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# Analysis of the axial electric field in a plasma-loaded-helix travelling wave tube*

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A helix type slow wave structure filled with plasma is immersed in a strong longitudinal magnetic field. Taking into account the effect of the plasma and the dielectric, the system is separated radially into three regions. By means of the sheath model and Maxwell equation, the distribution of the electromagnetic field is established. Using the boundary conditions of each region, the dispersion relation of the slow wave structure is derived. The trend of change for the radial profile of the axial electric field is analysed respectively in different plasma densities, plasma column radius and dielectric constant by numerical computation. Some useful results are obtained on the basis of the discussion.

**Keywords:** helix travelling wave tube, plasma, axial electric field
**PACC:** 5235, 5240D, 5275

## 1. Introduction

Helix travelling wave tube (TWT) is a kind of very important microwave apparatus which can provide high energy output and can work perfectly in a wide frequency range. It was first studied by Pierce and coworkers in 1940s.[1] Initially, the slow wave structure was a single helix structure. Subsequently, some variations of helix structure were developed such as ring-loop and ring-bar structure in order to enhance the output power, interaction impedance of fundamental wave and restrain the oscillation of backward wave. Recently, much attention has been paid to the study of many kinds of plasma-loaded microwave devices for efficiency enhancement.[2−7] It has been found experimentally that injection of plasma into the microwave devices may enhance the interaction and the output power. Plasma filling can also improve the transmission quality of electron beam, even make beam transmission without a guiding magnetic field. The All-Russian Institute (VEI), Hughes Research Laboratories and University of Maryland have reported research achievements in this area. At first their research focused on the backward wave oscillator (BWO). Then, research of plasma-filled TWT with sinusoidally corrugated slow wave structure has been reported. The output frequency and the operating modes of these devices are changed a little by the introduction of the plasma, but those changes are different when plasma is introduced in helix TWT. The electromagnetic wave is supported by the helix coupled with the electrostatic wave. Hybrid modes are formed at frequencies where the phase velocities of the two modes coincide. The axial electric field has been changed dramatically as the plasma density increases. In this paper, some useful results are obtained by theoretical analysis and numerical computations.

## 2. Physical model

The slow wave structure analysed is illustrated in Fig.1. A Cylindrical plasma column of radius $r_{\rm p}$ and electron density $n_{\rm p}$ is enclosed in a sheath helix of radius $r_{\rm h}$ and angle $\phi$. A dielectric is loaded inside the region between helix and a conducting waveguide of radius $r_{\rm w}$. For simplicity we assume the plasma is immersed in an infinitely strong confining magnetic field.

The dielectric tensor of the plasma can be given by

$$[\varepsilon] = \varepsilon_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\omega_{\rm p}^2/\omega^2 \end{bmatrix}, \qquad (1)$$

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where $\omega_{\rm p} = (e^2 n_{\rm p}/\varepsilon_0 m)^{1/2}$ is the plasma frequency; $e$ and $m$ are the electron charge and mass, respectively.

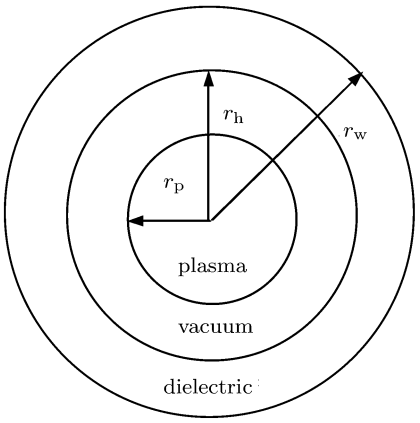



**Fig.1.** Cross section of the slow wave structure.

## 3. Fields in each region

By means of sheath model, the fields in each region can be written as[8]

In the plasma region ($0 \le r \le r_{\rm p}$):

$$\nabla_\perp^2 E_z - \Big(k_z^2 - \frac{\omega^2}{c^2}\Big)\Big(1 - \frac{\omega_{\rm p}^2}{\omega^2}\Big)E_z = 0\,, \tag{2}$$

where $k_z$ and $\omega$ are the axial wave number and frequency of the perturbation, respectively and $c$ is the velocity of light in vacuum.

$$\begin{aligned}
E_z &= A_1 I_0(T_1 r)\,,\\
E_r &= \frac{-{\rm i}k_z T_1}{T_2^2} A_1 I_1(T_1 r)\,,\\
E_\theta &= \frac{{\rm i}\omega\mu_0}{T_2} A_2 I_1(T_2 r)\,,\\
H_z &= A_2 I_0(T_2 r)\,,\\
H_r &= \frac{-{\rm i}k_z}{T_2} A_2 I_1(T_2 r)\,,\\
H_\theta &= \frac{-{\rm i}\omega\varepsilon_0 T_1}{T_2^2} A_1 I_1(T_1 r),
\end{aligned} \tag{3}$$

where

$$T_1^2 = \Big(k_z^2 - \frac{\omega^2}{c^2}\Big)\Big[1 - \frac{\omega_{\rm p}^2}{\omega^2}\Big]\,, \tag{4}$$

$$T_2^2 = k_z^2 - \frac{\omega^2}{c^2}\,. \tag{5}$$

In the vacuum region ($r_{\rm p} \le r \le r_{\rm h}$):

$$\nabla_\perp^2 E_z - \Big(k_z^2 - \frac{\omega^2}{c^2}\Big)E_z = 0\,, \tag{6}$$

$$\begin{aligned}
E_z &= A_3 I_0(T_2 r) + A_4 K_0(T_2 r)\,,\\
E_r &= \frac{-{\rm i}k_z}{T_2}[A_3 I_1(T_2 r) - A_4 K_1(T_2 r)]\,,\\
E_\theta &= \frac{{\rm i}\omega\mu_0}{T_2} A_2 I_1(T_2 r)\,,\\
H_z &= A_2 I_0(T_2 r)\,,\\
H_r &= \frac{-{\rm i}k_z}{T_2} A_2 I_1(T_2 r)\,,\\
H_\theta &= \frac{-{\rm i}\omega\varepsilon_0}{T_2}[A_3 I_1(T_2 r) - A_4 K_1(T_2 r)]\,.
\end{aligned} \tag{7}$$

In the dielectric region ($r_{\rm h} \le r \le r_{\rm w}$):

$$\nabla_\perp^2 E_z - (k_z^2 - \omega^2\varepsilon_{\rm d}\mu_{\rm d})E_z = 0\,, \tag{8}$$

$$\begin{aligned}
E_z &= A_5 I_0(T_3 r) + A_6 K_0(T_3 r)\,,\\
E_r &= \frac{-{\rm i}k_z}{T_3}[A_5 I_1(T_3 r) - A_6 K_1(T_3 r)]\,,\\
E_\theta &= \frac{{\rm i}\omega\mu_{\rm d}}{T_3}[A_7 I_1(T_3 r) - A_8 K_1(T_3 r)]\,,\\
H_z &= A_7 I_0(T_3 r) + A_8 K_0(T_3 r)\,,\\
H_r &= \frac{-{\rm i}k_z}{T_3}[A_7 I_1(T_3 r) - A_8 K_1(T_3 r)]\,,\\
H_\theta &= \frac{-{\rm i}\omega\varepsilon_{\rm d}}{T_3}[A_5 I_1(T_3 r) - A_6 K_1(T_3 r)]\,,
\end{aligned} \tag{9}$$

where

$$T_3^2 = k_z^2 - \omega^2\varepsilon_{\rm d}\mu_{\rm d}\,. \tag{10}$$

## 4. Dispersion relation

The boundary conditions in each region can be written as

$$\begin{aligned}
&E_z(r = r_{\rm p}^-) = E_z(r = r_{\rm p}^+)\,,\\
&H_\theta(r = r_{\rm p}^-) = H_\theta(r = r_{\rm p}^+)\,,\\
&E_z(r = r_{\rm h}^-) = E_z(r = r_{\rm h}^+)\,,\\
&E_z(r = r_{\rm h}^-) + E_\theta(r = r_{\rm h}^-)\cot\phi = 0\,,\\
&E_z(r = r_{\rm h}^+) + E_\theta(r = r_{\rm h}^+)\cot\phi = 0\,,\\
&H_z(r = r_{\rm h}^-) + H_\theta(r = r_{\rm h}^-)\cot\phi\,,\\
&\quad = H_z(r = r_{\rm h}^+) + H_\theta(r = r_{\rm h}^+)\cot\phi\,,\\
&E_z(r = r_{\rm w}) = 0\,,\\
&E_\theta(r = r_{\rm w}) = 0\,.
\end{aligned} \tag{11}$$

Substituting Eqs.(3), (7) and (9) into Eqs.(10), we can derive the relative amplitude coefficients of the fields in each region as

$$A_2 = \frac{{\rm i}T_2 {\rm tg}\,\phi}{\omega\mu_0}\frac{A_3 I_0(T_2 r_{\rm h}) + A_4 K_0(T_2 r_{\rm h})}{I_1(T_2 r_{\rm h})},$$

$$A_3 = \frac{A_1}{T_2}\frac{T_2 I_0(T_1 r_\mathrm{p}) K_1(T_2 r_\mathrm{p}) + T_1 I_1(T_1 r_\mathrm{p}) K_0(T_2 r_\mathrm{p})}{I_0(T_2 r_\mathrm{p}) K_1(T_2 r_\mathrm{p}) + I_1(T_2 r_\mathrm{p}) K_0(T_2 r_\mathrm{p})},$$

$$A_4 = \frac{A_1}{T_2}\frac{T_2 I_0(T_1 r_\mathrm{p}) I_1(T_2 r_\mathrm{p}) - T_1 I_1(T_1 r_\mathrm{p}) I_0(T_2 r_\mathrm{p})}{I_0(T_2 r_\mathrm{p}) K_1(T_2 r_\mathrm{p}) + I_1(T_2 r_\mathrm{p}) K_0(T_2 r_\mathrm{p})},$$

$$A_5 = \frac{A_3 I_0(T_2 r_\mathrm{h}) + A_4 K_0(T_2 r_\mathrm{h})}{I_0(T_3 r_\mathrm{h}) - I_0(T_3 r_\mathrm{w}) K_0(T_3 r_\mathrm{h})/K_0(T_3 r_\mathrm{w})},$$

$$A_6 = -\frac{I_0(T_3 r_\mathrm{w})}{K_0(T_3 r_\mathrm{w})} A_5,$$

$$A_7 = \frac{\mathrm{i} T_3 \mathrm{tg}\phi}{\omega \mu_2} \times \frac{A_5 I_0(T_3 r_\mathrm{h}) + A_6 K_0(T_3 r_\mathrm{h})}{I_1(T_3 r_\mathrm{h}) - I_1(T_3 r_\mathrm{w}) K_1(T_3 r_\mathrm{h})/K_1(T_3 r_\mathrm{w})},$$

$$A_8 = \frac{I_1(T_3 r_\mathrm{w})}{K_1(T_3 r_\mathrm{w})} A_7 . \tag{12}$$

Accordingly, the dispersion relation can be obtained and written as

$$\begin{aligned} T_2^2 \mathrm{tg}^2\phi \Bigg\{ & \frac{I_0(T_2 r_\mathrm{h})}{I_1(T_2 r_\mathrm{h})}[A_3 I_0(T_2 r_\mathrm{h}) + A_4 K_0(T_2 r_\mathrm{h})] \\ & - \frac{T_3}{T_2}\frac{\mu_0}{\mu_\mathrm{d}}\Big[I_0(T_3 r_\mathrm{h}) + \frac{I_1(T_3 r_\mathrm{w})}{K_1(T_3 r_\mathrm{w})} K_0(T_3 r_\mathrm{h})\Big] \\ & \times \frac{A_5 I_0(T_3 r_\mathrm{h}) + A_6 K_0(T_3 r_\mathrm{h})}{I_1(T_3 r_\mathrm{h}) - I_1(T_3 r_\mathrm{w}) K_1(T_3 r_\mathrm{h})/K_1(T_3 r_\mathrm{w})} \Bigg\} \\ = \omega^2 \varepsilon_0 \mu_0 \Bigg\{ & A_3 I_1(T_2 r_\mathrm{h}) - A_4 K_1(T_2 r_\mathrm{h}) - \frac{T_2}{T_3} \\ & \times \frac{\varepsilon_\mathrm{d}}{\varepsilon_0}[A_5 I_1(T_3 r_\mathrm{h}) - A_6 K_1(T_3 r_\mathrm{h})] \Bigg\}. \end{aligned} \tag{13}$$

## 5. Computation and conclusion

By numerical computation for dispersion equation (13), we can obtain the dispersion relation of the slow wave structure filled with plasma.[9] The relative amplitude of the axial electric field can be obtained from Eqs.(3), (7), (9) and (12) when the wave number is fixed.[10] Figures 2–4 illustrate respectively the distribution of the axial electric field with different plasma electron densities, plasma column radius and different dielectric constants. The parameters employed in the computations are given by $r_\mathrm{h} = 0.6$ cm, $r_\mathrm{w} = 1.0$ cm, $\phi = 6.0^0$, $\mu_\mathrm{d} = \mu_0$, $k_z = 12$ cm$^{-1}$.

In Fig.2, we use $r_\mathrm{p} = 0.45$ cm, $\varepsilon_\mathrm{d} = 2.0\varepsilon_0$. Five radial profiles of the axial electric field are shown corresponding to increasing plasma electron densities. It is apparent from the curves that the presence of plasma greatly enhances the value of the axial electric field.[11] As the plasma electron density increases, the value of the axial electric field also increases.

In Fig.3, $n_\mathrm{p} = 5.0 \times 10^{11}$ cm$^{-3}$, $\varepsilon_\mathrm{d} = 2.0\varepsilon_0$. Figure 3 shows the radial profiles of the axial electric field versus different plasma column radius. It is found that there is a best plasma column radius corresponding to the strongest axial electric field in the axes as the plasma electron density is fixed.

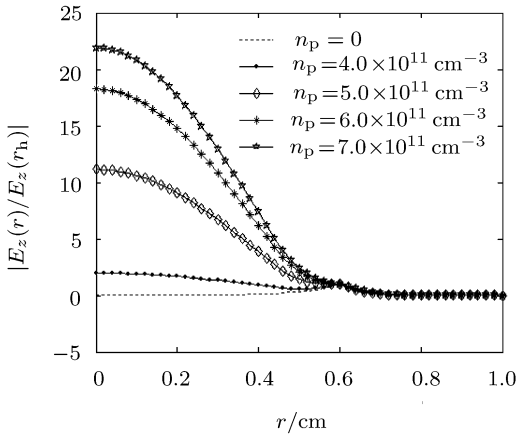



**Fig.2.** Five radial profiles of the axial electric field with different plasma electron densities.

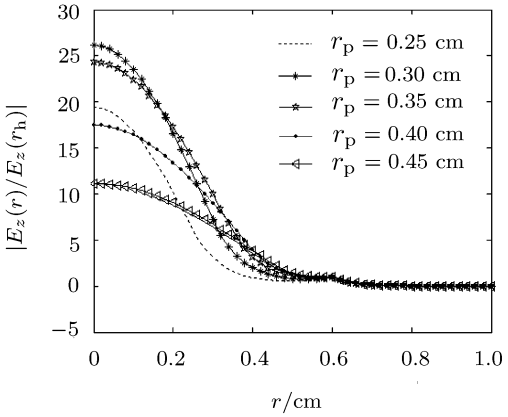



**Fig.3.** Five radial profiles of the axial electric field with different plasma column radii.

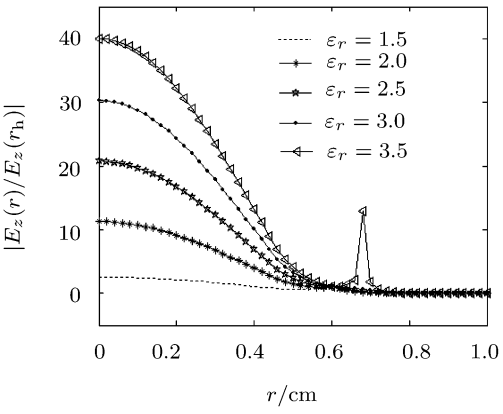



**Fig.4.** Five radial profiles of the axial electric field with different dielectric constants.

In Fig.4, $r_{\rm p} = 0.45$ cm, $n_{\rm p} = 5.0\times10^{11}$ cm$^{-3}$. The radial profiles of the axial electric field are changed dramatically as the dielectric constant increases. The enhancement of the dielectric constant can increase the axial electric field in the central region. The result is favourable to the case of using solid electron beam.

We have numerically analysed the axial electric field of a plasma-loaded helix slow wave structure in an infinitely large axial magnetic field. However, actually most travelling wave tubes operate in a finite magnetic field. Therefore the above derived from the theoretic analysis needs to be proved in practice.

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