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Linear optical implementation of optimal unambiguous discrimination among quantum states*

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In this paper, we present a linear optical scheme for optimal unambiguous discrimination among nonorthogonal quantum states in terms of the multiple-rail and polarization representation of a single photon. In our scheme, discriminated quantum states are expressed by using the spatial degree of freedom of a single photon while the polarization degree of freedom of the single photon is used to act as an auxiliary qubit. The optical components used in our scheme are only passive linear optical elements such as polarizing beam splitters, wave plates, polarizers, single photon detectors, and single photon source.

Keywords: unambiguous discrimination of quantum states, quantum measurements, linear optical elements, a single photon

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1. Introduction

Due to encoding information in quantum states, quantum communication theory provides a variety of fascinating potential applications in the transmission and processing of information. A fundamental property of quantum states that differs from the classical counterpart is that unknown quantum states cannot be cloned exactly; another property of them is that different nonorthogonal quantum states cannot be perfectly distinguished from each other. Stirred by the development of quantum information theory, the question arises concerning the discrimination from nonorthogonal quantum states in an optimal way. To answer this question, some strategies have been designed. An optimal strategy is required to be able to maximize the average successful probability for a known ensemble of states whose prior probabilities are given. Over the past few years, a great deal of work has been devoted to the unambiguous discrimination (UD) from nonorthogonal quantum states.[1−13] The UD gives two answers, a conclusive answer and an inconclusive answer, and it has never given any erroneous results.

The UD between two nonorthogonal quantum states with identical prior probabilities was first considered by Ivanovic in 1987,[1] and then a bound on the maximum successful probability was achieved by Dieks[2] and Peres,[3] separately. The problem with nonidentical prior probabilities was also considered by Jaeger and Shimony.[4] Subsequently it is generalized to the case of the discrimination among any number of states. Chefles and Barnett[5,6] found that the UD was possible whenever the N states in question were linearly independent, and obtained a maximum average probability with an additional constraint that the successful probabilities are the same for each state. In the optical experiments, the UD between two nonorthogonal polarization states was first carried out by Huttner *et al*,[7] and later by Clarke *et al.* [10] Based on the optical method in Ref.[14], Sun *et al* have proposed an optical implementation for the discrimination among three linearly independent states by using the multiple-rail representation of a qunit,[15] and their experimental demonstration has been given by Mohseni *et al.*[16] In this paper, we present a new optical implementation of the discrimination among different linearly independent states by using spatial and polarization degrees of freedom of a single photon,[17,18] which is the combination of multiple-rail

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representation of a qunit and the polarization representation of a qubit. In our scheme, we use only linear optical elements such as polarizing beam splitters (PBSs), wave plates, mirrors, photon detectors and a single photon source.

The rest of this paper is organized as follows. In Section 2, we describe the UD by extending the space through tensor product. In Section 3, we give our optical implementation of UD based on the combination of multiple-rail representation of a qunit and the polarization representation of a qubit. Finally, we give the conclusions drawn from the present study in Section 4.

2. Unambiguous discrimination

Unambiguous discrimination among quantum states is a probabilistic error-free discrimination. For a known set of linearly independent states $\{|\psi_k\rangle, k = 1, \ldots, N\}$ with the given prior probabilities $\{\eta_k\}$ in Hilbert space $\mathcal{H}$, the UD gives $N+1$ results: state 1, state 2, ..., state N, and the result 'do-not-know'; the procedure of UD can be described by a set of $N+1$ positive operators, which form a resolution of the identity

$$\sum_{k=0}^{N-1} \hat{A}_k^\dagger \hat{A}_k + \hat{A}_I^\dagger \hat{A}_I = \hat{I}, \tag{1}$$

where $\hat{I}$ is the identity operator, $\{\hat{A}_k\}$ correspond to the conclusive results, and $\hat{A}_I$ generates the inconclusive outcome. The state-discrimination measurement is called positive operator-valued measurement (POVM).

Neumark's theorem guarantees that an arbitrary POVM can always be realized by performing the orthogonal measurement in a larger space $\mathcal{K}$. By introducing another Hilbert space $\mathcal{A}$, $\mathcal{K}$ can be extended by making either a direct sum of $\mathcal{H}$ and $\mathcal{A}$, or a tensor product of them. And the UD is investigated in detail in Refs.[17,18] by performing the former expansion, it is reasonable to consider the latter one.

Suppose that we have a device of producing quantum signals. Each time it emits a signal in a state $|\psi_i\rangle$, which is one of a given set of nonorthogonal linearly independent states $\{|\psi_k\rangle, k = 1, \ldots, N\}$, with known prior probability η_i. In order to discriminate among the states in each emission, an auxiliary system prepared in state $|H\rangle$ is introduced, and entangled with the signal by choosing a unitary operator $\hat{U}$ that yields the following state transformation:

$$|\psi_i\rangle|H\rangle \to \sqrt{p_i}|e_i\rangle|V\rangle + |\gamma_i\rangle|H\rangle, \tag{2}$$

where i takes 1, 2, ..., and N. $|e_i\rangle$ is the orthogonal basis of $\mathcal{H}$, and $|\gamma_i\rangle$ lies in $\mathcal{H}$. $|H\rangle$ and $|V\rangle$ are orthogonal and form an auxiliary space $\mathcal{A}$. Afterwards a measurement is performed on the auxiliary system that is projected into $|H\rangle$ or $|V\rangle$. If the former occurs, the decision cannot be made; if the latter occurs, a further measurement on the system among $\{|e_k\rangle\}$ should be performed, with $|e_k\rangle$ corresponding to state $|\psi_k\rangle$. The average successful probability is determined by

$$P = \sum_{i=1}^{N} \eta_i p_i, \tag{3}$$

which is dependent on the successful probability p_i. The optimal UD is to maximize the average successful probability by choosing an appropriate unitary operator $\hat{U}$ that maps the states $\{|\gamma_i\rangle\}$ into some linear dependent states.

3. Implementing unambiguous discrimination with optical method

Before we present our implementation, let us introduce some optical elements: PBS and wave plates, which can couple two incident modes if the incident photon has a definite polarization in the multiple-rail representation of a qunit. A PBS has the property of transmitting photons with horizontal polarization and reflecting photons with the vertical polarization. Choosing the horizontal $|H\rangle$ and the vertical $|V\rangle$ as basis of the polarization qubit, for a PBS that is connected with two rails, it acts like a control-not gate,[19] that is, the polarization of photons controls the rail photons taking. The photonic polarization can also be manipulated with the aid of wave plates. The action of a half-wave plates (HWP) upon the basis states of the polarization qubit is described by the following unitary operator:

$$U_H(\alpha) = \begin{pmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{pmatrix}, \tag{4}$$

and the action of a quarter-wave plate (QWP) upon the basis states of the polarization qubit is given by the following unitary operator:

$$U_Q(\alpha) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 - \mathrm{i}\cos(2\alpha) & -\mathrm{i}\sin(2\alpha) \\ -\mathrm{i}\sin(2\alpha) & 1 + \mathrm{i}\cos(2\alpha) \end{pmatrix}, \tag{5}$$

where α is the angle between the wave plate axis and the horizontal direction.

Here, we give an example to show how PBSs, HWPs and QWPs couple two paths, which is the action of a beam splitter mixing two different modes. When a photon is incident upon either input-port of a beam splitter, the action of the beam splitter on this photon is the manipulation of a $U(2)$ transformation matrix

$$\hat{B}(\beta) = \begin{pmatrix} \cos\frac{\beta}{2} & \sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & -\cos\frac{\beta}{2} \end{pmatrix}. \tag{6}$$

If the photon has a polarized direction, since a beam splitter keeps the photonic polarization, in view of two degrees of freedom for our method, the beam splitter acts like performing a transformation of $\hat{B}(\beta)\otimes\hat{I}$, so by converting the information between spatial and polarization degrees of freedom, the action of the beam splitter can be replaced by the setup in Fig.1 where only PBSs and HWPs are used. In this setup, all the PBSs transmit $|H\rangle$ and reflect $|V\rangle$, and the angles of all HWPs between the wave plate axis and the horizontal direction are set as $\pi/4$, $\beta/4$ and $\pi/4$ in sequence.

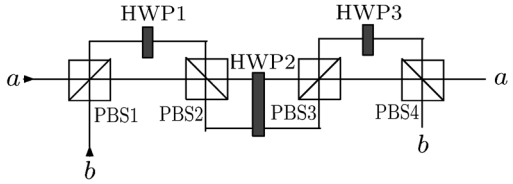


Fig.1. The setup for coupling two modes in the paths a and b. Here all PBSs transmit the state $|H\rangle$ and reflect the state $|V\rangle$, the direction angles of both HWP1 and HWP3 are $\pi/4$, the direction angle of HWP2 is set as $\beta/4$.

Since the arbitrary changes of the photonic polarization state can be performed with an HWP sandwiched between two QWPs,[17,20] we can realize all kinds of rotation gates and control-rotation gates by combining PBSs, HWPs and QWPs.

Now we present our optical realization setup by using a multiple-rail and polarization representation. Firstly, let us consider how to realize the UD between two linearly independent nonorthogonal quantum states given by

$$\begin{aligned} |\psi_1\rangle &= \cos\theta|0\rangle + \sin\theta|1\rangle, \\ |\psi_2\rangle &= \cos\theta|0\rangle - \sin\theta|1\rangle, \end{aligned} \tag{7}$$

which have prior identical probabilities. In order to discriminate between $|\psi_1\rangle$ and $|\psi_2\rangle$, we add an auxiliary qubit. The optimal UD is achieved by choosing the unitary operator in the following form:

$$\hat{U}(\theta) = \begin{pmatrix} \sqrt{1-\tan^2\theta} & 0 & 0 & -\sqrt{\tan^2\theta} \\ \sqrt{\frac{1}{2}\tan^2\theta} & 0 & \frac{1}{\sqrt{2}} & \sqrt{\frac{1}{2}(1-\tan^2\theta)} \\ 0 & 1 & 0 & 0 \\ \sqrt{\frac{1}{2}\tan^2\theta} & 0 & -\frac{1}{\sqrt{2}} & \sqrt{\frac{1}{2}(1-\tan^2\theta)} \end{pmatrix}, \tag{8}$$

which can be factorized into a product of the four unitary operators as follows:

$$\hat{U}(\theta) = \hat{B}_{12}\hat{B}_{14}\hat{B}_{23}\hat{B}_{34}, \tag{9}$$

where the operators $\hat{B}_{ij}$ have the following forms:

$$\hat{B}_{12} = \begin{pmatrix} \alpha & \sqrt{1-\alpha^2} & 0 & 0 \\ \sqrt{1-\alpha^2} & -\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \hat{B}_{14} = \begin{pmatrix} \beta & 0 & 0 & -\sqrt{1-\beta^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sqrt{1-\beta^2} & 0 & 0 & -\beta \end{pmatrix},$$

$$\hat{B}_{34} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -\sqrt{1-\gamma^2} \\ 0 & 0 & -\sqrt{1-\gamma^2} & -\gamma \end{pmatrix}, \quad \hat{B}_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{10}$$

with the three parameters determined by

$$\alpha = 2\sqrt{\frac{\cos 2\theta}{1+3\cos 2\theta}}, \quad \beta = -\sqrt{1-\frac{\tan^2\theta}{2}},$$

$$\gamma = -\sqrt{\frac{2\cos 2\theta}{1+3\cos 2\theta}}. \tag{11}$$

The operations $\hat{B}_{ij}$ are of control-rotation gates,

so we can realize them by using PBSs and wave plates. In Fig.2 presented is the optical implementation of these operations in terms of linear optical elements.

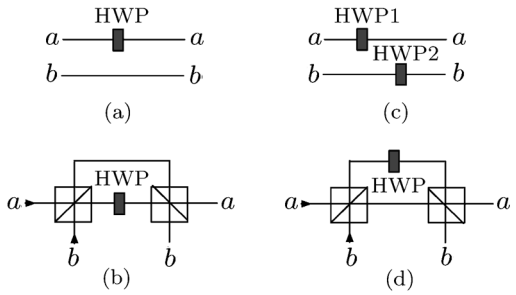


Fig.2. The setup for optically realizing the operations $\hat{B}_{ij}$ in Eq.(9) with a and b denoting the optical paths. (a) The operation $\hat{B}_{12}$; (b) The operation $\hat{B}_{34}$; (c) The operation $\hat{B}_{14}$; (d) The operation $\hat{B}_{23}$.

In view of the decomposition of the optimal UD unitary matrix (9), the setup of coupling two modes in Fig.1, and the optical realizations of the operations $\hat{B}_{ij}$ given in Fig.2, we present an optical setup for an optimal UD scheme of two linearly independent nonorthogonal quantum states given by Eq.7 in Fig.3. Here the basis of the Hilbert space in which $|\psi_1\rangle$ and $|\psi_2\rangle$ are expressed, is formed by the spatial degree of freedom of a single photon, which is labelled by 0 and 1 in Fig.3. The auxiliary system is composed of the polarization degree of freedom of this single photon.

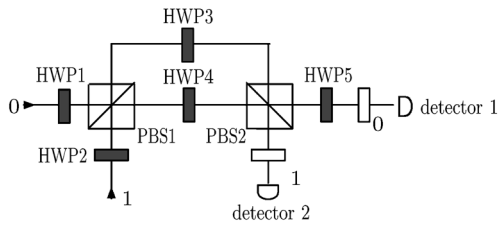


Fig.3. The setup for realizing the optimal UD for two nonorthogonal states. Two PBSs transmit the state $|H\rangle$ and reflect the state $|V\rangle$. Before detectors, two polarizers are used to filter the horizontal polarization. A successful case is indicated by the click of either detectors.

The elements used here are PBSs, HWPs, mirrors, polarizers and single photon detectors. Two PBSs transmit $|H\rangle$ and reflect $|V\rangle$. The setting angles of the HWP1 to HWP5 are denoted respectively by $\theta_1 = \pi/2$, $\theta_2 = \arccos\gamma/2$, $\theta_3 = \pi/4$, $\theta_4 = \arccos\beta/2$, and $\theta_5 = \arccos\alpha/2$. The two PBSs and five HWPs construct a unitary operator $\hat{U}(\theta)$. The measurement part is composed of two polarizers and two single photon detectors, and the polarizers are used to filter the horizontal polarization photons. A successful case is indicated by the click of either detectors. When detector 1 is clicked, the state emitted is $|\psi_1\rangle$, and when detector 2 is clicked, the state emitted is $|\psi_2\rangle$; when none of the two detectors clicks, it means the failure of the optical realization.

Next, we consider the discrimination among three given nonorthogonal states with identical prior probabilities. Suppose that the three linearly independent nonorthogonal states are given by

$$\begin{aligned} |\psi_1\rangle &= (\sqrt{2/3}, 0, \sqrt{1/3})^{\mathrm{T}}, \\ |\psi_2\rangle &= (0, \sqrt{1/3}, \sqrt{2/3})^{\mathrm{T}}, \\ |\psi_3\rangle &= (0, -\sqrt{1/3}, \sqrt{2/3})^{\mathrm{T}}, \end{aligned} \tag{12}$$

where T denotes the transport. To optimally discriminate among them, we again add an auxiliary qubit that is represented by the degree of freedom of polarization for a single photon. The unitary transformation to complete the optimal UD is a 6×6 unitary matrix which takes the following form:

$$\hat{U} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{\sqrt{8}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & \frac{1}{\sqrt{8}} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{8}} & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{2} & \frac{1}{\sqrt{8}} \end{pmatrix}. \tag{13}$$

It can be checked that the above unitary transformation of the optimal UD among three nonorthogonal states can be decomposed into the product of the following eight control-rotation operators:

$$\hat{U}(\theta) = \hat{C}_{12}\hat{C}_{14}\hat{C}_{16}\hat{C}_{23}\hat{C}_{34}\hat{C}_{36}\hat{C}_{45}\hat{C}_{56}, \tag{14}$$

where the eight control-rotation operators have the following matrix forms:

$$\hat{C}_{12} = \begin{pmatrix} -\sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \hat{C}_{14} = \begin{pmatrix} \sqrt{\frac{6}{7}} & 0 & 0 & \sqrt{\frac{1}{7}} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \sqrt{\frac{1}{7}} & 0 & 0 & -\sqrt{\frac{6}{7}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\hat{C}_{16} = \begin{pmatrix} -\sqrt{\frac{7}{8}} & 0 & 0 & 0 & 0 & -\sqrt{\frac{1}{8}} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\sqrt{\frac{1}{8}} & 0 & 0 & 0 & 0 & \sqrt{\frac{7}{8}} \end{pmatrix}, \quad \hat{C}_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

$$\hat{C}_{36} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{3}{7}} & 0 & 0 & -\sqrt{\frac{4}{7}} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\sqrt{\frac{4}{7}} & 0 & 0 & \sqrt{\frac{3}{7}} \end{pmatrix}, \quad \hat{C}_{56} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\ 0 & 0 & 0 & 0 & \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix},$$

$$\hat{C}_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \hat{C}_{45} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{16}$$

Similar to the control-rotation operations $\hat{B}_{ij}$ of the optimal UD between two nonorthogonal states, these control-rotation operations $\hat{C}_{ij}$ can also be realized in terms of PBSs and HWPs with appropriate direction angles. We present a linear optical scheme to realize the optimal UD among three nonorthogonal states in Fig.4. Here the basis of the Hilbert space in which $\{|\psi_i\rangle, i = 1, 2, 3\}$ are expanded is provided by the spatial degree of freedom of a single photon, which is labelled by 0, 1 and 2 in Fig.4, and the auxiliary system is composed of the polarization degree of freedom of this single photon.

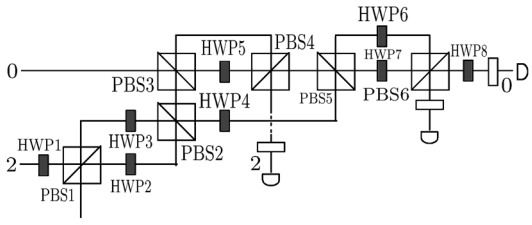


Fig.4. The setup of the optimal UD for three nonorthogonal states. Six PBSs transmit the state $|H\rangle$ and reflect the state $|V\rangle$, before detectors, three polarizers are used to filter the horizontal polarization component, a successful case is indicated by the click of one of the detectors.

All PBSs in Fig.4 transmit horizontal polarization photons and reflect vertical polarization photons, the setting angles of HWP2, HWP4 and HWP6 are the

same value $\pi/4$, and the direction angles of other five HWPs are given by

$$\cos 2\theta_1 = -\frac{1}{\sqrt{3}}, \quad \cos 2\theta_3 = -\sqrt{\frac{3}{7}}, \quad \cos 2\theta_8 = -\frac{1}{\sqrt{3}},$$
$$\cos 2\theta_5 = -\sqrt{\frac{7}{8}}, \quad \cos 2\theta_7 = \sqrt{\frac{6}{7}}. \tag{17}$$

In Fig.4, the six PBSs and eight HWPS are used to realize the optimal unitary operator given by Eq.(13). The following three polarizers realize the project measurement on the auxiliary system, and the three single photon detectors are for the project measurement on the discriminated system. When a photon enters the measurement part, it first encounters polarizers, which filter the horizontal polarization photons, and then goes to the single photon detectors. A successful UD is indicated by the click of one of these three detectors labelled by 0, 1 and 2 in Fig.4, when detector i is clicked, we can make our decision that the state emitted by the signal is $|\psi_i\rangle$ at this time, and none of the detectors clicking means the failure of the optical realization. In comparison with the scheme in Ref.[16], although both schemes employ the spatial and polarization degrees of freedom of a single photon, our setup contains only three rails, but their scheme needs four rails, which result in a different way of extending the space, and the working space in Ref.[16] can be regarded as a subspace chosen from the working space in dealing with discrimination among four states in the present paper.

4. Conclusions

We have proposed a linear optical scheme to realize the optimal unambiguous discrimination among nonorthogonal quantum states in terms of the multiple-rail and polarization representation of a single photon. In our scheme, discriminated quantum states are expressed by using the multiple-rail representation of a qunit in terms of the spatial degree of freedom of a single photon while the polarization degree of freedom of the single photon is used to act as an auxiliary qubit. The linear optical implementations of the optimal UD between two and among three linearly independent nonorthogonal quantum states are explicitly proposed by making use of only passive linear optical elements such as polarizing beam splitters, wave plates, polarizers, single photon detectors, and a single photon source. It can be expected that our method can be extended to discriminating among arbitrary number (more than two) of nonorthogonal linearly independent quantum states. Comparing our scheme with that previously proposed in Ref.[16] for the optimal UD, it is easy to find that the advantage of our method is that fewer light rails are required. With an existing single-photon source and quantum optical technology, our scheme can be realized in experiment. Our scheme can also be expected to have some applications in processing of quantum information.[21−24]

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