagation constant and k is the free space propagation constant. If $\beta_m^2 - k^2 > 0$, $F_m(\gamma_m r) = I_m(\gamma_m r)$ and the upper of the signal '$\pm$' or '$\mp$' is selected, while $\beta_m^2 - k^2 < 0$, $F_m(\gamma_m r) = J_m(\gamma_m r)$ and the lower case of the signal '$\pm$' or '$\mp$' is chosen. $J_m(\gamma_m r)$ and $I_m(\gamma_m r)$ represent the ordinary and modified Bessel functions of order m of first kind, respectively. A_m^{I} and B_m^{I} are the field constants.

B. Fields in groove region II

As proposed in Ref.[3], the arbitrary-shaped helical groove can be replaced by a series of linked rectangular areas, in which the theoretical solution of the wave equation can be obtained. A schematic diagram of such approximate representations of a groove by N steps is plotted in Fig.2, where, r_n and d_n represent the radius and width of the nth step. Obviously, as the number of the steps tends to infinity, the groove profile is precisely synthesized.

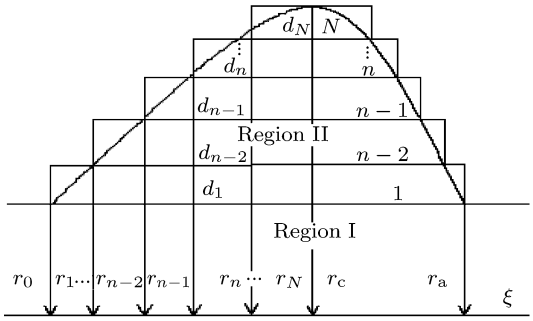


Fig.2. The approximate representation of a groove.

Then, the field expressions[4] in the nth area can be written as

$$\left.\begin{aligned}
E_{\xi(n)}^{\mathrm{II}} &= \left[A_n^{\mathrm{II}} J_v(kr) + B_n^{\mathrm{II}} J_{-v}(kr) \right] \mathrm{e}^{-\mathrm{j}v\theta}, \\
H_{\theta(n)}^{\mathrm{II}} &= -jY_0 \left[A_n^{\mathrm{II}} J_v'(kr) + B_n^{\mathrm{II}} J_{-v}'(kr) \right] \mathrm{e}^{-\mathrm{j}v\theta}, \\
H_{r(n)}^{\mathrm{II}} &= \frac{v}{r\omega\mu_0} \left[A_n^{\mathrm{II}} J_v(kr) + B_n^{\mathrm{II}} J_{-v}(kr) \right] \mathrm{e}^{-\mathrm{j}v\theta}, \\
E_{\theta(n)}^{\mathrm{II}} &= E_{r(n)}^{\mathrm{II}} = H_{\xi(n)}^{\mathrm{II}} = 0,
\end{aligned}\right\} \tag{2}$$

where A_n^{II} and B_n^{II} are the field amplitude factors. v is the angular propagation coefficient expressed as $v = \beta_0 L/2\pi$. Y_0 is the wave admittance in the free space. $J_{\pm v}$ is the normal Bessel function of the first kind and order $\pm v$, which indicates the mode excited here is oscillating in the r-direction. $J_{\pm v}'(kr)$ is the

first differential of $J_{\pm v}(kr)$ with respect to kr.

C. Dispersion equation

Besides these propagating modes, there are many higher-order evanescent modes at the connection place of two neighbouring areas, which are induced by the change of the step width d_n. The effect of these modes may be described as an equivalent susceptance by circuit theory. The normalized shunt susceptance[9] at $r = r_{n-1}$ is

$$B_{n-1} = \frac{2d_{n-1}}{\lambda_g}\left[\ln\left(\frac{1-\alpha^2}{4\alpha}\right)\left(\frac{1+\alpha}{1-\alpha}\right)^{\frac{1}{2}\left(\alpha+\frac{1}{\alpha}\right)} + 2\frac{A+A''+2C}{AA'-C^2} + \left(\frac{d_{n-1}}{4\lambda_g}\right)^2\left(\frac{1-\alpha}{1+\alpha}\right)^{4\alpha}\left(\frac{5\alpha^2-1}{1-\alpha^2}+\frac{4}{3}\frac{\alpha^2 C}{A}\right)^2\right], \tag{3}$$

where

$$\alpha = \frac{d_n}{d_{n-1}}, \quad A = \left(\frac{1+\alpha}{1-\alpha}\right)^{2\alpha}\frac{1+\sqrt{1-\left(\frac{d_{n-1}}{\lambda_g}\right)^2}}{1-\sqrt{1-\left(\frac{d_{n-1}}{\lambda_g}\right)^2}} - \frac{1+3\alpha^2}{1-\alpha^2},$$

$$A' = \left(\frac{1+\alpha}{1-\alpha}\right)^{2/\alpha}\frac{1+\sqrt{1-\left(\frac{d_n}{\lambda_g}\right)^2}}{1-\sqrt{1-\left(\frac{d_n}{\lambda_g}\right)^2}} - \frac{3+\alpha^2}{1-\alpha^2}, \quad C = \left(\frac{4\alpha}{1-\alpha^2}\right)^2.$$

Here, $\lambda_g = 2\pi/\beta_0$. So, we can obtain the recurrence relationship between the admittances of two neighbouring steps and the admittance at the groove opening by using the admittance matching condition.

The normalized admittance of propagating modes in the (n-1)th area at $r = r_{n-2}$ is

$$Y_{n-2,n-1} = \frac{1}{-jY_0}\frac{H^{\mathrm{II}}_{\theta(n-1)}(kr_{n-2})}{E^{\mathrm{II}}_{\xi(n-1)}(kr_{n-2})} = \frac{J'_v(kr_{n-2})+C_{n-1}J'_{-v}(kr_{n-2})}{J_v(kr_{n-2})+C_{n-1}J_{-v}(kr_{n-2})}, \tag{4}$$

where

$$C_{n-1} = B^{\mathrm{II}}_{n-1}/A^{\mathrm{II}}_{n-1}.$$

Similarly, the admittance in the (n-1)th area at $r = r_{n-1}$ is written as

$$Y_{n-1,n-1} = \frac{J'_v(kr_{n-1})+C_{n-1}J'_{-v}(kr_{n-1})}{J_v(kr_{n-1})+C_{n-1}J_{-v}(kr_{n-1})}. \tag{5}$$

Then, the matching condition of the normalized admittance at $r = r_{n-1}$ is

$$\bar{Y}_{n-1,n-1} = \bar{Y}_{n-1,n} + \bar{B}_{n-1}, \tag{6}$$

where $\bar{Y}_{n-1,n-1}$, $\bar{Y}_{n-1,n}$ and $\bar{B}_{n-1}$ can be obtained from Ref.[10],

$$\bar{Y}_{n-1,n-1} = -2\pi \mathrm{j} Y_0 Y_{n-1,n-1}(r_{n-1}/d_{n-1}),$$
$$\bar{Y}_{n-1,n} = -2\pi \mathrm{j} Y_0 Y_{n-1,n}(r_{n-1}/d_n),$$
$$\bar{B}_{n-1} = -2\pi \mathrm{j} Y_0 B_{n-1}(r_{n-1}/d_n).$$

Thus, one may have

$$\frac{Y_{n-1,n-1}}{d_{n-1}} = \frac{(Y_{n-1,n}+B_{n-1})}{d_n}. \tag{7}$$

Consequently, the recurrence relationship of the admittance can be obtained by substituting Eqs.(7) and (5) into Eq.(4),

$$Y_{n-2,n-1} = Q_v(kr_{n-1}, kr_{n-2}) \left[\frac{Y_{n-1,n} + B_{n-1} - \dfrac{d_n}{d_{n-1}} T_v(kr_{n-2}, kr_{n-1})}{Y_{n-1,n} + B_{n-1} - \dfrac{d_n}{d_{n-1}} Q_v(kr_{n-2}, kr_{n-1})} \right], \tag{8}$$

where

$$T_v(x,y) = \frac{J'_v(x)J'_{-v}(y) - J'_{-v}(x)J'_v(y)}{J'_v(x)J_{-v}(y) - J'_{-v}(x)J_v(y)}, \quad Q_v(x,y) = \frac{J_v(x)J'_{-v}(y) - J_{-v}(x)J'_v(y)}{J_v(x)J_{-v}(y) - J_{-v}(x)J_v(y)}.$$

Using Eq.(2), we can express C_N at the boundary $r = r_c$

$$C_N = -J_v(kr_c)/J_{-v}(kr_c).$$

Therefore,

$$\begin{aligned} Y_{N-1,N} &= \frac{J'_v(kr_{N-1}) + C_N J'_{-v}(kr_{N-1})}{J_v(kr_{N-1}) + C_N J_{-v}(kr_{N-1})} = \frac{J'_v(kr_{N-1})J_{-v}(kr_c) - J_v(kr_c)J'_{-v}(kr_{N-1})}{J_v(kr_{N-1})J_{-v}(kr_c) - J_v(kr_c)J_{-v}(kr_{N-1})} \\ &= Q_v(kr_c, kr_{N-1}). \end{aligned} \tag{9}$$

Moreover, the expression of $Y_{0,1}$ at $r = r_a$ can be obtained by recurrence relationship in Eq.(9) and $Y_{N-1,n}$.

The dispersion equation is derived

$$Y_{0,1} \pm \frac{\delta}{L} \sum_{m=-\infty}^{\infty} \frac{k}{\gamma_m} \left[\frac{F'_m(\gamma_m r)}{F_m(\gamma_m r)} - \frac{1}{k^2\gamma_m^2} \frac{F_m(\gamma_m r)}{F'_m(\gamma_m r)} \left(\frac{m\beta_m}{r_a} \mp \gamma_m^2 \mathrm{tg}\psi \right)^2 \right] \left(\frac{\sin \beta_m \delta/2}{\beta_m \delta/2} \right)^2 = 0. \tag{10}$$

Though Eq.(10) appears to be the same form as the dispersion equation in Ref.[3], $Y_{0,1}$ is different, which includes the effect of high–order evanescent modes. And when shunt susceptance $B = 0$, it will reduce to the formula in Ref.[3].

D. Interaction impedance

According to Pierce's definition, the interaction impedance of the mth space harmonic can be written as

$$K_c(m) = \frac{E_{zm}E^*_{zm}}{2\beta_m^2 P}, \tag{11}$$

where

$$E_{zm}(r_e) = \frac{\delta}{L} \left(\frac{\sin(\beta_m\delta/2)}{\beta_m\delta/2} \right) \frac{F_m(\gamma_m r_e)}{F_m(\gamma_m r_a)} [J_v(kr_a) + C_1 J_{-v}(kr_a)] A_1^{II} \cos\psi. \tag{12}$$

Here $E_{zm}(r_e)$ is the longitudinal electric field amplitude of the m th harmonic at the position of the electron beam and E^*_{zm} is its conjugate value. P is the total power and can be calculated from

$$P = \sum_{m=-\infty}^{\infty} P^{I}(m) + \sum_{n=1}^{N} P^{II}(n), \tag{13}$$

where

$$\begin{aligned} P^{I}(m) &= \frac{1}{2} \int_0^{r_a} \left[E_r^I H_\phi^{*I} - E_\phi^I H_r^{*I} \right] 2\pi r \mathrm{d}r \\ &= \pi Y_0 \left(\frac{\delta}{L} \right)^2 \left(\frac{\sin(\beta_m\delta/2)}{\beta_m\delta/2} \right)^2 \left(\frac{\beta_m}{k} \right) [J_v(kr_a) + C_1 J_{-v}(kr_a)](A_1^{II})^2 r_a^2 \\ &\quad \times \left\{ \frac{1}{4} \left[\left(\frac{k}{\gamma_m} \right)^2 + \left(\frac{m\beta_m}{\gamma_m^2 r_a} \mp \tan\psi \right)^2 \left(\frac{F_m(\gamma_m r_a)}{F'_m(\gamma_m r_a)} \right)^2 \right] \left[\frac{F^2_{m+1}(\gamma_m r_a) + F^2_{m-1}(\gamma_m r_a)}{F^2_m(\gamma_m r_a)} \right. \right. \\ &\quad \left. \left. - \frac{F_{m+2}(\gamma_m r_a) + F_{m-2}(\gamma_m r_a)}{F_m(\gamma_m r_a)} \right] - m \frac{(\beta_m^2 + k^2)}{\beta_m \gamma_m^3} \frac{[m\beta_m/(\gamma_m^2 r_a) \mp \tan\psi]}{r_a^2} \frac{F_m(\gamma_m r_a)}{F'_m(\gamma_m r_a)} \right\}, \end{aligned} \tag{14}$$

$$
\begin{aligned}
P^{\mathrm{II}}(n) &= \frac{1}{2} d_{\mathrm{n}} \int_{n-1}^{r_{\mathrm{n}}} E_{\xi}^{\mathrm{II}}(n) \times H_{r}^{\mathrm{II}}(n) \mathrm{d}r \\
&= \frac{Y_0}{4} d_n (A_n^{\mathrm{II}})^2 \times \Bigg\{ \left[J_{-v}(kr) + C_n J_v(kr) \right]^2 \\
&\quad \times kr \Bigg\{ \left[J_{(v+1)}(kr) - C_{n+1} J_{-(v+1)}(kr) \right] \times \frac{\partial}{\partial v} \left[J_{-v}(kr) + C_n J_v(kr) \right] \\
&\quad - \left[J_{-v}(kr) + C_n J_v(kr) \right] \times \frac{\partial}{\partial v} \left[J_{(v+1)}(kr) - C_n J_{-(v+1)}(kr) \right] \Bigg\} \Bigg\} \begin{matrix} r = r_n \\ r = r_{n-1} \end{matrix} .
\end{aligned} \tag{15}
$$

Here, $A_n^{\mathrm{II}} = A_1^{\mathrm{II}} \prod_{i=2}^{n} \frac{[J_v'(kr_{i-1}) + C_{i-1} J_{-v}'(kr_{i-1})]}{J_v'(kr_{i-1}) + C_i J_{-v}'(kr_{i-1})}$, which can be derived by employing the continuous condition of H_θ at the interface of two neighbouring steps.

3. Results and discussion

To solve the transcendental equation (10), a general MATLAB computer program is employed, in which the N can take arbitrary number. In practical calculations, 30 rectangular steps are used with the relative error less than 10^{-6}. Based on the present theory, we have figured out the physical dimensions of two helical-grooved waveguides, with the ridge-loaded rectangular groove and the hole-gap groove shown in Fig.3. Here, the inner and outer radius are 1.25 and 1.8cm, respectively, the groove opening width is 0.2cm and the pitch is 0.6cm. The two HGWs have been manufactured by the technique proposed in Ref.[6], and the dispersion properties of them have been measured by the resonance method.[11]

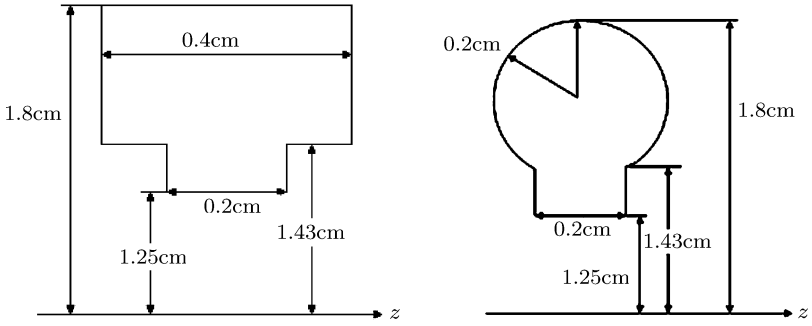


Fig.3. The profiles and physical dimensions of two grooves.

As illustrated in Figs.4 and 5, the dispersion properties are obtained for the ridge-loaded HGW and the hole-gap shaped HGW by using this theory, the former approach[3] and the experimental data, respectively. It can be seen clearly that the calculation result using the present theory is in a better agreement with the experimental data. This is due to the addition of the effect of high–order evanescent modes in the groove. Therefore, the present theory may give more accurate prediction on the velocity of the electromagnetic wave, for determining the working voltage of a travelling-wave tube to meet the requirement of velocity synchronization.

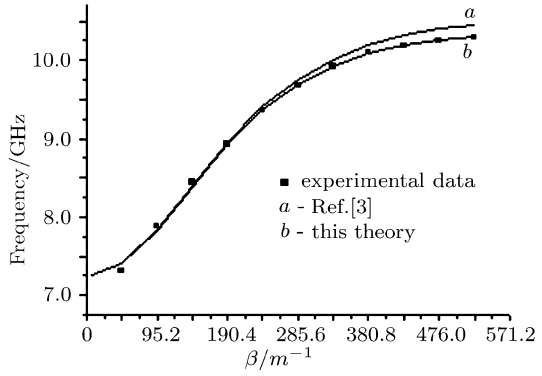


Fig.4. Dispersion characteristic of ridge-loaded helical-grooved waveguide.

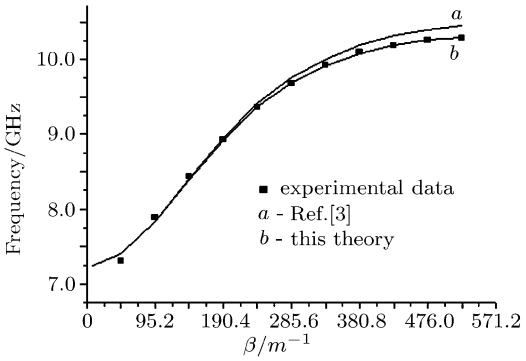


Fig.5. Dispersion characteristic of hole-gap shaped helical-grooved waveguide.

The interaction impedance indicates the coupling intensity between the wave and the electron beam. The interaction impedances for the two HGWs have been calculated by using the present theory and the former one, respectively; and the results are shown in Fig.6. There exists some discrepancy at the high-frequency part, indicating the influence of the evanescent waves.

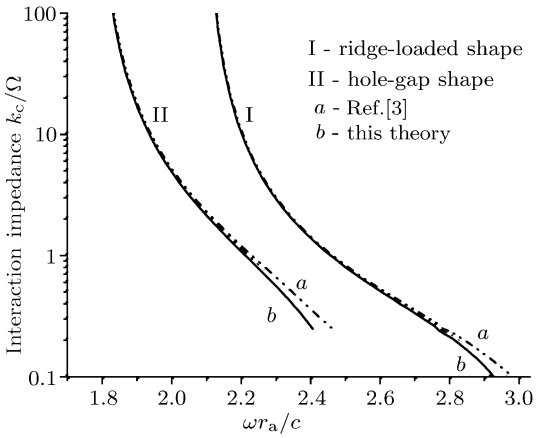


Fig.6. Interaction impedance of ridge-loaded and hole-gap shaped HGWs.

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