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The CP^1 nonlinear sigma model with Chern–Simons term in the Faddeev–Jachiw quantization formalism*

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Using the Faddeev–Jackiw (FJ) quantization method, this paper treats the CP^1 nonlinear sigma model with Chern–Simons term. The generalized FJ brackets are obtained in the framework of this quantization method, which agree with the results obtained by using the Dirac's method.

Keywords: Faddeev–Jackiw quantization method, CP^1 nonlinear sigma model, Chern–Simons theories, constrained systems

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1. Introduction

Recently there has been an upsurge of interest in the study of the nonlinear sigma model in (2+1) dimensions, and in the effect of the Faddeev–Jackiw (FJ) quantization method. The O(3) nonlinear sigma model was studied by the addition of a Hopf term or by the introduction of a Chern–Simons term: both have revealed the occurrence of fractional spin and statistics.[1−7] The CP^1 nonlinear sigma model is studied by including the Chern–Simons term[8] or including the Chern–Simons term and a Maxwell term:[9,10] the results are both fractional spin and statistics. The FJ quantization method is widely applied in the constrained system as a self-dual field,[11] electromagnetic field,[12] Yang–Mills field,[12] supersymmetry,[13,14] supergravity,[15] nonlinear sigma model,[16] and complex scalar field coupled to the Abel Chern–Simons term.[17] In this paper we will study the CP^1 nonlinear sigma model in the framework of the FJ quantization method; the results agree with the results obtained by using the Dirac method.[8] It is shown that this method is quite straightforward and elegant for this system.

2. FJ quantization method

We start this section by reviewing very briefly the FJ quantization method[18] in field theories, for this method as necessarily describing a constrained system, the Lagrangian density contains the term which is a first order in time derivative.

We consider a system whose Lagrangian is given by

$$L = a_i(\phi)\dot{\phi}_i - V(\phi), \qquad (i = 1, 2, \cdots, n), \qquad (1)$$

where $\phi_i = \phi_i(x) = \phi_i(t, \boldsymbol{x})$ are field variables. Based on the canonical one-form $a_i(\phi)$, the symplectic matrix $f_{ij}(x, y)$ is

$$f_{ij}(x, y) = \frac{\delta a_j(y)}{\delta \phi_i(x)} - \frac{\delta a_i(x)}{\delta \phi_j(y)}, \qquad (2)$$

where $\phi_i(t, \boldsymbol{x})$ represents any field belonging to the symplectic set.

When the symplectic matrix is singular and leads to constraints,[19−21] the symplectic potential would become symplectic potential density, which have to be integrated. The constraints are expressed as

$$\Omega_\alpha = \int \mathrm{d}x(\nu_\alpha(t, \boldsymbol{x})^{\mathrm{T}} \frac{\delta}{\delta \phi_\alpha(t, \boldsymbol{x})} \times \int \mathrm{d}y V(t, \boldsymbol{y}) = 0 \quad (\alpha = 1, 2, \cdots, m), \qquad (3)$$

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where $\nu_\alpha(t,\boldsymbol{x})$ is zero-model which is multiplied by the matrix $f_{ij}(x)$ to equal zero.

The quantities Ω_α are the constraints in the FJ symplectic formalism, and they are introduced in the Lagrangian by using Lagrangian multipliers:

$$L_i = a_i(\phi)\dot{\phi}_i - \lambda^{(\alpha)}\Omega_\alpha - V^{(0)}(\phi)\,. \tag{4}$$

At this point one can run the symplectic algorithm once again. Enlarging the configuration space by considering the set of variables $(\phi_i, \xi^{(\alpha)})$, by redefining the $\lambda^{(\alpha)}$ variables, relating to $\xi^{(\alpha)}$ we can set

$$\lambda^{(\alpha)} = -\dot{\xi}^{(\alpha)}\,. \tag{5}$$

Therefore, the first-iterated Lagrangian is written as

$$L^{(1)} = a_i(\phi)\dot{\phi}_i + \Omega_{(\alpha)}\dot{\xi}^{(\alpha)}\,, \tag{6}$$

where

$$V^{(1)}(\phi) = V^{(0)}(\phi)|_{\Omega_{(\alpha)}=0}\,. \tag{7}$$

In terms of the new set of dynamical variables, the symplectic matrix is completed which can be written as

$$f_{ij}^{(1)} = \begin{pmatrix} \bar{f}_{ij} & \dfrac{\delta\Omega_{(\alpha)}}{\delta\phi_j} \\ \dfrac{\delta\Omega_{(\alpha)}}{\delta\phi_i} & 0 \end{pmatrix}, \tag{8}$$

where $\bar{f}_{ij}$ represents the square nonsingular matrix (2) constructed from the original symplectic set of field variables. The term $\delta\Omega_{(\alpha)}/\delta\phi_j$ represents a rectangular matrix. This iterative procedure modifies the symplectic matrix until all the nonorthogonal zero-modes have been eliminated. If the symplectic matrix is nonsingular, the generalized FJ brackets is given by[16]

$$\begin{aligned}[\phi_i,\phi_j] &= (\bar{f}_{ij})^{-1}(x,y) \\ &- \int \mathrm{d}z\mathrm{d}w\Big\{\Big[\int \mathrm{d}u(\bar{f}_{ik})^{-1}(x,u)\cdot\frac{\delta\Omega_b(z)}{\delta\phi^k(u)}\Big] \\ &\times \int \mathrm{d}v\Big[(\bar{f}_{ij})^{-1}(z,v)\cdot\frac{\delta\Omega_a(w)}{\delta\phi^l(v)}\Big]G^{ab}(w,y)\Big\}\,,\end{aligned} \tag{9}$$

where

$$\int \mathrm{d}z\,\Omega_{ab}(x,z)G^{bc}(z,y) = \delta_a^c\delta(x-y)\,, \tag{10}$$

$$\Omega_{ab}(x,z) = \int \mathrm{d}y\mathrm{d}v\frac{\delta\Omega_a(x)}{\delta\phi_j(y)}(\bar{f}_{ij})^{-1}(y,v)\frac{\delta\Omega_b(z)}{\delta\phi_i(v)}\,. \tag{11}$$

If the original matrix is nonsingular then the generalized brackets can be obtained directly from its inversed matrix.

3. The CP[1] nonlinear sigma model with Chern–Simons term in the FJ quantization formalism

We consider the gauged CP[1] nonlinear sigma model with Chern–Simons term in (2+1) dimensions, the Lagrangian density of the system is given by[8]

$$\mathcal{L} = \frac{1}{\gamma_0}(D_\mu Z_k)^*(D^\mu Z_k) + \frac{\theta}{4\pi^2}\varepsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda\,, \tag{12}$$

where γ_0 is coupling constant, $k = 1, 2$ and z_k is a two-component complex field which satisfies the constraint,

$$Z_k^* Z_k = |Z_1|^2 + |Z_2|^2 = 1\,, \tag{13}$$

where A_μ is the Chern–Simons gauge field, and the covariant derivative $D_\mu = \partial_\mu - \mathrm{i}A_\mu$. The Levi-Civita symbol $\varepsilon_{\mu\nu\lambda} = 1$ and $g_{\eta\nu} = (1,-1,-1)$. We take Cho and Kimm's cue from Ref.[22], and the Lagrangian of the system can be written as

$$\begin{aligned}\mathcal{L}^{(0)} &= \frac{1}{\gamma_0}(D_\mu Z_k)^*(D^\mu Z_k) \\ &+ \frac{\theta}{4\pi^2}\varepsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda + \lambda(Z_k Z_k^* - 1)\,.\end{aligned} \tag{14}$$

From Eq.(14) it is easy to write the first-order Lagrangian by introducing the canonical conjugate momentum π_k and $\bar{\pi}_k$ with respect to the field Z_k and Z_k^*,

$$\pi_k = \frac{\partial\mathcal{L}^{(0)}}{\partial\dot{Z}_k} = \frac{1}{\gamma_0}(D_0 Z_k)^*\,, \tag{15}$$

$$\bar{\pi}_k = \frac{\partial\mathcal{L}^{(0)}}{\partial\dot{Z}_k^*} = \frac{1}{\gamma_0}(D_0 Z_k)\,. \tag{16}$$

Because the Lagrangian density (14) is already of first order in the Chern–Simons gauge field A_μ, in the Lagrangian of the FJ formalism no new variables appear. The initial set of symplectic variables defining the extended space is given by the set $(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_0, A_i, \lambda)$, and so the starting Lagrangian density is written in first-order form as follows:

$$\begin{aligned}\mathcal{L}^{(0)} &= \pi_k\dot{Z}_k + \bar{\pi}_k\dot{Z}_k^* - \frac{\theta}{4\pi^2}\varepsilon^{ij}A_i\dot{A}_j \\ &+ V^{(0)}(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_0, A_i, \lambda)\,,\end{aligned} \tag{17}$$

where the starting symplectic potential is given by

$$\begin{aligned}V^{(0)}&(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_0, A_i, \lambda) \\ &= -\gamma_0\pi_k\bar{\pi}_k + \frac{1}{\gamma_0}(D_\mu Z_k)^*(D^i Z_k)\end{aligned}$$

$$+\frac{\theta}{2\pi^2}\varepsilon^{ij}A_0\partial_i A_j - \lambda(Z_k Z_k^* - 1) - A_0 J_0\,, \quad (18)$$

where

$$J_0 = \mathrm{i}(Z_k\pi_k - Z_k^*\bar{\pi}_k)\,. \quad (19)$$

In the case under consideration the symplectic matrix $f^{(0)}(x,y)$ is constructed from the original set of field variables $(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_0, A_i, \lambda)$, which is given by

$$f^{(0)}(x,y) = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dfrac{\theta}{2\pi^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \delta(x-y)\,. \quad (20)$$

From above equation (20) it is easy to see that there are two zero-modes, the corresponding zero-modes are given by

$$\nu_1 = (0,0,0,0,\nu_{A_0},0,0)\,, \quad (21a)$$

$$\nu_2 = (0,0,0,0,0,0,\nu_\lambda)\,, \quad (21b)$$

where ν_{A_0} and ν_λ are arbitrary functions. The constraints are evaluated from Eq.(3),

$$\Omega_1 = -J_0 + \frac{\theta}{2\pi^2}\varepsilon^{ij}\partial_i A_j = 0\,, \quad (22)$$

$$\Omega_2 = 1 - Z_k Z_k^* = 0\,. \quad (23)$$

According to the symplectic algorithm, the constraints are introduced in the Lagrangian density by using Lagrangian multipliers, and enlarging the space by considering the set of variables $(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_i, \xi_{A_0}, \xi_\lambda)$. Therefore, the first-iterated Lagrangian density is written as

$$\mathcal{L}^{(1)} = \pi_k\dot{Z}_k + \bar{\pi}_k\dot{Z}_k^* - \frac{\theta}{4\pi^2}\varepsilon^{ij}A_i\dot{A}_j + \dot{\xi}_{A_0}\Big(-J_0 + \frac{\theta}{2\pi^2}\varepsilon^{ij}\partial_i A_j\Big) + \dot{\xi}_\lambda(1 - Z_k Z_k^*) + V^{(1)}(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_i, \xi_{A_0}, \xi_\lambda)\,, \quad (24)$$

where

$$V^{(1)}(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_i, \xi_{A_0}, \xi_\lambda) = V^{(0)}|_{\Omega_i=0}\,. \quad (25)$$

According to the new set of dynamical variables $(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_i, \xi_{A_0}, \xi_\lambda)$, the variables of canonical one-form are

$$a_{Z_k} = \pi_k\,, \quad a_{Z_k^*} = \bar{\pi}_k\,, \quad a_{\pi_k} = 0\,, \quad a_{\bar{\pi}_k} = 0\,, \quad a_{A_i} = -\frac{\theta}{4\pi^2}\varepsilon^{ij}A_i\,,$$
$$\xi_{A_0} = -J_0 + \frac{\theta}{2\pi^2}\varepsilon^{ij}A_i\,, \quad a_{\xi_\lambda} = 1 - Z_k Z_k^*\,, \quad (26)$$

and the first-iterated symplectic matrix is written as

$$f^{(1)}(x,y) = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & -\mathrm{i}\pi_k & -Z_k^* \\ 0 & 0 & 0 & -1 & 0 & \mathrm{i}\bar{\pi}_k & -Z_k \\ 1 & 0 & 0 & 0 & 0 & -\mathrm{i}Z_k & 0 \\ 0 & 1 & 0 & 0 & 0 & \mathrm{i}Z_k^* & 0 \\ 0 & 0 & 0 & 0 & \dfrac{\theta}{2\pi^2}\varepsilon^{ij} & \dfrac{\theta}{2\pi^2}\varepsilon^{ij}\partial_i^x & 0 \\ \mathrm{i}\pi_k & -\mathrm{i}\bar{\pi}_k & \mathrm{i}Z_k & -\mathrm{i}Z_k^* & -\dfrac{\theta}{2\pi^2}\varepsilon^{ij}\partial_i^x & 0 & 0 \\ Z_k^* & Z_k & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \delta(x-y)\,. \quad (27)$$

The modified symplectic matrix after the first iteration is completed which is again singular. As it can be seen, there is one new zero-mode associated to this matrix and it is written in terms of one new arbitrary quantity ν_3

$$\nu_3 = (0,0,Z_k^*\nu_3, Z_k\nu_3, 0, 0, \nu_3)\,. \quad (28)$$

A new constraint can be evaluated from Eq.(3), which is written as

$$\Omega_3 = Z_k^* \bar{\pi}_k + Z_k \pi_k = 0\,. \tag{29}$$

The set of symplectic variables defining the extended space is taken place by the set $(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_i, \xi_{A_0}, \xi_\lambda, \xi_{\xi_\lambda})$. The Lagrangian density is given by

$$\begin{aligned}\mathcal{L}^{(2)} = {} & \pi_k \dot{Z}_k + \bar{\pi}_k \dot{Z}_k^* - \frac{\theta}{4\pi^2}\varepsilon^{ij} A_i \dot{A}_j + \dot{\xi}_{A_0}\Big(-J_0 + \frac{\theta}{2\pi^2}\varepsilon^{ij}\partial_i A_j\Big) + \dot{\xi}_\lambda(1 - Z_k Z_k^*) \\ & + \dot{\xi}_{\xi_\lambda}(Z_k^* \bar{\pi}_k + Z_k \pi_k) + V^{(2)}(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_i, \xi_{A_0}, \xi_\lambda, \xi_{\xi_\lambda})\,,\end{aligned} \tag{30}$$

where

$$V^{(2)}(Z_k, Z_k^*, \pi_k, \bar{\pi}_k, A_i, \xi_{A_0}, \xi_\lambda, \xi_{\xi_\lambda}) = V^{(1)}|_{\Omega_i = 0} = -\gamma_0 \pi_k \bar{\pi}_k + \frac{1}{\gamma_0}(D_\mu Z_k)^*(D_i Z_k)\,, \tag{31}$$

where $l = 1, 2, 3$. The new variables of canonical one-form are

$$\begin{aligned} & a_{Z_k} = \pi_k\,, \qquad a_{Z_k^*} = \bar{\pi}_k\,, \qquad a_{\pi_k} = 0\,, \qquad a_{\bar{\pi}_k} = 0\,, \quad a_{A_i} = -\frac{\theta}{4\pi^2}\varepsilon^{ij} A_i\,, \\ & \xi_{A_0} = -J_0 + \frac{\theta}{2\pi^2}\varepsilon^{ij} A_i\,, \qquad a_{\xi_\lambda} = 1 - Z_k Z_k^*\,, \qquad a_{\xi_\lambda} = Z_k^* \bar{\pi}_k + Z_k \pi_k\,. \end{aligned} \tag{32}$$

The second-iterated symplectic matrix is given by

$$f^{(2)}(x,y) = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & -\mathrm{i}\pi_k & -Z_k^* & \pi_k \\ 0 & 0 & 0 & -1 & 0 & \mathrm{i}\bar{\pi}_k & -Z_k & \bar{\pi}_k \\ 1 & 0 & 0 & 0 & 0 & -\mathrm{i}Z_k & 0 & Z_k \\ 0 & 1 & 0 & 0 & 0 & \mathrm{i}Z_k^* & 0 & Z_k^* \\ 0 & 0 & 0 & 0 & \frac{\theta}{2\pi^2}\varepsilon^{ij} & \frac{\theta}{2\pi^2}\varepsilon^{ij}\partial_j^x & 0 & 0 \\ \mathrm{i}\pi_k & -\mathrm{i}\bar{\pi}_k & \mathrm{i}Z_k & -\mathrm{i}Z_k^* & -\frac{\theta}{2\pi^2}\varepsilon^{ij}\partial_i^x & 0 & 0 & 0 \\ Z_k^* & Z_k & 0 & 0 & 0 & 0 & 0 & 0 \\ -\pi_k & -\bar{\pi}_k & -Z_k & -Z_k^* & 0 & 0 & 0 & 0 \end{pmatrix} \delta(x-y)\,. \tag{33}$$

The matrix $f^{(2)}(x,y)$ is again singular. But based on the zero-mode of the symplectic matrix $f^{(2)}(x,y)$, no new constraint is found in the FJ formalism. The singularity of matrix $f^{(2)}(x,y)$ can be eliminated by means of a gauge-fixing term added to the Lagrangian density. The gauge-fixing term plays the role of a new constraint, which is given by

$$\Omega_4 = \partial^i A^i = 0\,. \tag{34}$$

Consequently, the set of constraints include Eqs.(22), (23), (29) and (34). Now, from Eq.(11) we obtain the matrix elements $\Omega_{ab}(x,y)$ as follows:

$$\Omega_{14} = -\nabla^2 \delta(x-y)\,, \tag{35a}$$

$$\Omega_{23} = -2 Z_k Z_k^*\,, \tag{35b}$$

$$\Omega_{32} = 2 Z_k Z_k^*\,, \tag{35c}$$

$$\Omega_{41} = \nabla^2 \delta(x-y)\,, \tag{35d}$$

the other matrix elements are all zero. According to Eqs.(35a–35d) and (10), the matrix G_{ab} can be found

$$G_{ab}(x,y) = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2\nabla_x^2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2\nabla_x^2} & 0 & 0 & 0 \end{pmatrix} \delta(x-y)\,. \tag{36}$$

From Eqs.(8) and (20), we obtain

$$\bar{f}_{in}^{-1} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\pi^2}{\theta}\varepsilon^{ij} \end{pmatrix} \delta(x-y)\,. \tag{37}$$

According to Eqs.(9), (22), (23), (29), (34), (39) and (37), we can obtain the nonvanishing set of equal-time generalized FJ brackets

$$[Z_k(x), \pi_l(y)] = \Big[\delta_{kl} - \frac{1}{2} Z_k(x) Z_k^*(y)\Big]\delta(x-y), \quad (38a)$$

$$[Z_k^*(x), \pi_l(y)] = -\frac{1}{2} Z_k^*(x) Z_k^*(y)\delta(x-y)\,, \quad (38b)$$

$$[\pi_k(x), \pi_l(y)] = \frac{1}{2}[\pi_k(x) Z_l^*(y) - Z_k^*(x)\pi_l(y)]\delta(x-y)\,. \quad (38c)$$

$$[\pi_k(x), \bar{\pi}_l(y)] = \frac{1}{2}[\pi_k(x) Z_l(y) - Z_k^*(x)\bar{\pi}_l(y)]\delta(x-y)\,, \quad (38d)$$

$$[Z_k(x), A_i(y)] = -\frac{\mathrm{i}2\pi^2}{\theta} Z_k(x) \frac{1}{\nabla_x^2} \varepsilon^{ij} \partial_j^x \delta(x-y)\,. \quad (38e)$$

The generalized FJ brackets correspond to the Dirac brackets of the model.[8] This shows that the FJ quantization formalism equals Dirac's canonical quantization formalism[23] in the system. Because the FJ quantization formalism is based on the Lagrangian and the Dirac canonical quantization formalism is based on the Hamiltonian, it shows[24,25] that the method is more direct and elegant than Dirac's method. The equivalence of the both methods needs to be discussed more.

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