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Synchronization of hyperchaotic Lorenz system based on passive control

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Synchronization of a hyperchaotic Lorenz system is discussed using passive control. Based on the properties of a passive system, a passive controller is designed and the synchronization between two hyperchaotic Lorenz systems under different initial conditions is realized. Simulation results show the proposed synchronization method to be effective.

Keywords: hyperchaotic Lorenz system, passive control, asymptotically stabilized

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1. Introduction

Hyperchaos, which has more than one positive Lyapunov exponent, has increasingly aroused the interest of all due to its great potential in technological applications in many fields, such as secure communication, laser, and so on. So, the generation, control, synchronization and application of hyperchaos have recently become a hot topic for the research in this regard.[1−7] A large variety of hyperchaotic systems have been presented over the past a few decades.[3−7] The hyperchaotic Chua circuit[3] and Rössler system[5] are two representative hyperchaotic systems. Recently, several new hyperchaotic systems have been proposed. Li *et al* proposed a new hyperchaotic system through introducing an additional state to a third-order generalized Lorenz chaotic system.[6] Chen *et al* proposed a new hyperchaotic system through adopting a state feedback control to Lü's chaotic system.[7,8] These hyperchaotic systems are new and have their own properties. Therefore, it is very important to further explore the control and synchronization of these new hyperchaotic systems for the engineering applications.

Since the phenomenon of chaotic synchronization was first revealed by Pecora and Carroll,[9] various effective synchronization schemes have been proposed to achieve chaotic synchronization. They are adaptive control,[10,11] backstepping control,[12] linear and nonlinear feedback control,[13,14] active control,[15,16] impulsive control,[17,18] and so on. In this paper, based on the properties of a passive system, the synchronization between two hyperchaotic Lorenz systems is studied. The essential conditions are derived under which the erroneous dynamical system could be equivalent to a passive system and globally asymptotically stabilized at a zero equilibrium point via smooth state feedback.

The rest of this paper is organized as follows. In Section 2, based on the properties of a passive system, a rather simple conclusion about controlling an erroneous dynamical system is drawn for stabilizing the system at a zero equilibrium point through a feedback control. In Section 3 a brief description of a hyperchaotic Lorenz system is given and the approach is applied to this system. Section 4 is devoted to numerical simulations for illustration and verification of our result. Finally, some concluding remarks and comments are given in Section 5.

2. The theory of passive control

Consider the following differential equation:

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{1}$$

where $x \in X \subset R^n$ is the state variable. $f(x)$ and $g(x)$ are the smooth vector fields. $u(t) \in U$ is the input. $f(0) = 0$ and $h(x)$ is a smooth mapping.

Definition 1 The system (1) is a minimum phase system if $L_{\rm g}h(0)$ is nonsingular and $x = 0$ is one of asymptotically stabilized equilibrium points of $f(x)$.

Definition 2 The system (1) is passive if the following two conditions are satisfied:
1). $f(x)$ and $g(x)$ exist and are smooth vector fields, $h(x)$ is a smooth mapping.
2). For any $t \geq 0$, there is a real value β satisfying the inequality

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau \geq \beta, \tag{2}$$

or there are a real value β and $\rho > 0$ satisfying the inequality

$$\int_0^t u^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho y^{\mathrm{T}}(\tau)y(\tau)\mathrm{d}\tau. \tag{3}$$

Let $z = \vartheta(x)$, and the system (1) will be changed into the following generalized form:

$$\begin{cases} \dot{z} = f_0(z) + p(z,y)y, \\ \dot{y} = b(z,y) + a(z,y)u, \end{cases} \tag{4}$$

where $a(z,y)$ is nonsingular for any (z,y).

Theorem 1 If the system (1) is a minimum phase system and has a relative degree $[1,\ 1,\ \cdots]$ at $x = 0$, the system (4) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point through the local feedback control as follows:

$$u = a(z,y)^{-1}\left[-b^{\mathrm{T}}(z,y) - \frac{\partial}{\partial z}W(z)p(z,y) - \alpha y + v\right], \tag{5}$$

where $W(z)$ is the Lyapunov function of $f_0(z)$, α is a positive real value and v is an external signal that is connected with the reference input.

Proof
Suppose that

$$V(z,y) = W(z) + \frac{1}{2}y^2, \tag{6}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}V(z,y) &= \frac{\partial W(z)}{\partial z}\dot{z} + y\dot{y} \\ &= \frac{\partial W(z)}{\partial z}f_0(z) + \frac{\partial W(z)}{\partial z}p(z,y)y \\ &\quad + yb(z,y) + ya(z,y)u. \end{aligned} \tag{7}$$

Because the system (4) is a minimum phase system, the inequality is obtained to be

$$\frac{\partial W(z)}{\partial z}f_0(z) \leq 0, \tag{8}$$

so

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}V(z,y) \leq\ & \frac{\partial W(z)}{\partial z}p(z,y)y \\ & + yb(z,y) + ya(z,y)u. \end{aligned} \tag{9}$$

Substituting Eq.(5) into the inequality (9), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V(z,y) \leq -\alpha y^2 + vy. \tag{10}$$

Performing integration on both sides of the inequality (10) yields

$$V(z,y) - V(z_0,y_0) \leq -\int_0^t \alpha y(\tau)^2\mathrm{d}\tau + \int_0^t v(\tau)y(\tau)\mathrm{d}\tau. \tag{11}$$

For $V(z,y) \geq 0$, let $V(z_0,y_0) = \mu$, the above inequality can be rewritten as

$$\int_0^t v(\tau)y(\tau)\mathrm{d}\tau + \mu \geq \int_0^t \alpha y(\tau)^2\mathrm{d}\tau + V(z,y) \geq \int_0^t \alpha y(\tau)^2\mathrm{d}\tau. \tag{12}$$

According to Definition 2, the system (4) is a passive system. Because $W(z)$ is radially unbounded, it follows from Eq.(6) that $V(z,y)$is also radially unbounded, so that a closed-loop system is in a bounded state stable for $[z^{\mathrm{T}}, y]^{\mathrm{T}}$. This means that we can use the local feedback control described in the form (5) to regulate the system (4) to an equilibrium point.

3. Synchronization between two identical hyperchaotic Lorenz systems using passive control

Recently, Li *et al* introduced a new hyperchaotic Lorenz system through introducing an additional state in a third-order generalized Lorenz chaotic system. The nonlinear differential equations that describe this

hyperchaotic system, as a drive system, are

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = bx_1 + cx_2 - x_1x_3 + x_4, \\ \dot{x}_3 = -dx_3 + x_1x_2, \\ \dot{x}_4 = -kx_1. \end{cases} \tag{13}$$

where x_1, x_2, x_3 and x_4 are state variables, and a, b, c, d and k are system parameters. Taking $a = 35$, $b = 7$, $c = 12$, $d = 3$ and $k = 5$ for example, the hyperchaotic attractors are obtained, and depicted in Figs.1(a)–1(b) while the Lyapunov exponents of this chaotic system are obtained to be $\lambda_1 = 0.41$, $\lambda_2 = 0.20$, $\lambda_3 = 0.00$ and $\lambda_4 = -26.7$.

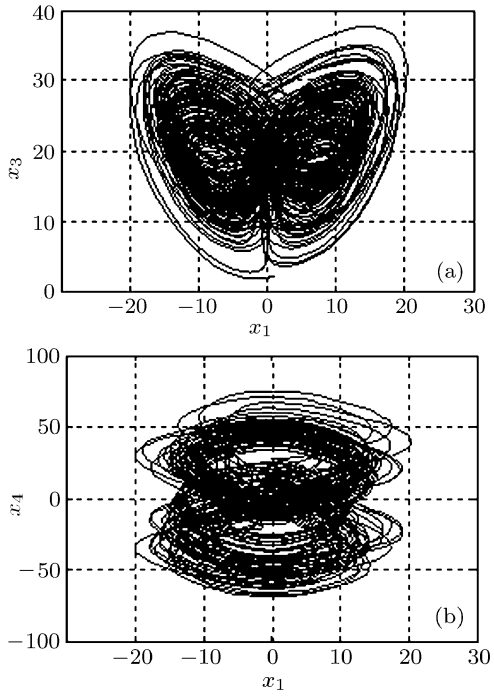


Fig.1. The phase portrait of hyperchaotic Lorenz system (a) x_3 vs x_1, (b) x_4 vs x_1.

The response system is described by the following equations:

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1), \\ \dot{y}_2 = by_1 + cy_2 - y_1y_3 + y_4 + u, \\ \dot{y}_3 = -dy_3 + y_1y_2, \\ \dot{y}_4 = -ky_1. \end{cases} \tag{14}$$

The erroneous dynamical system between the drive system (13) and response system (14) can be expressed as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1), \\ \dot{e}_2 = be_1 + ce_2 - e_1e_3 - x_1e_3 - x_3e_1 + e_4 + u, \\ \dot{e}_3 = -de_3 + e_1e_2 + x_1e_2 + x_2e_1, \\ \dot{e}_4 = -ke_1. \end{cases} \tag{15}$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$ and $e_4 = y_4 - x_4$. The aim is to determine the controller u for stabling the erroneous dynamical system (15) at a zero equilibrium point.

Theorem 2 If we choose the controller as follows:

$$u = -(a+b-x_3+\frac{dM^2}{4})e_1-(c+\alpha)e_2-(1-\frac{a^2}{k})e_4, \tag{16}$$

the erroneous dynamical system (15) will be asymptotically stabilized at a zero equilibrium point and the two hyperchaotic Lorenz systems with different initial conditions will be synchronized with each other.

Proof Let $z_1 = e_1$, $z_2 = e_3$, $z_3 = e_4$ and $y = e_2$ the system (15) can be rewritten as

$$\begin{cases} \dot{z}_1 = a(y - z_1), \\ \dot{z}_2 = -dz_2 + z_1y + x_1y + x_2z_1, \\ \dot{z}_3 = -kz_1, \\ \dot{y} = bz_1 + cy - z_1z_2 - x_1z_2 - x_3z_1 + z_3 + u. \end{cases} \tag{17}$$

so

$$\begin{cases} \dot{z} = f_0(z) + p(z,y)y, \\ \dot{y} = b(z,y) + a(z,y)u, \end{cases} \tag{18}$$

where

$$\begin{cases} f_0(z) = [-az_1 \quad -dz_2 + x_2z_1 \quad -kz_1]^{\mathrm{T}}, \\ p(z,y) = [a \quad x_1 + z_1 \quad 0]^{\mathrm{T}}, \\ b(z,y) = bz_1 + cy - z_1z_2 - x_1z_2 - x_3z_1 + z_3, \\ a(z,y) = 1. \end{cases} \tag{19}$$

With each chaotic or hyperchaotic system having its own bound, the following inequality can be obtained:

$$|x_2| \le x_{2\max} = M = 22.4927. \tag{20}$$

Let

$$W(z) = \frac{1}{2}(z_1 - \frac{a}{k}z_3)^2 + \frac{1}{2}z_2^2 + \frac{dM^2}{8a}z_1^2, \tag{21}$$

$$\begin{aligned} \dot{W}(z) &= \left(z_1 - \frac{a}{k}z_3\right)\left(\dot{z}_1 - \frac{a}{k}\dot{z}_3\right) + z_2\dot{z}_2 + \frac{dM^2}{4a}z_1\dot{z}_1 \\ &= -dz_2^2 + x_2z_1z_2 - \frac{dM^2}{4}z_1^2 \\ &= -d\left(z_2 - \frac{x_2}{2}z_1\right)^2 - \left(\frac{dM^2}{4} - \frac{dx_2^2}{4}\right)z_1^2. \end{aligned} \tag{22}$$

From the inequality (20), the following inequality is obtained:

$$\dot{W}(z) \le 0. \tag{23}$$

So, $f_0(z)$ is globally asymptotically stable. Accordingly to Theorem 1, the feedback control is

$$u = -\left(a + b - x_3 + \frac{dM^2}{4}\right) z_1 - (c + \alpha) y - \left(1 - \frac{a^2}{k}\right) z_3 + v, \tag{24}$$

namely

$$u = -\left(a + b - x_3 + \frac{dM^2}{4}\right) e_1 - (c + \alpha) e_2 - \left(1 - \frac{a^2}{k}\right) e_4 + v, \tag{25}$$

When we let $v = 0$, the erroneous dynamical system (15) will be asymptotically stabilized at a zero equilibrium point and the two hyperchaotic Lorenz systems with different initial conditions will be synchronized with each other.

4. Simulation results

In this section, the fourth-order Runge–Kutta method is used to solve the systems of differential equations (13) and (14), with a step size being 0.001 in numerical simulation. Choose the value of α as 3 and initial conditions: $x_1(0) = 1$, $x_2(0) = -1$, $x_3(0) = 2$, and $x_4 = -3$ for the driving system (13), and $y_1(0) = 0.1$, $y_2(0) = -3$, $y_3(0) = -2$, and $y_4(0) = -0.5$ for the response system (14). After 20 s, the controller is added to the response system. As expected, one can observe that the trajectories of the response system asymptotically approach those of the drive system as illustrated in Figs.2(a)–2(d), and the norm $||e||$ of erroneous vector eventually converges to zero after the controller is activated as shown in Fig.2(e). The simulation result implies that the two identical hyperchaotic Lorenz systems are synchronized with each other, and validates the effectiveness of the proposed method.

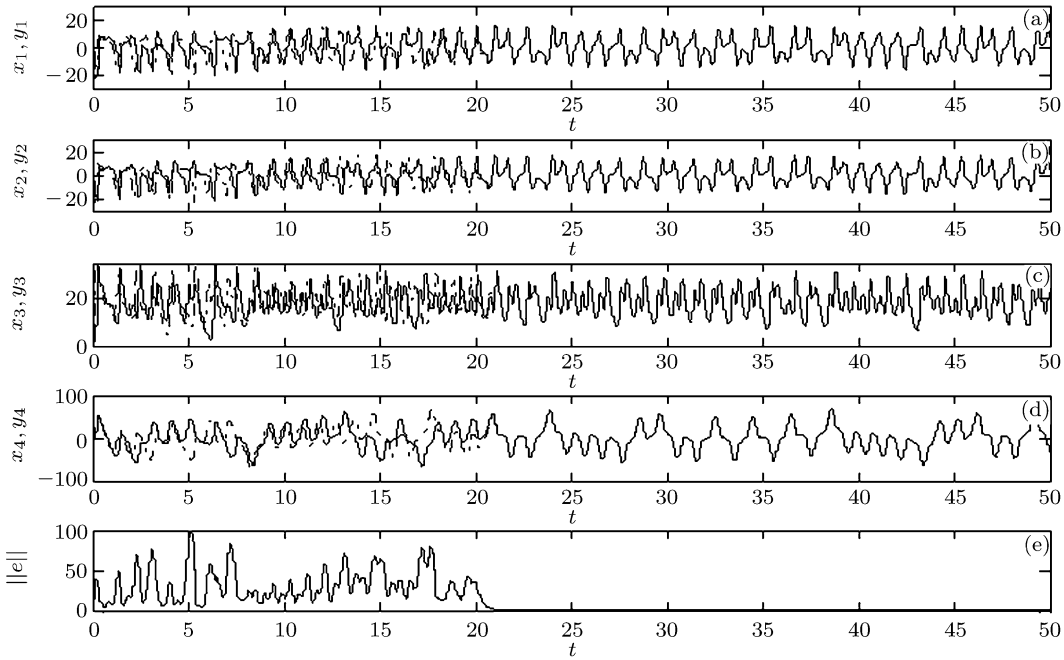


Fig.2. Synchronization between two identical hyperchaotic Lorenz system with each other. (a) Time waveforms of the states x_1 (solid) and y_1 (dashed), (b) Time waveforms of the states x_2 (solid) and y_2 (dashed), (c) Time waveforms of the states x_3 (solid) and y_3 (dashed), (d) Time waveforms of the states x_4 (solid) and y_4 (dashed). (e) Time waveform of the norm $||e||$ of error vector

5. Conclusions

Based on the properties of a passive system, a feedback controller for the synchronization between two hyperchaotic Lorenz systems is derived. The effectiveness of the proposed synchronization method has been demonstrated by numerical simulation results. In addition, this method can be applied to the synchronization between two other chaotic or hyperchaotic systems with different initial conditions.

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