Vol 15 No 9, September 2006
1009-1963/2006/15(09)/1953-07

Chinese Physics

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Influence of quantum degeneracy on the performance of a gas Stirling engine cycle*

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(Received 8 February 2006; revised manuscript received 21 March 2006)

Based on the state equation of an ideal quantum gas, the regenerative loss of a Stirling engine cycle working with an ideal quantum gas is calculated. Thermal efficiency of the cycle is derived. Furthermore, under the condition of quantum degeneracy, several special thermal efficiencies are discussed. Ratios of thermal efficiencies versus the temperature ratio and volume ratio of the cycle are made. It is found that the thermal efficiency of the cycle not only depends on high and low temperatures but also on maximum and minimum volumes. In a classical gas state the thermal efficiency of the cycle is equal to that of the Carnot cycle. In an ideal quantum gas state the thermal efficiency of the cycle is smaller than that of the Carnot cycle. This will be significant for deeper understanding of the gas Stirling engine cycle.

Keywords: Stirling engine cycle, ideal quantum gas, regenerative characteristics, thermal efficiency

PACC: 0530, 0570L, 0720

1. Introduction

Recently, quantum-mechanical cycles have been intensively studied. The concept of quantum heat engines was proposed and its optimum performance was discussed in some papers.[1−9] Some quantum heat engine and refrigeration cycles working with an ideal quantum gas, in which the quantum degeneracy of gas is considered, have been investigated.[10−18] Some significant results have been obtained.

It is well known that a Stirling engine cycle working with an ideal classical gas may possess the condition of perfect regeneration through the use of a reversible regenerator. The thermal efficiency of the cycle may attain that of a reversible Carnot engine cycle in the same temperature range. This conclusion is only valid under the condition of high temperature and low density. When the gas temperature is low enough or the gas density is high enough, the thermodynamic behaviours of an ideal classical gas will lose validity. Quantum degeneracy of gas will become important, which will affect the performance of thermodynamic cycle. Therefore, it is significant to research the performance of a Stirling engine cycle working with an ideal quantum gas. It will have many new characteristics different from those of classical thermodynamic cycles.

In the present paper, we will investigate the influence of quantum degeneracy on the performance of a gas Stirling engine cycle. Firstly, the thermodynamic properties of an ideal quantum gas are given, based on the equation of state of an ideal quantum gas. Secondly, the inherent regenerative losses of a Stirling engine cycle working with an ideal quantum gas are analysed, and the expressions for the thermal efficiencies of the cycle are derived. Finally, the details of expressions for the thermal efficiencies of the cycle are discussed under the different condition of quantum degeneracy. The characteristic curves of the ratios of the thermal efficiencies to the Carnot efficiencies versus the temperature and volume ratios of the cycle are generated. Therefore, the influence of quantum degeneracy on the performance of a gas Stirling engine cycle is revealed.

2. Thermodynamic properties of an ideal quantum gas

Based on quantum statistics, the pressure and the number density of an ideal Fermi gas are given by[19]

$$P = \frac{gkT}{\lambda^3} f_{5/2}(z) \tag{1}$$

*Project supported by the National Natural Science Foundation of China (Grant No 10465003), the Natural Science Foundation of Jiangxi Province, China (Grant No 0412011) and Science Foundation of Jiangxi Education Bureau, China.

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and

$$n = \frac{N}{V} = \frac{1}{\nu} = \frac{g}{\lambda^3} f_{3/2}(z), \tag{2}$$

where g is a weight factor that arises from the "internal structure" of the particles (such as spin), $\lambda = h/(2\pi mkT)^{1/2}$ is the mean thermal wavelength of the particles, h is Planck's constant, m is the rest mass of the particles, k is Boltzmann's constant, T is temperature of the gas, N is the total number of the particles, and V is the volume of the gas, ν is the occupied volume of the particle, $z = \exp(\mu/kT)$ is the fugacity of the gas, μ is the chemical potential of the gas, while

$$f_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1}\mathrm{d}x}{z^{-1}\mathrm{e}^x + 1}$$

is called the Fermi function, and $\Gamma(n)$ is the Gamma function.

For an ideal Bose gas, the pressure and the number density are expressed as

$$P = \frac{kT}{\lambda^3} g_{5/2}(z) \tag{3}$$

and

$$n = \frac{N - N_0}{V} = \frac{1}{\lambda^3} g_{3/2}(z), \tag{4}$$

where N_0 is the number of the particles in ground state, while

$$g_n(z) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1}\mathrm{d}x}{z^{-1}\mathrm{e}^x - 1}$$

is called the Bose function. The equation of state for an ideal quantum gas may be written as

$$P = nkTF(z), \tag{5}$$

where $F(z)$ is called the correction factor which is defined for a Fermi gas as

$$F(z) = \frac{f_{5/2}(z)}{f_{3/2}(z)}. \tag{6}$$

The correction factor for a Bose gas is

$$F(z) = \frac{g_{5/2}(z)}{g_{3/2}(z)}, \quad T > T_0, \tag{7}$$

where T_0 is the temperature of Bose–Einstein condensation. It is defined as

$$T_0(\nu) = A/\nu^{2/3}, \tag{8}$$

where $A = h^2 \Big/ (2\pi mk\Big[\zeta(3/2)\Big]^{2/3})$, $\zeta(n)$ is Riemann–Zeta function.

The internal energy, entropy and the specific heat at constant volume of an ideal quantum gas are given by

$$U = \frac{3}{2} NkTF(z), \tag{9}$$

$$S = Nk\left[\frac{5}{2}F(z) - \ln z\right] \tag{10}$$

and

$$C_V = \left(\frac{\partial U}{\partial T}\right)_\nu = \frac{3}{2} Nk \frac{\mathrm{d}}{\mathrm{d}T}[TF(T,\nu)], \tag{11}$$

where $F(T,\nu)$ is a function of temperature T and volume ν.

From Eqs.(10) and (11), the quantities of heat exchange in isothermal and isometric processes can be determined by

$$Q_{\mathrm{if}} = T(S_\mathrm{f} - S_\mathrm{i}) = NkT\left\{\frac{5}{2}[F(z_\mathrm{f}) - F(z_\mathrm{i})] - (\ln z_\mathrm{f} - \ln z_\mathrm{i})\right\} \tag{12}$$

and

$$Q_{\mathrm{if}}^{\nu} = \int_\mathrm{i}^\mathrm{f} C_V(T,\nu)\mathrm{d}T = \frac{3}{2} Nk[T_\mathrm{f} F(T_\mathrm{f},\nu) - T_\mathrm{i} F(T_\mathrm{i},\nu)], \tag{13}$$

where indices i and f refer to initial and final states respectively. On the basis of Eqs.(12) and (13), one can analyse the regenerative characteristics and the thermal efficiency of engine cycle working with an ideal quantum gas.

3. A gas Stirling engine cycle

A Stirling engine cycle is composed of two isothermal and two isometric processes. The entropy–temperature diagram of a quantum Stirling engine cycle is shown in Fig.1. Q_L and Q_H are the amounts of heat exchanges between the working substance and

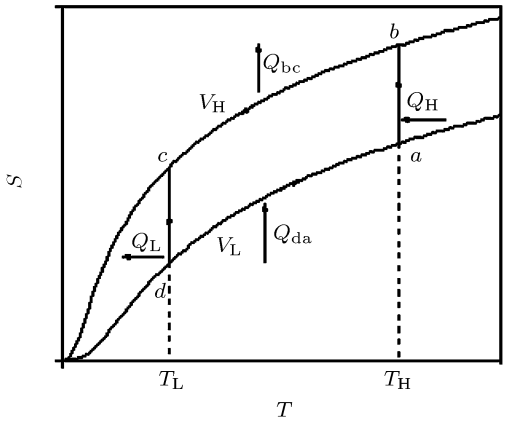


Fig.1. The entropy–temperature ($S-T$) diagram of a quantum Stirling engine cycle

heat reservoir at temperature T_L and T_H, respectively, during the two isothermal processes. In order to improve the performance of the cycle, a regenerator is often applied in the two isometric processes. Q_{bc} and Q_{da} are the amounts of heat exchanges between the working substance and the regenerator respectively, during the two isometric processes. V_L and V_H are, respectively, the minimum and maximum volumes of the gas. All heats Q_L, Q_H, Q_{bc}, and Q_{da} are positive. By using Eqs.(12) and (13), the amounts of heat exchanges in the various processes of quantum Stirling engine cycle may be expressed by

$$Q_L = \frac{5}{2}NkT_L[F(z_c) - F(z_d)] - NkT_L(\ln z_c - \ln z_d), \tag{14}$$

$$Q_H = \frac{5}{2}NkT_H[F(z_b) - F(z_a)] - NkT_H(\ln z_b - \ln z_a), \tag{15}$$

$$Q_{da} = \int_d^a C_V(T,\nu_H)\mathrm{d}T = \frac{3}{2}Nk[T_H F(z_a) - T_L F(z_d)] \tag{16}$$

and

$$Q_{bc} = \int_c^b C_V(T,\nu_L)\mathrm{d}T = \frac{3}{2}Nk[T_H F(z_b) - T_L F(z_c)], \tag{17}$$

where z_a, z_b, z_c, and z_d are the fugacity of the gas in the states a, b, c, and d, respectively. With the help of Eqs.(14) – (17), the work output can be derived:

$$\begin{aligned} W &= Q_H - Q_L + Q_{da} - Q_{bc} \\ &= NkT_H\left[F(z_b) - F(z_a) - \ln z_b + \ln z_a\right] + NkT_L\left[F(z_d) - F(z_c) - \ln z_d + \ln z_c\right]. \end{aligned} \tag{18}$$

The quantities of $\mathrm{TF}(T,\nu)$ versus the temperature for two different volumes are shown in Fig.2. Making

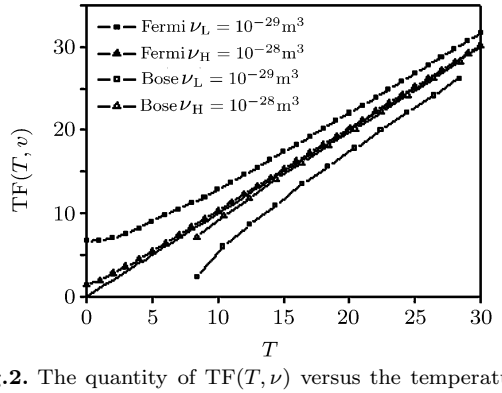


Fig.2. The quantity of $\mathrm{TF}(T,\nu)$ versus the temperature for two different volumes ($\nu_L = 10^{-29}\mathrm{m}^3$ and $\nu_H = 10^{-28}\mathrm{m}^3$).

use of Eq.(11) and comparing it with the slopes of the two curves, one can obtain the following relationships in Fermi gas (^{3}He) and in Bose gas (^{4}He):

$$C_V^F(T,\nu_H) > C_V^F(T,\nu_L) \tag{19}$$

and

$$C_V^B(T,\nu_H) < C_V^B(T,\nu_L), \quad T > T_0(\nu_L), \tag{20}$$

respectively. Equation (19) indicates that in a Fermi Stirling cycle the amount of heat exchange Q_{da} transferred from the regenerator is smaller than that of heat exchange Q_{bc} transferred into the regenerator, the redundant heat in the regenerator can only be released into the cold reservoir in a timely manner. If not, the temperature of the regenerator would be changed so that the regenerator would not be operated normally. The amount of heat absorbed from hot reservoir Q_H does not change. Thus, regenerative loss of the Fermi Stirling engine cycle, $\Delta Q = Q_{da} - Q_{bc}$, is less than zero, i.e.

$$\Delta Q = Q_{\rm da} - Q_{\rm bc} = \frac{3}{2}Nk\left\{T_{\rm H}\Big[F(z_{\rm a}) - F(z_{\rm b})\Big] + T_{\rm L}\Big[F(z_{\rm c}) - F(z_{\rm d})\Big]\right\} < 0. \tag{21}$$

From Eqs.(15) and (18), the thermal efficiency can be obtained by

$$\eta^{\rm F} = \frac{W}{Q_{\rm H}} = \frac{T_{\rm H}\Big[F(z_{\rm b}) - F(z_{\rm a}) - \ln z_{\rm b} + \ln z_{\rm a}\Big] + T_{\rm L}\Big[F(z_{\rm d}) - F(z_{\rm c}) - \ln z_{\rm d} + \ln z_{\rm c}\Big]}{(5/2)T_{\rm H}\Big[F(z_{\rm b}) - F(z_{\rm a})\Big] - T_{\rm H}(\ln z_{\rm b} - \ln z_{\rm a})}. \tag{22}$$

On the other hand, Eq.(20) means that in a Bose Stirling cycle the amount of heat exchange $Q_{\rm da}$ transferred from the regenerator is larger than that of heat exchange $Q_{\rm bc}$ transferred into the regenerator. The inadequate heat in the regenerator can only be compensated from the hot reservoir in a timely manner. If not, the temperature of the regenerator would be changed and the regenerator would not be operated normally. Heat absorbed from hot reservoir will increase from $Q_{\rm H}$ to $Q'_{\rm H}$. Thus, the regenerative loss of the Bose Stirling engine cycle is larger than zero, i.e. $\Delta Q = Q_{\rm da} - Q_{\rm bc} > 0$. From Eqs.(15), (18) and (21), heat absorbed from the hot reservoir and the thermal efficiency can be obtained:

$$\begin{aligned} Q'_{\rm H} &= Q_{\rm H} + \Delta Q \\ &= NkT_{\rm H}\Big[F(z_{\rm b}) - F(z_{\rm a})\Big] + \frac{3}{2}NkT_{\rm L}\Big[F(z_{\rm c}) - F(z_{\rm d})\Big] - NkT_{\rm H}(\ln z_{\rm b} - \ln z_{\rm a}), \end{aligned} \tag{23}$$

and

$$\eta^{\rm B} = \frac{W}{Q'_{\rm H}} = \frac{T_{\rm H}\Big[F(z_{\rm b}) - F(z_{\rm a}) - \ln z_{\rm b} + \ln z_{\rm a}\Big] + T_{\rm L}\Big[F(z_{\rm d}) - F(z_{\rm c}) - \ln z_{\rm d} + \ln z_{\rm c}\Big]}{T_{\rm H}\Big[F(z_{\rm b}) - F(z_{\rm a})\Big] + \frac{3}{2}T_{\rm L}\Big[F(z_{\rm c}) - F(z_{\rm d})\Big] - T_{\rm H}(\ln z_{\rm b} - \ln z_{\rm a})}. \tag{24}$$

On the basis of Eqs.(22) and (24), one can discuss the results in the following three special cases.

4. Discussion

1) Under the condition of strong gas degeneracy, i.e. low temperature and high density, one can derive the fugacity, correction factor, regenerative loss and thermal efficiency of a Fermi Stirling cycle in the first approximation, respectively:

$$\ln z = \frac{T_{\rm F}(\nu)}{T} - \frac{\pi^2}{12}\frac{T}{T_{\rm F}(\nu)}, \tag{25}$$

$$F(T,\nu) = \frac{2T_{\rm F}(\nu)}{5T} + \frac{\pi^2 T}{6T_{\rm F}(\nu)}, \tag{26}$$

$$\Delta Q = \frac{\pi^2}{4D}Nk(T_{\rm H}^2 - T_{\rm L}^2)(\nu_{\rm L}^{2/3} - \nu_{\rm H}^{2/3}) < 0, \tag{27}$$

and

$$\eta^{\rm F} = \frac{W}{Q_{\rm H}} = \left(1 - \frac{T_{\rm L}}{T_{\rm H}}\right)\frac{T_{\rm H} + T_{\rm L}}{2T_{\rm H}} < \eta_{\rm c}, \tag{28}$$

where $T_F(\nu) = D/\nu^{2/3}$ is the Fermi temperature, $D = \left(3\pi^2\hbar^3\right)^{2/3}/(2km)$ is a constant, $\nu_{\rm H}$ and $\nu_{\rm L}$ are respectively the maximum and minimum volumes occupied by an ideal Fermi particle, and $\eta_{\rm c} = 1 - T_{\rm L}/T_{\rm H}$ is thermal efficiency of a Carnot heat engine cycle.

2) Under the condition of weak gas degeneracy, i.e. high temperature and low density, one can derive the fugacity, correction factor, regenerative loss and thermal efficiency of a Fermi Stirling cycle in the first approximation, respectively:

$$z = 4\sqrt{2}E/\left(T^{3/2}\nu\right)\Big[1 + 2E/\left(T^{3/2}\nu\right)\Big], \tag{29}$$

$$F\left(T,\nu\right) = 1 + E\frac{1}{T^{3/2}\nu}, \tag{30}$$

$$\Delta Q = \frac{3}{2}NkE\left(\nu_{\rm L}^{-1} - \nu_{\rm H}^{-1}\right)\left(1/T_{\rm H}^{1/2} - 1/T_{\rm L}^{1/2}\right) < 0, \tag{31}$$

and

$$\eta^{\mathrm{F}} = \frac{(T_{\mathrm{H}} - T_{\mathrm{L}})\ln(\nu_{\mathrm{H}}/\nu_{\mathrm{L}}) + E(1/\nu_{\mathrm{H}} - 1/\nu_{\mathrm{L}})(1/T_{\mathrm{L}}^{1/2} - 1/T_{\mathrm{H}}^{1/2})}{T_{\mathrm{H}}\ln(\nu_{\mathrm{H}}/\nu_{\mathrm{L}}) + (1/2)E(1/\nu_{\mathrm{H}} - 1/\nu_{\mathrm{L}})/T_{\mathrm{H}}^{1/2}}, \tag{32}$$

where $E = h^3/32(\pi mk)^{3/2}$ is a constant. Similarly, one can derive the fugacity, correction factor, regenerative loss and thermal efficiency of a Bose Stirling cycle in the first approximation, respectively:

$$z = 4\sqrt{2}\left(G/T^{3/2}\nu\right)\left[1 - 2G/\left(T^{3/2}\nu\right)\right], \tag{33}$$

$$F(T,\nu) = 1 - G/\left(T^{3/2}\nu\right), \tag{34}$$

$$\Delta Q = \frac{3}{2}NkG\left(1/\nu_{\mathrm{L}} - 1/\nu_{\mathrm{H}}\right)\left(1/\nu_{\mathrm{L}}^{1/2} - 1/\nu_{\mathrm{H}}^{1/2}\right) > 0, \tag{35}$$

and

$$\eta^{\mathrm{B}} = \frac{(T_{\mathrm{H}} - T_{\mathrm{L}})\ln(\nu_{\mathrm{H}}/\nu_{\mathrm{L}}) + G(1/\nu_{\mathrm{H}} - 1/\nu_{\mathrm{L}})\left(1/T_{\mathrm{L}}^{1/2} - 1/T_{\mathrm{H}}^{1/2}\right)}{T_{\mathrm{H}}\ln(\nu_{\mathrm{H}}/\nu_{\mathrm{L}}) + G(1/\nu_{\mathrm{L}} - 1/\nu_{\mathrm{H}})\Big/\left(1/T_{\mathrm{H}}^{1/2} + 3/2T_{\mathrm{L}}^{1/2}\right)}, \tag{36}$$

where $G = h^3/16(\pi mk)^{3/2}$ is a constant. The temperature of Bose – Einstein condensation is the lower bound of that of a Bose gas in the Bose Stirling cycle. Thus, one can only discuss the case under the condition of weak gas degeneracy.

When the temperature is high enough, Eqs.(32) and (36) may be simplified

$$\eta^{\mathrm{F}} = \eta^{\mathrm{B}} = 1 - T_{\mathrm{L}}/T_{\mathrm{H}} = \eta_{\mathrm{c}}. \tag{37}$$

In this case, an ideal quantum gas becomes an ideal classical gas and the working substance is always in classical gas state throughout the cycle.

Ratios of thermal efficiencies versus the temperature ratio for three different volume ratios, $r_\nu = \nu_{\mathrm{H}}/\nu_{\mathrm{L}}$, are shown in Fig.3, while the minimum volume and high temperature are fixed at $\nu_{\mathrm{L}} = 10^{-29}\mathrm{m}^3$ and $T_{\mathrm{H}} = 177\mathrm{K}$, respectively. The maximum values

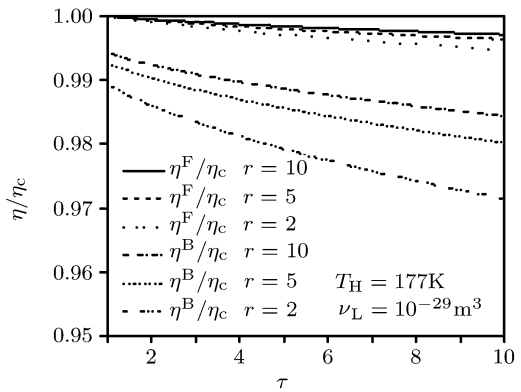


Fig.3. Ratios η/η_{c} of thermal efficiencies versus the temperature ratio τ for three different volume ratios r_ν. The minimum volume and high temperature are given as $\nu_{\mathrm{L}} = 10^{-29}\mathrm{m}^3$ and $T_{\mathrm{H}} = 177\mathrm{K}$, respectively. $T_{\mathrm{L}} > T_{\mathrm{F}}(\nu_{\mathrm{L}}) = 16.6\mathrm{K}$.

of τ are restricted by $\tau_{\max} = T_{\mathrm{H}}/T_{\mathrm{F}}(\nu_{\mathrm{L}})$ for a Fermi cycle and $\tau_{\max} = T_{\mathrm{H}}/T_0(\nu_{\mathrm{L}})$ for a Bose cycle, where $T_{\mathrm{F}}(\nu_{\mathrm{L}}) = 16.6\mathrm{K}$ and $T_0(\nu_{\mathrm{L}}) = 8.4\mathrm{K}$. Thermal efficiency decreases with the increase of τ, and the thermal efficiency of Fermi cycle is larger than that of Bose cycle. It is noted from Fig.3 that for low values of τ the thermal efficiency of the Fermi Stirling cycle is approximately equal to that of the Carnot engine cycle.

Ratios of thermal efficiencies versus the volume ratio for a constant maximum volume value, $\nu_{\mathrm{H}} = 10^{-28}\mathrm{m}^3$, and a constant maximum volume value, $\nu_{\mathrm{L}} = 10^{-29}\mathrm{m}^3$, are shown in Fig.4, while high and

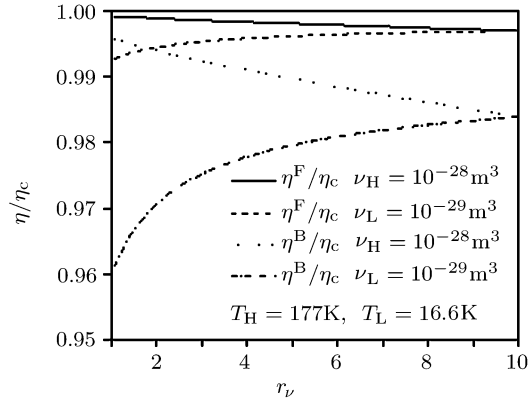


Fig.4. Ratios of thermal efficiencies versus the volume ratio. The high and low temperatures are given as $T_{\mathrm{H}} = 177\mathrm{K}$ and $T_{\mathrm{L}} = 16.6\mathrm{K}$, respectively.

low temperatures are fixed at $T_{\mathrm{H}} = 177\mathrm{K}$ and $T_{\mathrm{L}} = F_{\mathrm{F}}(\nu_{\mathrm{L}}) = 16.6\mathrm{K}$. For a constant ν_{H}, thermal efficiency decreases with the increase of r_ν. For a constant ν_{L}, thermal efficiency increases with the increase of r_ν. Therefore, it is important to know that the thermal efficiency can be improved by increasing the high volume.

3) Under the condition of $T_{\rm H} \gg T_{\rm F}(\nu_{\rm L})$ and $T_{\rm L} \ll T_{\rm F}(\nu_{\rm H})$, one can derive the fugacity, correction factor, regenerative loss and thermal efficiency of a Fermi Stirling cycle in the first approximation, respectively:

$$z(T_{\rm H},\nu) = 4\sqrt{2}E\Big/(T_{\rm H}^{3/2}\nu)\left[1+2E\Big/\left(T_{\rm H}^{3/2}\nu\right)\right], \tag{38}$$

$$F(T_{\rm H},\nu) = 1+E/\left(T_{\rm H}^{3/2}\nu\right), \tag{39}$$

$$\ln z(T_{\rm L},\nu) = \frac{T_{\rm F}(\nu)}{T_{\rm L}} - \frac{\pi^2}{12}\frac{T_{\rm L}}{T_{\rm F}(\nu)}, \tag{40}$$

$$F(T_{\rm L},\nu) = \frac{2T_{\rm F}(\nu)}{5T_{\rm L}} + \frac{\pi^2 T_{\rm L}}{6T_{\rm F}(\nu)}, \tag{41}$$

$$\Delta Q = \frac{3}{5}NkD\left(\nu_{\rm H}^{-2/3}-\nu_{\rm L}^{-2/3}\right) + \frac{3}{2}NkE\left(1/\nu_{\rm L}-1/\nu_{\rm H}\right)\Big/T_{\rm H}^{1/2} + \frac{\pi^2}{4D}NkT_{\rm L}^2\left(\nu_{\rm H}^{2/3}-\nu_{\rm L}^{2/3}\right), \tag{42}$$

and

$$\eta^{\rm F} = \frac{T_{\rm H}\ln(\nu_{\rm H}/\nu_{\rm L}) - (3/5)D\left(1/\nu_{\rm L}^{2/3}-1/\nu_{\rm H}^{2/3}\right) - E(1/\nu_{\rm H}-1/\nu_{\rm L})/T_{\rm H}^{1/2} + \left(\pi^2/4D\right)T_{\rm L}^2\left(\nu_{\rm L}^{2/3}-\nu_{\rm H}^{2/3}\right)}{T_{\rm H}\ln(\nu_{\rm H}/\nu_{\rm L}) + (1/2)E(1/\nu_{\rm H}-1/\nu_{\rm L})/T_{\rm H}^{1/2}}. \tag{43}$$

Ratios of thermal efficiencies versus the temperature ratio for three different volume ratios are shown in Fig.5, while minimum volume and high temperature are given as $\nu_{\rm L} = 10^{-29}{\rm m}^3$ and $T_{\rm H} = 177{\rm K}$, respectively. The minimum value of τ is restricted by $\tau_{\rm min} = T_{\rm H}/T_{\rm F}(\nu_{\rm H})$. When τ goes to infinity, thermal efficiency goes to its upper limit, which is lower than unity. Ratios of thermal efficiencies versus the volume ratio for a constant maximum volume value, $\nu_{\rm H} = 10^{-28}{\rm m}^3$, and a constant minimum volume value, $\nu_{\rm L} = 10^{-29}{\rm m}^3$, are shown in Fig.6, while high and low temperatures are given by $T_{\rm H} = 177{\rm K}$ and $T_{\rm L} = 1{\rm K}$. For a constant $\nu_{\rm H}$, thermal efficiency decreases with increasing r_ν. For a constant $\nu_{\rm L}$, thermal efficiency increases with increasing r_ν. The situations in this figure are very similar to those in Fig.4.

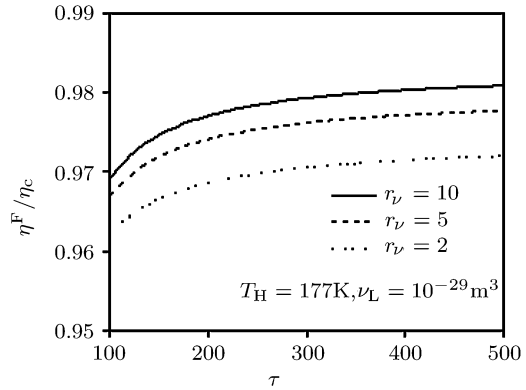


Fig.5. Ratios of thermal efficiencies $\eta^{\rm F}/n_{\rm c}$ versus the temperature ratio τ for three different volume ratios. The minimum volume and high temperature are given as $\nu_{\rm L} = 10^{-29}{\rm m}^3$ and $T_{\rm H} = 177{\rm K}$, respectively. $T_{\rm L} < T_{\rm F}(\nu_{\rm H}) = 3.57{\rm K}$.

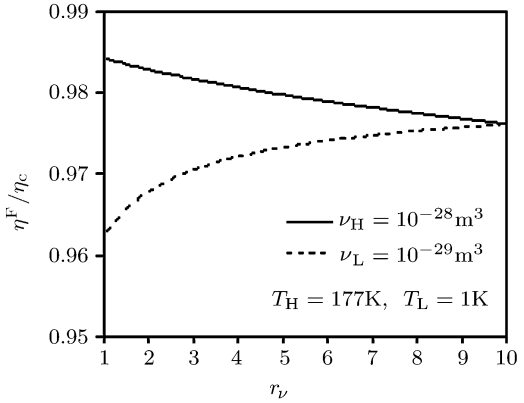


Fig.6. Ratios $\eta^{\rm F}/\eta_{\rm c}$ of thermal efficiencies versus the volume ratio r_ν. The high and low temperatures are given as $T_{\rm H} = 177{\rm K}$ and $T_{\rm L} = 1{\rm K}$, respectively.

In the zeroth approximation, the working gas is in the classical gas state at temperatures near $T_{\rm H}$ while it is in the completely degenerate gas state at temperatures near $T_{\rm L}$. In this case, Eqs.(38)–(43) may be simplified to

$$z(T_{\rm H},\nu) = 4\sqrt{2}E\Big/\left(T_{\rm H}^{3/2}\nu\right), \tag{44}$$

$$F(T_{\rm H},\nu) = 1, \tag{45}$$

$$\ln z(T_{\rm L},\nu) = \frac{T_{\rm F}(\nu)}{T_{\rm L}}, \tag{46}$$

$$F(T_{\rm L},\nu)=\frac{2T_{\rm F}(\nu)}{5T_{\rm L}}, \tag{47}$$

$$\Delta Q=\frac{3}{5}NkD\left(\nu_{\rm H}^{-2/3}-\nu_{\rm L}^{-2/3}\right), \tag{48}$$

and

$$\eta^{\rm F}=1-(3/5)D\left(1/\nu_{\rm L}^{2/3}-1/\nu_{\rm H}^{2/3}\right)/\left[T_{\rm H}\ln(\nu_{\rm H}/\nu_{\rm L})\right], \tag{49}$$

respectively.

5. Conclusions

It can be seen clearly from the results obtained above that the Stirling engine cycle working with an ideal quantum gas does not possess the condition of perfect regeneration, and the thermal efficiency of the cycle is smaller than that of the Carnot cycle in the same temperature range due to the quantum degeneracy of an ideal gas. It is worth noting that in a strong gas degeneracy, the thermal efficiency of the Fermi Stirling engine cycle is only a function of high and low temperatures. In a weak gas degeneracy, the thermal efficiency of the quantum gas engine cycle not only depends on high and low temperatures but also the maximum and minimum volumes. Meanwhile, the thermal efficiency of a Fermi cycle is larger than that of a Bose cycle. At a sufficiently high temperature, an ideal quantum gas will turn to be an ideal classical gas. Therefore, the thermal efficiency of the quantum Stirling engine cycle is equal to that of a Carnot engine cycle.

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