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Resolution and noise in ghost imaging with classical thermal light*

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The resolution and classical noise in ghost imaging with a classical thermal light are investigated theoretically. For ghost imaging with a Gaussian Schell model source, the dependences of the resolution and noise on the spatial coherence of the source and the aperture in the imaging system are discussed and demonstrated by using numerical simulations. The results show that an incoherent source and a large aperture will lead to a good image quality and small noise.

Keywords: ghost imaging, thermal light, signal-to-noise rate
PACC: 4230V, 4225K, 4250

1. Introduction

Imaging a quantum state,[1] or using a quantum source for imaging an object[2] has attracted much attention in recent years. Particularly the topic of ghost imaging has aroused the interest of all.[3−7] Ghost imaging is a kind of correlated imaging technique. It relies on the correlation properties between two spatially separated beams. The ghost imaging is proposed and studied by the use of entangled photon pairs created by spontaneous parametric down conversion (SPDC). One of the entangled photons propagates through a known (reference) optical imaging system, and the other travels through an unknown (test) optical imaging system in which an object is located. By measuring the coincidence rate of these photon pairs at the reference and test detectors, one can obtain the image of the object as a function of the transverse position of the reference photon.

Recently, there has been much interest in whether it is possible to realize ghost imaging by using the classically correlated sources.[8−16] It is reported in Ref.[8] that only quantum entangled sources can be used to realize the ghost imaging, but the classical light sources cannot be used to produce the image of the object. However, the experiment shows that the classically correlated beams can also be used to produce a ghost image.[9,10] Moreover, recent theoretical results indicate that even classical thermal source can be used to realize ghost imaging.[11−14] So the ghost imaging can be realized in both classical statistical optics[11] and quantum optics.[12] Experimental evidences of the ghost imaging realized by using a classical thermal light have been reported.[15−17]

Resolution and noise are two most important factors to characterize an imaging system. There is some work to study the resolution in ghost imaging with a thermal light,[18] but many subjects remain to be studied: for example, the noise properties. In this paper, we theoretically investigate the classical noise in ghost imaging with a thermal light. Based on the theory of classical statistical optics,[19] we calculate the point spread function and the variance of the ghost image.

According to Ref.[19], the shot noise is the dominant noise source for true thermal sources. But for pseudo-thermal light sources, the classical noise is important and can be dominant. As several recent experiments are performed with pseudo-thermal sources,[15−17] in this paper our main interest is focused on the noise from the classically random fluctuations of the thermal source, and the possible quantum fluctuations from the detectors are not considered.

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2. Theoretical description

Let us consider the ghost imaging scheme given in Fig.1. An incoherent thermal beam S is divided into two correlated beams by the beam splitter BS. The two beams then propagate through two different imaging systems.

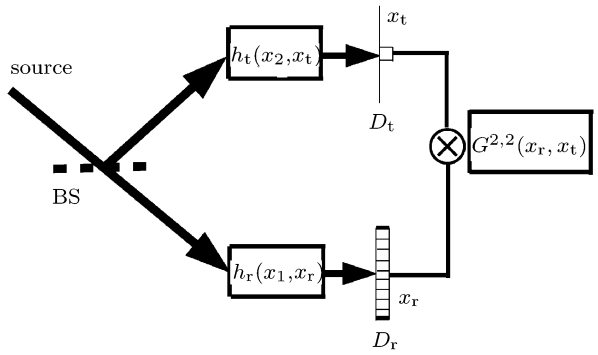


Fig.1. A setup of ghost imaging with a classical thermal light. A thermal source beam is divided by the beam splitter BS into two correlated beams. One transmits through the test system $h_t(x_2,x_t)$ of an unknown object, and the other through the reference system with a known $h_r(x_1,x_r)$. Two detectors D_t and D_r record the intensity distribution. A correlator is used to measure the correlation function of intensity fluctuations, which gives the ghost image of the object.

In many cases, the classical thermal light can be modelled by a complex circular Gaussian random process with zero mean,[19] so any correlation properties of the thermal light can be expressed with the second order correlation function,

$$\Gamma(x_1,x_2)=\langle E(x_1)E^*(x_2)\rangle\,. \tag{1}$$

Here, $\langle\cdots\rangle$ denotes the statistical average. After the propagation of the light, the optical field in the two different detectors are

$$E_k(x_k)=\int \mathrm{d}x\,E(x)h_k(x,x_k),\quad k=\mathrm{r}\text{ or }\mathrm{t}, \tag{2}$$

where $h_r(x,x_r)$ and $h_t(x,x_t)$ are the impulse response functions of the reference and test imaging systems, respectively. Thus, the intensity distribution in the test or reference detector is

$$\langle I_k(x_k)\rangle=\int \mathrm{d}x_1\mathrm{d}x_2\,\Gamma(x_1,x_2)\times h_k(x_1,x_k)h_k^*(x_2,x_k)\,, \tag{3}$$

while the ghost image obtained from the correlation between the intensity fluctuations in the two detectors takes the form[11]

$$G(x_r,x_t)=\langle\Delta I_r(x_r)\Delta I_t(x_t)\rangle$$
$$=\Big|\int \mathrm{d}x_1\mathrm{d}x_2\,\Gamma(x_1,x_2)h_r(x_1,x_r)h_t^*(x_2,x_t)\Big|^2, \tag{4}$$

where $\Delta I_k(x_k)=I_k(x_k)-\langle I_k(x_k)\rangle$, with $k=\mathrm{t}$ or r.

To see the resolution in ghost imaging with a classical thermal light, we study a specific imaging system whose setup is similar to that in Ref.[12]. The test imaging system is composed of an object characterized by a transmission function $t(x)$, a lens, and a detector (D_t). The lens is located at a focal distance f from both the object and D_t. Any optical imaging system may contain different optical elements, such as lens and apertures. If the size of the lens is much larger than the object, h_t has the form[20]

$$h_t(x_t,x)=-\frac{\mathrm{i}}{\lambda f}t(x)\exp\Big(-\frac{2\pi\mathrm{i}}{\lambda f}x_t x\Big), \tag{5}$$

where λ is the wavelength. In the reference imaging system, a lens is located at a distance $2f$ from both the source and the detector D_r. Then h_r has the form[20]

$$h_r(x_r,x)=\frac{1}{4\lambda^2f^2}P\Big(\frac{x_r+x}{2\lambda f}\Big)\times\exp\Big(\frac{\mathrm{i}\pi}{2\lambda f}(x_r^2+x^2)\Big), \tag{6}$$

where $P(u)$ is the Fourier transform of the pupil function ($p(x)$) of the lens. If the size of the aperture is D, then $P(u)=\mathrm{sinc}(uD)$. So a large aperture will lead to a good image resolution in the reference system.

Substituting Eqs.(5) and (6) into Eq.(4) and assuming a point-like test detector (located at $x_t=0$), we have

$$G(x_r,0)=\Big|\int \mathrm{d}x_1\mathrm{d}x_2\,\Gamma(x_1,x_2)t^*(x_1)P\Big(\frac{x_r+x_2}{2\lambda f}\Big)\times\exp\Big(\frac{\mathrm{i}\pi}{2\lambda f}(x_r^2+x_2^2)\Big)\Big|^2. \tag{7}$$

If the object is a single point located at $x_1=0$, and $t(x_1)=\delta(x_1)$, then the corresponding image represents the point spread function,

$$H(x_r)=\Big|\int \mathrm{d}x_2\,\Gamma(0,x_2)P\Big(\frac{x_r+x_2}{2\lambda f}\Big)\times\exp\Big(\frac{\mathrm{i}\pi}{2\lambda f}(x_r^2+x_2^2)\Big)\Big|^2. \tag{8}$$

The point spread function $H(x_r)$ characterizes the resolution of the ghost imaging system. The integrand in Eq.(8) contains both the source contribution (Γ) and the reference imaging system contribution (P).

Suppose that, as (done) that in Ref.[21], the incident thermal beam can be described by the Gaussian Schell model,

$$\Gamma(x_1,x_2)=\exp\Big(-\frac{x_1^2+x_2^2}{2a^2}\Big)\exp\Big(-\frac{(x_1-x_2)^2}{b^2}\Big). \tag{9}$$

Here, a represents the size of the source and b the transverse coherent length. In this case, the point spread function is assumed to be

$$H(x_{\rm r}) = \Big|\int {\rm d}x_2 \exp\Big[-\Big(\frac{1}{2a^2}+\frac{1}{b^2}\Big)x_2^2\Big] \times P\Big(\frac{x_{\rm r}+x_2}{2\lambda f}\Big)\exp\Big(\frac{{\rm i}\pi(x_{\rm r}^2+x_2^2)}{2\lambda f}\Big)\Big|^2. \quad (10)$$

Obviously, the resolution depends on the aperture size ($P(x)$), the source size (a), and the transverse coherent length (b). Large aperture and small source size are favourable for resolution. Moreover, poor coherence (small b) results in better resolution! Generally, b is much smaller than a for a thermal light source, so the transverse coherent length affects the image resolution more significantly.

Now, we turn to the study of the fluctuation in $G(x_{\rm r}, x_{\rm t})$, measured by

$$\Delta G(x_{\rm r}, x_{\rm t}) = \times\sqrt{\langle[\Delta I_{\rm r}(x_{\rm r})\Delta I_{\rm t}(x_{\rm t})]^2\rangle - [G(x_{\rm r}, x_{\rm t})]^2}. \quad (11)$$

To obtain $\langle[\Delta I_{\rm r}(x_{\rm r})\Delta I_{\rm t}(x_{\rm t})]^2\rangle$, the correlations up to eighth order in field are needed. On the assumption that the source is modelled by a complex circular Gaussian random process with zero mean, all higher order correlation functions can be expressed in terms of second order correlation $\Gamma(x_1, x_2)$.[19] In our case, we have

$$\langle E(x_1)E(x_2)E(x_3)E(x_4)E^*(y_1)E^*(y_2)E^*(y_3)E^*(y_4)\rangle = \sum_{\pi}\Gamma(x_1, y_{i1})\Gamma(x_2, y_{i2})\Gamma(x_3, y_{i3})\Gamma(x_4, y_{i4}), \quad (12)$$

where $\sum_{\pi}$ denotes the summation over all 24 possible permutations of the subscripts $i1, i2, i3, i4$. By direct calculations, we find

$$\langle[\Delta I_{\rm r}(x_{\rm r})\Delta I_{\rm t}(x_{\rm t})]^2\rangle = [\langle I_{\rm r}(x_{\rm r})\rangle\langle I_{\rm t}(x_{\rm t})\rangle + 2G(x_{\rm r}, x_{\rm t})]^2\,. \quad (13)$$

It is easy to verify that $\Delta G(x_{\rm r}, x_{\rm t}) > \sqrt{3}G(x_{\rm r}, x_{\rm t})$. Thus, in a single shot measurement, the noise is always larger than the signal, and the object cannot be resolved. In practical experiments as done in Ref.[16], thousands of images are measured and averaged to give a high-quality signal. Suppose that there are N shot measurements, then the averaged signal is

$$\bar{G}(x_{\rm r}, x_{\rm t}) = G(x_{\rm r}, x_{\rm t})\,, \quad (14)$$

but the variance of $\bar{G}(x_{\rm r}, x_{\rm t})$ will be $\sqrt{N}$ times smaller,

$$\overline{\Delta G}(x_{\rm r}, x_{\rm t}) = \frac{\Delta G(x_{\rm r}, x_{\rm t})}{\sqrt{N}}\,. \quad (15)$$

The signal-to-noise ratio will therefore be enhanced by a factor of $\sqrt{N}$.

3. Results and discussion

In the following, we give numerical simulations to demonstrate the obtained results. The ghost imaging scheme, with Eqs.(5) and (6) at the fixed values of $f = 100$ mm, $\lambda = 650$ nm, and $N = 1000$, is used throughout the simulation. The thermal light is assumed to be of the Gaussian Schell model (Eq.(9)). A double slit is used as the object, with each slit having a width $w = 0.05$ mm and the separation between them being $d = 1$ mm. The lens aperture size D, source size a, and transverse coherent length b are chosen to be variable parameters in our simulation.

Depicted in Fig.2 are for $a = 2$ mm, $b = 0.05$ mm, and $D = 10$ mm, the conditional correlation, $I_0(x_{\rm r}) = G(x_{\rm r}, 0)$ (dashed curve) and its fluctuation, $\Delta I_0(x_{\rm r}) = \overline{\Delta G}(x_{\rm r}, 0)$ (see Eq.(15)) (solid curves). Both curves are normalized with a maximum value of $\Delta I_0(x_{\rm r})$. It is clearly shown that for the given parameters the classical thermal light gives out a good image quality in terms of both resolution and noise level.

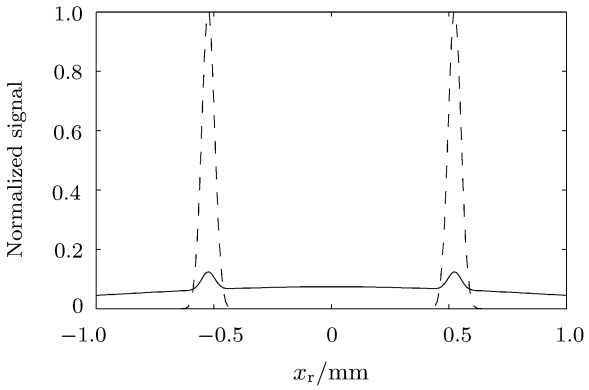


Fig.2. Normalized conditional correlation $I_0(x_{\rm r})$ (dashed line) and its fluctuation $\Delta I_0(x_{\rm r})$ (solid line) as a function of $x_{\rm r}$. Parameters are $N = 1000$, $f = 100$ mm, $\lambda = 650$ nm, $a = 2$ mm, $b = 0.05$ mm, and $D = 10$ mm.

Figure 3 shows the same quantities of Fig.2 but with a smaller value of source size $a = 0.7$ mm. As inferred from Eq.(10), a reduced a value would lead to a reduction in image resolution. However, since $b \ll a$, the resolution is affected mainly by b. Therefore, the change in resolution in Fig.3 is not quite prominent as compared with that in Fig.2. The parameter a affects the noise level rather significantly; and at the present value of a, there is shown in Fig.3 the solid curve with

an additional noise peak being at $x_r = 0$, which is different from any test signal position.

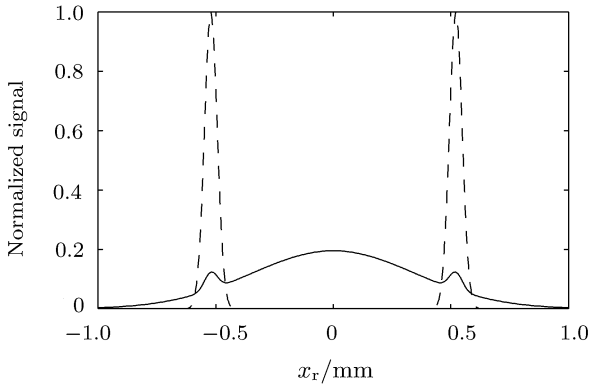


Fig.3. The same as Fig.2, but with a reduced source size of $a = 0.7$ mm.

The source coherence, i.e. the transverse coherent length b here, plays a very important role in ghost imaging with a classical thermal light. With the coherent length increasing from $b = 0.05$ mm in Fig.2 to $b = 0.2$ mm in Fig.4, both the resolution and noise are deteriorated. The images of two slits in Fig.4 are also significantly deviated from their original shapes. The fluctuation also increases as compared with that in Fig.2. We have thus demonstrated that the source with poor coherence is in favour of both the image quality and SNR.

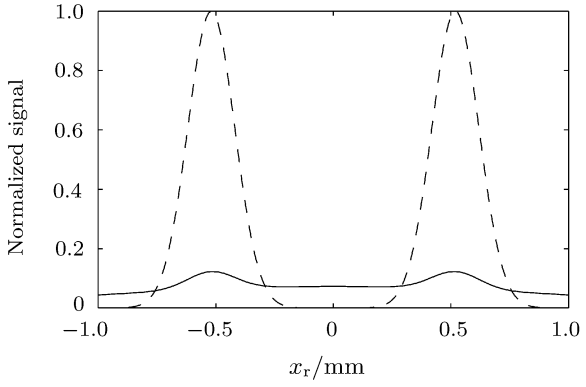


Fig.4. The same as Fig.2, but with an increased transverse coherence length of $b = 0.2$ mm.

As can be seen from Eq.(10), the aperture of the imaging system can also affect the image quality. Used in Fig.5 for demonstration is an aperture lens of $D = 2$ mm, or one fifth of that used in Fig.2. Small aperture makes the resolution of reference imaging system degraded; and due to Eq.(10), the total image signal resolution is also degraded. Shown in Fig.5 is not only the noise increasing in comparison with Fig.2, but also some undesired lobes appearing. Therefore, an appropriately large lens is essential to the performance of reference imaging system.

The noise in the ghost imaging with a thermal light is quite different from that in the case of entangled two-photon ghost imaging, which has been studied in Ref.[22]. The reason is that the entangled two-photon state is a kind of highly nonclassical quantum state. Using the symbols in Ref.[22], instead of Eq.(12), we have

$$\langle \hat{a}_t^\dagger(x_1)\hat{a}_r^\dagger(y_1)\hat{a}_t(x_2)\hat{a}_r(y_2)\hat{a}_t^\dagger(x_3)\hat{a}_r^\dagger(y_3)\hat{a}_t(x_4)\hat{a}_r(y_4)\rangle = \varphi^*(x_1, y_1)\varphi(x_4, y_4)\delta(x_3 - x_2)\delta(y_3 - y_2)\,. \quad (16)$$

This equation makes the results in Ref.[22] greatly different from our results in this paper.

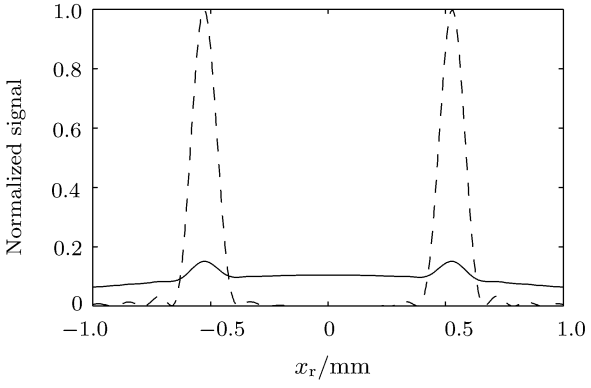


Fig.5. The same as Fig.2, but with a reduced lens aperture of $D = 2$ mm.

4. Conclusions

We have theoretically investigated the resolution and classical noise in ghost imaging with a classical thermal light. Rigorous results of the resolution and noise are presented. For ghost imaging with a Gaussian Schell model source, poor spatial coherence leads to a good resolution and small noise. Using large apertures can increase the resolution and decrease the noise at the same time. We must point out that our results of noise are applicable only to pseudo-thermal sources. For the very recent ghost imaging experiment with a true thermal light,[23] quantum shot noise has to be taken into consideration, which is our future work.

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