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Controlling projective synchronization in coupled chaotic systems*

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In this paper, a new method for controlling projective synchronization in coupled chaotic systems is presented. The control method is based on a partially linear decomposition and negative feedback of state errors. Firstly, the synchronizability of the proposed projective synchronization control method is proved mathematically. Then, three different representative examples are discussed to verify the correctness and effectiveness of the proposed control method.

Keywords: chaos, chaos synchronization, projective synchronization
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1. Introduction

Since the discussion of chaos synchronization in the early 1990s,[1,2] this important subject has been attracting much attention from scientists and engineers.[3−11] Different types of synchronization phenomena have been observed and studied in a variety of chaotic systems, such as identical systems synchronization,[1,2,11] phase synchronization,[3,4] lag synchronization,[5,6] generalized synchronization,[7,8] and so on. If a nonlinear system can be written as $\dot{\boldsymbol{x}}_{\rm m} = \boldsymbol{A}(z)\boldsymbol{x}_{\rm m}$, $\dot{z} = g(\boldsymbol{x}_{\rm m}, z)$, the state vector $\boldsymbol{x}_{\rm m}$ has a linear form related to its time derivative, the matrix $\boldsymbol{A}(z)$ depends only on the variable z, then the system is said to be a partially-linear system. Taking z as the coupling variable, the response system can be written as $\dot{\boldsymbol{x}}_{\rm s} = \boldsymbol{A}(z)\boldsymbol{x}_{\rm s}$. In the coupled partially linear systems, it was found that the two identical systems (drive and response) could be synchronized to a scaling factor.[12] This type of chaotic synchronization phenomenon is referred to as projective synchronization (PS). Although there exists a linear relation between the coupled systems, PS could not be classified as generalized synchronization because the largest Lyapunov exponent of the response system is non-negative even when synchronization is achieved. It has been also found that the scaling factor of PS is sensitive to the initial values of the two coupled systems.[12] Then, some criteria for the occurrence of projective synchronization in chaotic systems have been established.[13,14] Recently, a digital communication scheme has been proposed based on controlled PS,[15] which shows that a chaos-based communication technique for transmitting M-ary digital signals using controlled PS of coupled chaotic systems is superior over conventional and other chaos-based digital communication techniques in the sense of a low bit error rate.

Several control methods have already been developed to achieve PS in coupled chaotic systems, including controlling PS in the so-called partially-linear chaotic systems and nonpartially-linear chaotic systems.[10,16,17] In this paper, a new control method for controlling PS in coupled chaotic systems is presented, which can be applied to coupled partially-linear chaotic systems as well as coupled nonpartially-linear chaotic systems.

The rest of this paper is organized as follows. First, the new PS control method is put forward and proved mathematically. Then, it is shown that PS is achieved under the proposed control in three examples: the partially-linear chaotic Lorenz system and the nonpartially-linear chaotic Chua circuit and Chen system.

2. Proposed control method for projective synchronization

A coupled chaotic system is described as follows:

$$\dot{\boldsymbol{x}}_{\rm m} = \boldsymbol{f}(\boldsymbol{x}_{\rm m}, z)\,, \quad \dot{z} = g(\boldsymbol{x}_{\rm m}, z)\,, \quad \dot{\boldsymbol{x}}_{\rm s} = \boldsymbol{f}(\boldsymbol{x}_{\rm s}, z) + \boldsymbol{u}\,, \tag{1}$$

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where $\boldsymbol{x}_{\rm m}, \boldsymbol{x}_{\rm s} \in R^{n-1}$ denote the $(n-1)$-dimensional state variables of the drive and the response systems, respectively, $z \in R^1$ is a one-dimensional coupling variable, which is the same in both the drive system and the response system, and $\boldsymbol{u} \in R^{n-1}$ is an $(n-1)$-dimensional vector controller.

Decompose the subsystem $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, z)$ into two parts:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(z)\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x}, z)\,, \tag{2}$$

where the $(n-1)\times(n-1)$ matrix

$$\boldsymbol{A}(z) = \begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1(n-1)} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{i1} & \cdots & a_{ii} & \cdots & a_{i(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{(n-1)1} & \cdots & \cdots & \cdots & a_{(n-1)(n-1)} \end{bmatrix}$$

only depends on the variable z, and $\boldsymbol{\Phi}(\boldsymbol{x}, z)$ is a nonlinear vector function of $\boldsymbol{x}$ and z.

Theorem 1

If the vector controller in the coupled system (1) takes the form of

$$\boldsymbol{u} = \alpha^* \boldsymbol{\Phi}(\boldsymbol{x}_{\rm m}, z) - \boldsymbol{\Phi}(\boldsymbol{x}_{\rm s}, z) + \boldsymbol{M}(\alpha^* \boldsymbol{x}_{\rm m} - \boldsymbol{x}_{\rm s})\,,$$

where α^* is the assigned scaling factor and

$$\alpha^* \in R, \alpha^* \neq 0\,, \qquad \boldsymbol{M} = \begin{bmatrix} k_1 & & \\ & \ddots & \\ & & k_{n-1} \end{bmatrix},$$

satisfying the condition

$$k_i > a_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}|\,, \quad i = 1, 2, \ldots, n-1\,,$$

where a_{ii} and a_{ij} are elements of matrix $\boldsymbol{A}(z)$, then PS can be achieved between the drive and the response systems in the coupled system (1).

Proof Define the error vector of PS as

$$\boldsymbol{e} = \alpha^* \boldsymbol{x}_{\rm m} - \boldsymbol{x}_{\rm s}\,. \tag{3}$$

One then has

$$\dot{\boldsymbol{e}} = \alpha^* \dot{\boldsymbol{x}}_{\rm m} - \dot{\boldsymbol{x}}_{\rm s}\,. \tag{4}$$

Substituting Eq.(1) into Eq.(4), one gets

$$\dot{\boldsymbol{e}} = \alpha^* \boldsymbol{f}(\boldsymbol{x}_{\rm m}, z) - \boldsymbol{f}(\boldsymbol{x}_{\rm s}, z) - \boldsymbol{u}\,. \tag{5}$$

It follows from Eq.(2) that $\dot{\boldsymbol{x}} = \boldsymbol{A}(z)\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x}, z)$. So, Eq.(5) can be written as

$$\dot{\boldsymbol{e}} = \alpha^*(\boldsymbol{A}(z)\boldsymbol{x}_{\rm m} + \boldsymbol{\Phi}(\boldsymbol{x}_{\rm m}, z)) - (\boldsymbol{A}(z)\boldsymbol{x}_{\rm s} + \boldsymbol{\Phi}(\boldsymbol{x}_{\rm s}, z)) - \boldsymbol{u} = \boldsymbol{A}(z)(\alpha^* \boldsymbol{x}_{\rm m} - \boldsymbol{x}_{\rm s}) + \alpha^* \boldsymbol{\Phi}(\boldsymbol{x}_{\rm m}, z) - \boldsymbol{\Phi}(\boldsymbol{x}_{\rm s}, z) - \boldsymbol{u}\,. \tag{6}$$

When the vector controller $\boldsymbol{u}$ takes the form described in the theorem, one obtains

$$\dot{\boldsymbol{e}} = \boldsymbol{A}(z)(\alpha^* \boldsymbol{x}_{\rm m} - \boldsymbol{x}_{\rm s}) - \boldsymbol{M}(\alpha^* \boldsymbol{x}_{\rm m} - \boldsymbol{x}_{\rm s}) = (\boldsymbol{A}(z) - \boldsymbol{M})\boldsymbol{e}$$

$$= \begin{bmatrix} a_{11} - k_1 & a_{12} & \cdots & \cdots & a_{1(n-1)} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{i1} & \cdots & a_{ii} - k_i & \cdots & a_{i(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{(n-1)1} & \cdots & \cdots & \cdots & a_{(n-1)(n-1)} - k_{n-1} \end{bmatrix} \boldsymbol{e}\,. \tag{7}$$

According to the Gershgorin theorem, the eigenvalues of matrix $(\boldsymbol{A}(z) - \boldsymbol{M})$ are located in the following discs:

$$|\lambda - a_{ii} + k_i| \leq \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}|\,,$$

$$i = 1, 2, \ldots, n-1\,. \tag{8}$$

When

$$k_i > a_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}|\,, \quad \text{let} \quad \Delta = k_i - \Big(a_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}|\Big)\,,$$

one has $\Delta > 0$, and k_i can be written as

$$k_i = a_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}| + \Delta\,,$$

so that

$$|\lambda - a_{ii} + k_i| = \left|\lambda + \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}| + \Delta\right| \leq \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}|\,. \quad (9)$$

The disc described by Eq.(9) is located in the open left-half complex plane. When all $k_i, i = 1, 2, \ldots, n-1$, satisfy the condition

$$k_i > a_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}|\,.$$

All discs described by Eq.(8) are located in the open left-half complex plane, which implies that all eigenvalues of $(\boldsymbol{A}(z) - \boldsymbol{M})$ are located in the open left-half complex plane, so $\boldsymbol{e} = 0$ is the only asymptotically stable state of system (7), i.e. $\lim\limits_{t\to\infty} \boldsymbol{e} = \lim\limits_{t\to\infty} (\alpha^* \boldsymbol{x}_{\rm m} - \boldsymbol{x}_{\rm s}) = 0$. That is, the drive system and the response system will achieve projective synchronization.

Since $\boldsymbol{A}(z)$ depends only on the variable z, which is finite when the drive system is in the chaotic state, the value of $a_{ii} + \sum\limits_{\substack{j=1 \\ j\neq i}}^{n-1} |a_{ij}|$ is finite, implying that the above proposed PS control method is feasible.

For coupled partially-linear chaotic systems,

$$\dot{\boldsymbol{x}}_{\rm m} = \boldsymbol{A}(z)\boldsymbol{x}_{\rm m}, \quad \dot{z} = g(\boldsymbol{x}_{\rm m}, z)\,, \quad \dot{\boldsymbol{x}}_{\rm s} = \boldsymbol{A}(z)\boldsymbol{x}_{\rm s}\,,$$

it should be noticed that whether or not this system satisfies the condition of PS established in Refs.[13] and [14], PS is always a solution of the coupled systems under certain initial conditions. For example, when the initial condition satisfies $\boldsymbol{x}_{\rm s}(0) = \alpha^* \boldsymbol{x}_{\rm m}(0)$, one has $\dot{\boldsymbol{x}}_{\rm s} = \boldsymbol{A}(z)\boldsymbol{x}_{\rm s}(0) = \alpha^* \boldsymbol{A}(z)\boldsymbol{x}_{\rm m}(0) = \alpha^* \dot{\boldsymbol{x}}_{\rm m}$, hence PS can be achieved in the coupled systems. To stabilize the scaling factor to an assigned value independent of the initial condition of the coupled partially-linear systems, one can use a controller described in Theorem 1. When PS is achieved, since $\alpha^* \boldsymbol{x}_{\rm m} = \boldsymbol{x}_{\rm s}$, the equivalent vector controller embedded in the response system disappears.

For coupled nonpartially-linear chaotic systems,

$$\dot{\boldsymbol{x}}_{\rm m} = \boldsymbol{A}(z)\boldsymbol{x}_{\rm m} + \boldsymbol{\Phi}(\boldsymbol{x}_{\rm m}, z),$$
$$\dot{z} = g(\boldsymbol{x}_{\rm m}, z)\,, \quad \dot{\boldsymbol{x}}_{\rm s} = \boldsymbol{A}(z)\boldsymbol{x}_{\rm s} + \boldsymbol{\Phi}(\boldsymbol{x}_{\rm s}, z)\,,$$

it should be noted that PS is not a solution of the original coupled chaotic systems in general. For achieving PS in the coupled chaotic systems, therefore, a controller described in Theorem 1 is needed. When PS is achieved in such coupled nonpartially-linear chaotic systems, the equivalent vector controller given by $\boldsymbol{u} = \alpha^* \boldsymbol{\Phi}(\boldsymbol{x}_{\rm m}, z) - \boldsymbol{\Phi}(\boldsymbol{x}_{\rm s}, z)$ may not vanish.

3. Analysis of specific examples

In this section, the proposed projective synchronization control method is applied to one partially-linear chaotic system and two nonpartially-linear chaotic systems. The partially-linear chaotic system considered here is the Lorenz chaotic system[18] and the two nonpartially-linear chaotic systems are Chua's circuit[19] and Chen's system,[20] respectively.

The Lorenz system[18] is described by

$$\begin{cases} \dot{x}_{\rm m} = \sigma(y_{\rm m} - x_{\rm m})\,, \\ \dot{y}_{\rm m} = (\mu - z)x_{\rm m} - y_{\rm m}\,, \\ \dot{z} = x_{\rm m} y_{\rm m} - \rho z \end{cases} \quad (10)$$

with $\sigma = 10$, $\mu = 60$, $\rho = 8/3$. Taking z as the coupling variable, the Lorenz system can be written in the form of $\dot{\boldsymbol{x}} = \mathrm{A}(z)\boldsymbol{x}$, $\dot{z} = g(\boldsymbol{x}, z)$. This shows that it is a partially-linear chaotic system with

$$\boldsymbol{A}(z) = \begin{bmatrix} -\sigma & \sigma \\ \mu - z & -1 \end{bmatrix}.$$

Since

$$\frac{\partial f_1}{\partial x_{\rm m}} + \frac{\partial f_2}{\partial y_{\rm m}} = -\sigma - 1 < 0\,,$$

PS can be achieved in the coupled Lorenz systems without control.[13]

To stabilize the coupled systems to an assigned scaling factor α^*, independent of the initial condition of the coupled systems, the vector controller described in Theorem 1 is applied, so that the controlled response system is given by

$$\begin{cases} \dot{x}_{\rm s} = \sigma(y_{\rm s} - x_{\rm s}) + k_1(\alpha^* x_{\rm m} - x_{\rm s})\,, \\ \dot{y}_{\rm s} = (\mu - z)x_{\rm s} - y_{\rm s} + k_2(\alpha^* y_{\rm m} - y_{\rm s})\,. \end{cases} \quad (11)$$

Numerical simulation shows that $23 < z < 93$. According to Theorem 1, when $k_1 > -\sigma + \sigma = 0$ and $k_2 > -1 + |\mu - z| > 36$, PS with the assigned scaling factor α^* can be achieved in the coupled chaotic systems (10) and (11).

Simulation results are shown in Fig.1, where r and θ denote the state variables in cylindrical coordinates (r, θ, z) and the indices m and s denote the master system and the slave system, respectively, where $r = \sqrt{x^2 + y^2}$, $\mathrm{tg}(\theta) = y/x$. When PS is achieved, we have $r_\mathrm{s}\cos(\theta_\mathrm{s} - \theta_\mathrm{m})/r_\mathrm{m} = \alpha^*$ and the vector controller $\boldsymbol{u} = \boldsymbol{0}$, as pointed out above.

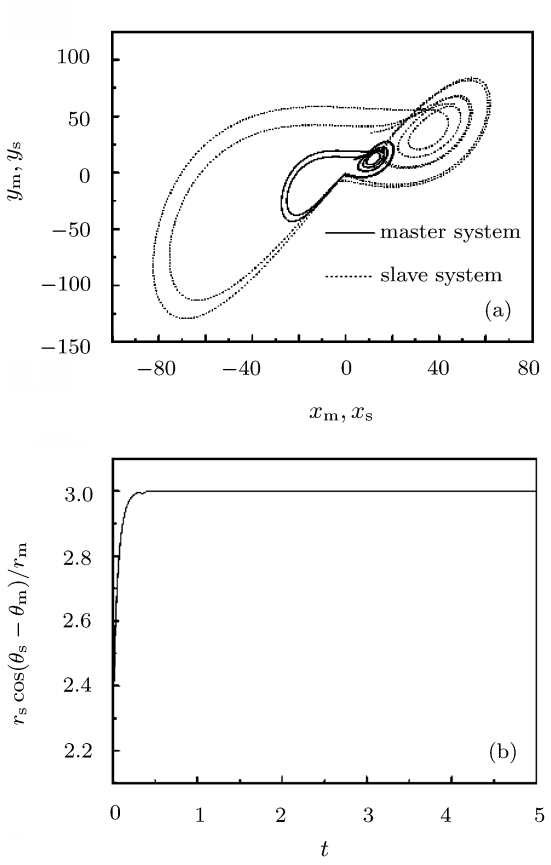


Fig.1. Controlled PS for the Lorenz system: (a) synchronization trajectories (in the $x - y$ plane) with the assigned scaling factor $\alpha^* = 3$ and $k_1 = 0$, $k_2 = 40$, (b) the variation of the ratio with respect to time.

Now, consider Chua's circuit,[19] described by

$$\begin{cases} \dot{x}_\mathrm{m} = \alpha[y_\mathrm{m} - x_\mathrm{m} - f(x_\mathrm{m})], \\ \dot{y}_\mathrm{m} = x_\mathrm{m} - y_\mathrm{m} + z, \\ \dot{z} = -\beta y_\mathrm{m}\,, \end{cases} \tag{12}$$

where $f(x) = bx + 0.5(a - b)(|x + 1| - |x - 1|)$, $\alpha = 9.78$, $\beta = 14.97$, $a = -1.31$, $b = -0.75$.

When z is the coupling variable, since the system is not a partially-linear system, PS is not a solution of the coupled Chua circuits without control. For achieving PS in the coupled Chua circuits, one can add a controller in the response system. In view of Eq.(2), the system can be decomposed as $\dot{\boldsymbol{x}} = \boldsymbol{A}(z)\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x}, z), \dot{z} = g(\boldsymbol{x}, z)$, where

$$\boldsymbol{A}(z) = \begin{bmatrix} -\alpha & \alpha \\ 1 & -1 \end{bmatrix},$$

$$\boldsymbol{\Phi}(\boldsymbol{x}, z) = \begin{bmatrix} -\alpha f(x) \\ z \end{bmatrix},$$

$$g(\boldsymbol{x}, z) = -\beta y\,.$$

The controlled response system can be written as

$$\begin{cases} \dot{x}_\mathrm{s} = \alpha[y_\mathrm{s} - x_\mathrm{s} - f(x_\mathrm{s})] + u_1\,, \\ \dot{y}_\mathrm{s} = x_\mathrm{s} - y_\mathrm{s} + z + u_2\,. \end{cases} \tag{13}$$

When the vector controller takes the form described in Theorem 1, that is

$$\begin{aligned} \boldsymbol{u} &= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ &= \begin{bmatrix} \alpha(f(x_\mathrm{s}) - \alpha^* f(x_\mathrm{m})) + k_1(\alpha^* x_\mathrm{m} - x_\mathrm{s}) \\ (\alpha^* - 1)z + k_2(\alpha^* y_\mathrm{m} - y_\mathrm{s}) \end{bmatrix}, \end{aligned} \tag{14}$$

and k_1, $k_2 > 0$, PS can be achieved in the coupled system (12) and (13). When PS is achieved in this coupled system, one has $\alpha^* \boldsymbol{x}_\mathrm{m} = \boldsymbol{x}_\mathrm{s}$, so the equivalent vector controller in the response system is

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \alpha(f(x_\mathrm{s}) - \alpha^* f(x_\mathrm{m})) \\ (\alpha^* - 1)z \end{bmatrix}. \tag{15}$$

Figure 2 is the simulation results of coupled Chua circuits (12) and (13), where r and θ are defined as in Example 1. It can be seen in Fig.2(b) that the bigger the k_1, k_2, the faster the convergence speed.

Finally, consider Chen's system,[20] described by

$$\begin{cases} \dot{x}_\mathrm{m} = z y_\mathrm{m} - b x_\mathrm{m}\,, \\ \dot{y}_\mathrm{m} = (c - a)z + c y_\mathrm{m} - x_\mathrm{m} z\,, \\ \dot{z} = a(y_\mathrm{m} - z)\,, \end{cases} \tag{16}$$

with $a = 35$, $b = 3$, $c = 28$. Taking z as a coupling variable, the system is not a partially-linear system, so PS is not a solution of the coupled Chen systems

without control.

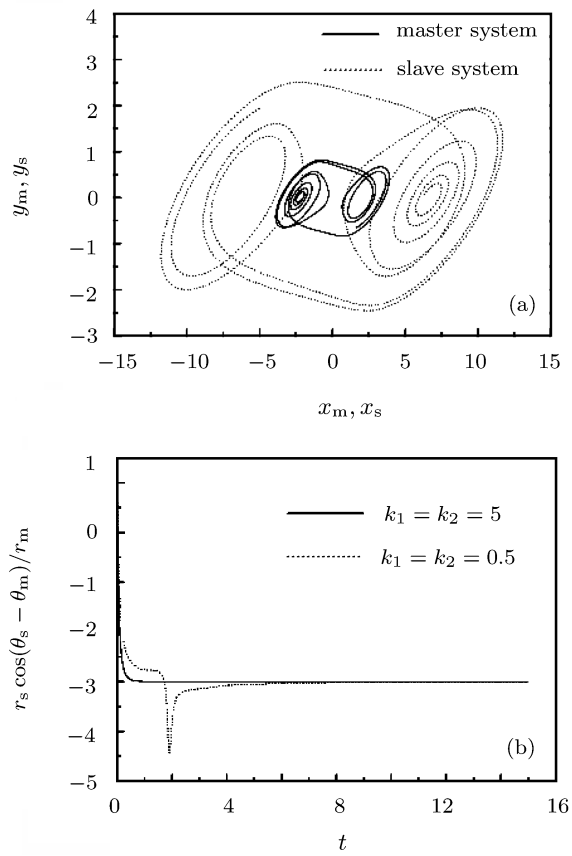


Fig.2. Controlled PS for Chua's circuit: (a) synchronization trajectories (in the $x - y$ plane) with the assigned scaling factor $\alpha^* = -3$ and $k_1 = k_2 = 0.5$, (b) variation of the ratio with different convergence rates.

One can decompose Chen's system into two parts,

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(z)\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x}, z), \dot{z} = g(\boldsymbol{x}, z)\,,$$

where

$$\boldsymbol{A}(z) = \begin{bmatrix} -b & z \\ -z & c \end{bmatrix},$$

$$\boldsymbol{\Phi}(\boldsymbol{x}, z) = \begin{bmatrix} 0 \\ (c-a)z \end{bmatrix},$$

$$g(\boldsymbol{x}, z) = a(y - z)\,.$$

To achieve PS in the coupled Chen systems, the controlled response system is taken as

$$\begin{cases} \dot{x}_s = z y_s - b x_s + u_1\,, \\ \dot{y}_s = (c-a)z + c y_s - x_s z + u_2\,, \end{cases} \tag{17}$$

where the controllers take the form described in Theorem 1, namely

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_1(\alpha^* x_m - x_s) \\ (\alpha^* - 1)(c-a)z + k_2(\alpha^* y_m - y_s) \end{bmatrix}. \tag{18}$$

Numerical simulation shows that $|z| < 25$. In this case, when $k_1 > 22$ and $k_2 > 53$, according to Theorem 1, PS can be achieved in the coupled Chen systems.

Simulation results are shown in Fig.3, where r and θ are defined as in Example 1.

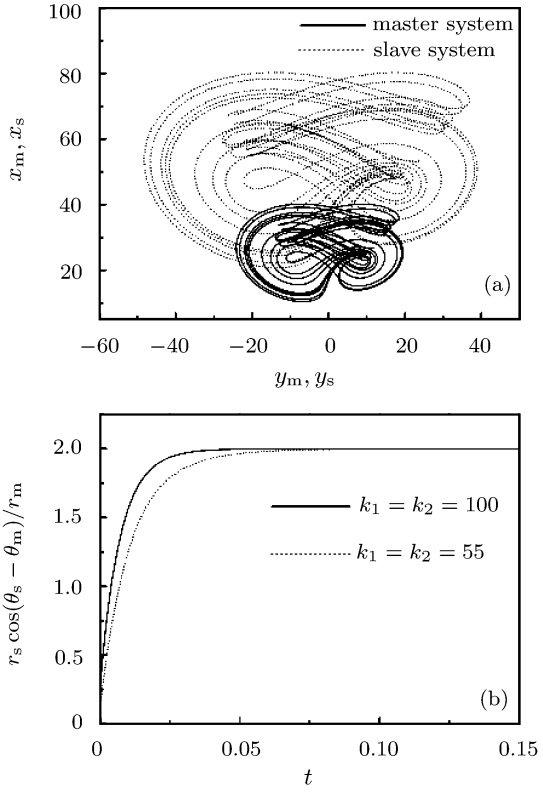


Fig.3. Controlled PS in the coupled Chen systems: (a) synchronization trajectories (in the $x - y$ plane) with the assigned scaling factor $\alpha^* = 2$, $k_1 = k_2 = 55$; (b) variation of the ratio with different convergence rates.

It is clear from Fig.3(b) that the time to achieve PS can be shortened with the increase of the feedback gains k_1 and k_2. When PS is achieved in the coupled Chen systems (16) and (17), the equivalent

vector controller in the response system (17) is

$$\boldsymbol{u} = \begin{bmatrix} 0 \\ (\alpha^* - 1)(c - a)z \end{bmatrix}. \tag{19}$$

4. Conclusion

In this paper, a new method for controlling projective synchronization in coupled chaotic systems has been presented. The proposed control method is based on a partially-linear decomposition and negative feedback of state errors. The projective synchronizability of the coupled chaotic systems has been proved mathematically and verified by three different but representative examples, including one partially-linear and two nonpartially-linear chaotic systems.

It should be noted that the proposed control method can be applied to a large class of chaotic systems.

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