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Effects of initial frequency chirp on the linear propagation characteristics of the exponential optical pulse*

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In this paper, the linear propagation characteristics of the exponential optical pulse with initial linear and nonlinear frequency chirp are numerically studied in a single mode fibre for $\beta_2 < 0$. It can be found that the temporal full width at half maximum and time-bandwidth product of exponential pulse monotonically increase with the increase of propagation distance and decrease with the increase of linear chirp C for $C < 0.5$, go through an initial decreasing stage near $\zeta = 1$, then increase with the increase of propagation distance and linear chirp C for $C \geq 0.5$. The broadening of pulses with negative chirp is faster than that with positive chirp. The exponential pulse with linear chirp gradually evolves into a near-Gaussian pulse. The effect of nonlinear chirp on waveform of the pulse is much greater than that of linear chirp. The temporal waveform breaking of exponential pulse with nonlinear chirp is first observed in linear propagation. Furthermore, the expressions of the spectral width and time-bandwidth product of the exponential optical pulse with the frequency chirp are given by use of the numerical analysis method.

Keywords: fibre optics, frequency chirp, numerical analysis, exponential optical pulse
PACC: 4280M, 4281D, 0260

1. Introduction

Laser pulses are extensively applied to optical communication, ultrafast optics, and so on, and generally have frequency chirp which has great effects on characteristics of the pulse.[1−20] The previous research on exponential optical pulse is limited to the case of $T \geq 0$.[21−23] The double exponential pulse in the case of $-\infty < T < \infty$ has proven very difficult to analytically investigate and is little known. In this paper, we numerically study propagation characteristics of the double exponential pulse with initial linear and nonlinear frequency chirp in a single mode fibre for $\beta_2 < 0$, compare with those of a Gaussian pulse, and give the expressions for the spectral width and time-bandwidth product of the exponential optical pulse with the frequency chirp.

2. Characteristics of chirped exponential optical pulse

The normalized incident field of exponential optical pulse with linear chirp is given by

$$U(0,T) = \exp(-\frac{|T|}{T_0})\exp(-\frac{\mathrm{i}CT^2}{2T_0^2}), \quad (-\infty < T < \infty) \tag{1}$$

where T is the time, T_0 is the half-width at e^{-2}-intensity point, C is frequency chirp parameter. Then, the full width at half maximum (FWHM) in place of T_0 from Eq.(1) is given by

$$\Delta T = \ln 2 T_0 \approx 0.693 T_0. \tag{2}$$

The pulse phase is

$$\phi = -\frac{CT^2}{2T_0^2}. \tag{3}$$

Equation (3) shows that the phase of pulse is zero for $T = 0$, the frequency chirp parameter C is two times in magnitude as many as the phase at $T = T_0$. The frequency chirp

$$\delta\omega = -\frac{\partial\phi}{\partial T} = \frac{CT}{T_0^2} \tag{4}$$

is called up-chirp or positive-chirp for $C > 0$, down-chirp or negative-chirp for $C < 0$. It is called the linear frequency chirp that the instantaneous frequency linearly changes from the leading to the trailing edge.

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The frequency spectra of linearly chirped exponential pulse is given by

$$U(0,\omega) = \int_{-\infty}^{\infty} U(0,T)\exp(\mathrm{i}\omega T)\mathrm{d}T = \int_{-\infty}^{\infty} \exp(\frac{|T|}{T_0})\exp(-\frac{\mathrm{i}CT^2}{2T_0^2})\exp(\mathrm{i}\omega T)\mathrm{d}T. \tag{5}$$

Integrating Eq.(5) for unchirped exponential pulse ($C = 0$), one obtains

$$U(0,\omega) = \frac{2T_0}{1+\omega^2 T_0^2}. \tag{6}$$

The spectral FWHM of unchirped exponential pulse

$$\Delta\nu_0 = \frac{\sqrt{\sqrt{2}-1}}{\pi T_0} \approx 0.205/T_0, \tag{7}$$

which is given from Eq.(6). From Eqs.(2) and (7), one obtains the time-bandwidth product of unchirped exponential pulse

$$\Delta T \cdot \Delta\nu_0 = \ln 2T_0 \cdot \frac{\sqrt{\sqrt{2}-1}}{\pi T_0} \approx 0.142, \tag{8}$$

which is much less than 0.441 of unchirped Gaussian pulse or 0.315 of unchirped hyperbolic secant pulse.

We obtain the frequency spectra and expressions of the spectral FWHM of exponential pulse with different chirp by numerically solving Eq.(5). Figure 1 shows that the spectral FWHM of exponential and Gauss pulse varies with the chirp. In the figure the x coordinate is the frequency chirp parameter C, y coordinate is spectral FWHM normalized to $1/T_0$.

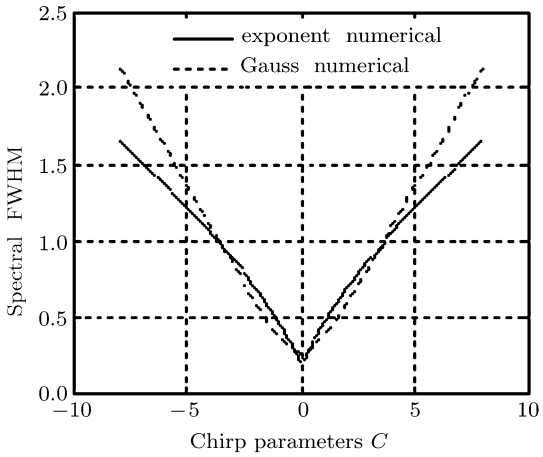


Fig.1. Variations of spectral FWHM of the exponential and gaussian pulses with frequency chirp parameter C by numerical approach, the solid curve is that of exponential pulse, the dotted curve is that of Gaussian pulse.

The solid curve is the variation of spectral FWHM of exponential pulse with chirp C by numerical approach, which is well fitted by

$$\Delta\nu = (0.001|C|^3 - 0.0203|C|^2 + 0.2779|C| + 0.205)/T_0 \quad (|C| \le 8); \tag{9}$$

the dotted curve is that of the Gaussian pulse, which is in accord with $\Delta\nu = \sqrt{\ln 2(1+C^2)}/\pi t_0$. It is shown from Fig.1 that the spectral FWHM of the exponential pulse is less than that of the Gaussian pulse for $|C| \le 0.3$, it is greater than that of the Gaussian pulse for $0.3 < |C| \le 3.52$, and less than that of the Gaussian pulse for $|C| > 3.52$. The larger that the value of $|C|$ is, the greater the difference between the spectral FWHM of the exponential pulse and that of the Gaussian pulse.

From Eqs.(2) and (9), one obtains the time-bandwidth product of exponential pulse with chirp parameter C,

$$\Delta T \cdot \Delta\nu = 0.0007|C|^3 - 0.0141|C|^2 + 0.1926|C| + 0.142, \quad (|C| \le 8). \tag{10}$$

Comparing with $\Delta t \cdot \Delta\nu = 2\ln 2\sqrt{1+C^2}/\pi$ of the Gaussian pulse, we know that the time-bandwidth product of exponential pulse increases more slowly than that of Gaussian pulse with increase of $|C|$. Equation (10) gives a new method to estimate the temporal waveform of the pulse.

3. Effects of initial linear frequency chirp on propagation characteristics

We numerically study the linear propagation characteristics of the optical pulse by use of the split-step Fourier method according to the normalized linear propagation equation

$$\mathrm{i}\frac{\partial U}{\partial \zeta} + \frac{1}{2}\frac{\partial^2 U}{\partial T^2} = 0, \tag{11}$$

where U is incident field, ζ is propagation distance which normalized to the dispersion length (L_{D}), T is the time which normalized to T_0. The input exponential and Gaussian pulses are respectively

$$U(0,T) = \exp(-|T|)\exp(-\frac{\mathrm{i}CT^2}{2}) \tag{12}$$

and

$$U(0,T) = \exp(-\frac{(1+\mathrm{i}C)T^2}{2}). \tag{13}$$

3.1. Variations of temporal width and time-bandwidth product of the pulse with propagation distance

Figure 2 shows that the temporal FWHM varies with the normalized propagation distance. The solid curves (1–3) are for the exponential pulses with $C = 0$, 1 and -1 respectively. The dotted curves (4–6) are for Gaussian pulse with $C = 0$, 1, and -1 respectively.

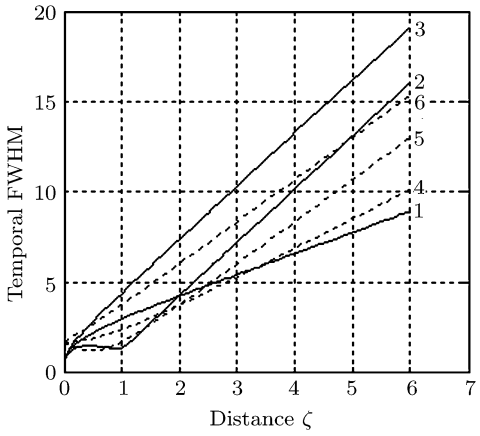


Fig.2. Variations of the temporal FWHM of two pulses with the normalized propagation distance for linear chirp. Solid curves (1–3) for the exponential pulse with $C = 0, 1, -1$, dotted curves (4–6) for Gaussian pulse with $C = 0, 1, -1$.

It can be found from the numerical results that the temporal FWHM of exponential pulse monotonically increases with the increase of propagation distance and decreases with the increase of C for $C < 0.5$, goes through an initial decreasing stage, then increases with increase of propagation distance and C for $C \geq 0.5$. The larger the value of C, the more the temporal FWHM initially decreases near $\zeta = 1$. This indicates that the initial positive chirp can induce compression of the exponential pulse.

Variations of temporal FWHM of the exponential pulse with propagation distance and chirp are faster than those of Gaussian pulse for $C < 0$ or $C \geq 0.5$, and slower than that of Gaussian pulse for $0 \leq C < 0.5$. The broadening of pulses with negative chirp is faster than that with positive chirp. This shows that the effects of the negative chirp on temporal width of the two pulses are greater than those of the positive chirp.

The variations of time-band product of the two linearly chirped pulses with propagation distance are similar to those of temporal width because the frequency spectra of the two pulses are unchanged in propagation.

3.2. Variation of the pulse peak with propagation distance

Figure 3 shows that the peaks of the exponential and Gaussian pulse vary with the normalized propagation distance in the cases of different linear chirp. In the figure the y-coordinate is the peak of temporal waveform. The solid curves (1–3) are the results of exponential pulse studied numerically for $C = 0$, 1 and -1 respectively. The dotted curves (4–6) are those of Gaussian pulse for $C = 0$, 1, and -1 respectively.

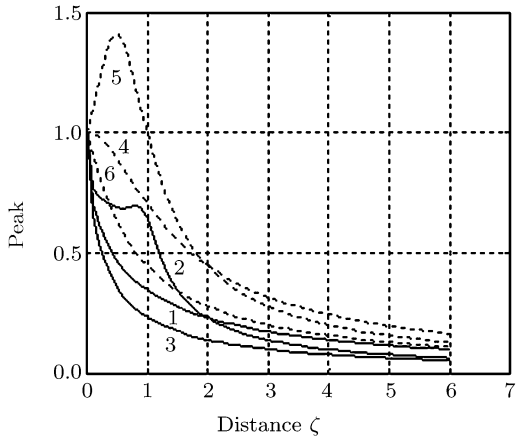


Fig.3. Variations of two pulses peaks with the normalized propagation distance for linear chirp. Solid curves (1–3) for the exponential pulse with $C = 0, 1, -1$, dotted curves (4–6) for Gaussian pulse with $C = 0, 1, -1$.

It can be found from the numerical results that the peak of exponential pulse monotonically decreases with the increase of propagation distance and in-

creases with the increase of C for $C < 0.5$, goes through an initial increasing stage, then decreases for $C \geq 0.5$. The larger the value of C is, the more the peak initially increases near $\zeta = 1$. The change of the peak of exponential pulse is faster than that of Gaussian pulse for $C < 0.5$, as shown as curves 1, 3, 4 and 6. It indicates that the exponential pulse is not appropriate to linear propagation. The initial increase near $\zeta = 1$ of Gaussian pulse is higher than that of exponential pulse, as shown as curves 2 and 5.

3.3. The evolution of temporal waveform of linearly chirped exponential optical pulse

The temporal waveforms of the exponential pulses with the initial linear chirp $C = -1$ and 1 after propagating over $6L_{\mathrm{D}}$ are shown in Fig.4. In the figure the x-coordinate is the time which normalized to T_0, and the y-coordinate is the normalized intensity of the temporal waveform.

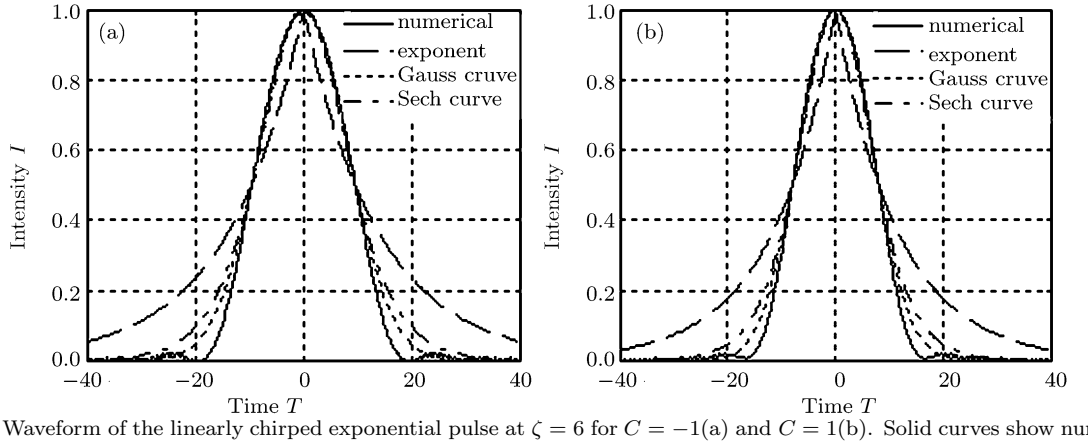


Fig.4. Waveform of the linearly chirped exponential pulse at $\zeta = 6$ for $C = -1$(a) and $C = 1$(b). Solid curves show numerical waveform of exponential pulse. Dashed curve shows waveform of exponential pulse with the same temporal FWHM as the numerical curve, the dotted curve shows that of Gaussian pulse, the dash-dotted curve shows that of hyperbolic secant pulse.

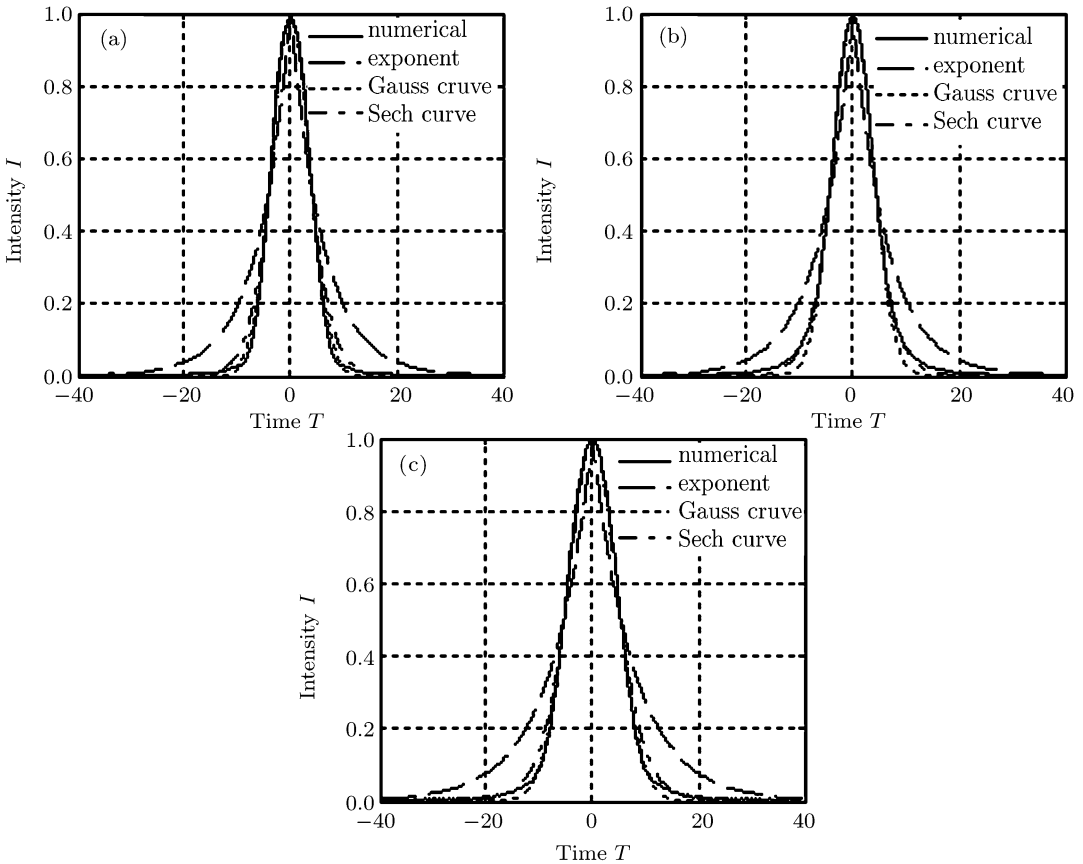


Fig.5. Waveforms of linearly chirped exponential pulse in the case of $C = 0.5$ during propagation at $\zeta = 1$(a), $\zeta = 3$(b), and $\zeta = 6$(c). The coordinates and legends are the same as those of Fig.4.

It can be found from the numerical results that exponential pulse with $C < 0$ and $C \geq 1.4$ evolves into a near-Gaussian pulse with dispersive waves at $\zeta = 1$, it always maintains near Gaussian pulse for $1 < \zeta \leq 6$. The solid curve in Fig.4(a) is the waveform of the exponential pulse at $\zeta = 6$ for $C = -1$.

The pulse with $0.7 \leq C < 1.4$ evolves into near hyperbolic secant pulse at $\zeta = 1$, it becomes near Gaussian pulse with dispersive waves at $\zeta = 3$, then remains for $3 < \zeta \leq 6$. Figure 4(b) shows the waveform of exponential pulse at $\zeta = 6$ for C=1. The longer the distance propagates, the larger the value of $|C|$ is, the stronger the dispersive waves are.

The exponential pulse with $0.4 \leq C < 0.7$ evolves into a near-Gaussian pulse at $\zeta = 1$, it becomes a near-hyperbolic secant pulse at $\zeta = 3$ and evolves into a near-Gaussian pulse at $\zeta = 6$, as shown as the solid curve in Fig.5. The exponential pulse with $0 \leq C \leq 0.3$ evolves into a near-Gaussian pulse at $\zeta = 1$, it becomes a near-hyperbolic secant pulse at $\zeta = 6$ and evolves into a near-Gaussian pulse at $\zeta = 11$.

The Gaussian pulse is the analytic solution of linear propagation equation (11) and a stable transmission mode, and remains unchanged in profile during propagation, which is consistent with the numerical results. However, exponential and hyperbolic secant pulses are not analytic solution of linear propagation equation (11), cannot stably propagate, and finally evolve into a Gaussian pulse.

4. Effects of initial nonlinear chirp on temporal waveform of exponential optical pulse

Waveform breaking is firstly observed if a perturbation of nonlinear chirp near the edges of the exponential pulse is added during linear propagation. We numerically study the variation of temporal waveforms of the pulse during linear propagation with initial nonlinear chirp (the corresponding nonlinear phase:

$$|T| \geq 2.1875,\ \Phi = 3; |T| < 2.1875,\ \Phi = -CT^2/2, \tag{14}$$

where the temporal window is 80, the number of sampling is 256, the time interval of sampling is 0.3125, the edge of pulse is at $|T| = 7 \times 0.3125 = 2.1875$).

The temporal waveforms of the exponential pulses with the initial nonlinear chirp described by Eq.(14) for $C = -1$ and $\zeta = 6$ are shown in Fig.6 (a). The coordinates and the legends are the same as those in Fig.4.

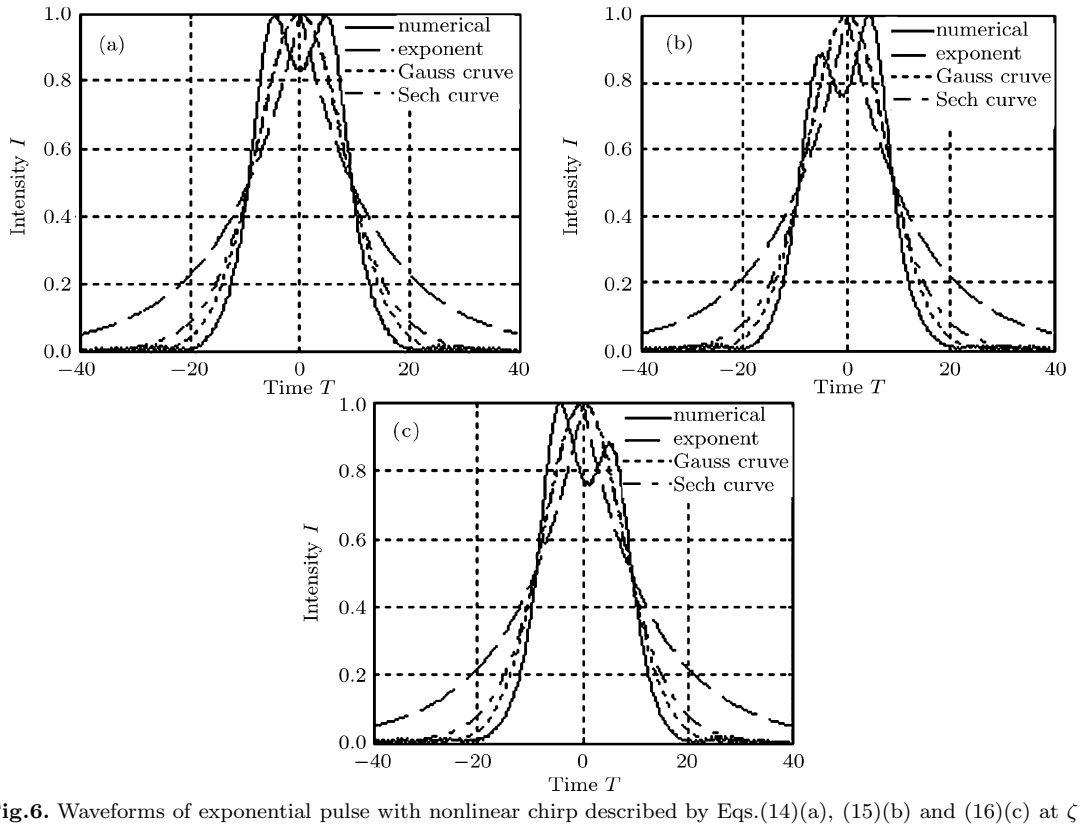


Fig.6. Waveforms of exponential pulse with nonlinear chirp described by Eqs.(14)(a), (15)(b) and (16)(c) at $\zeta = 6$ for $C = -1$. The coordinates and legends are the same as those of Fig.4.

It can be found from the numerical results that there is a symmetrical structure with dispersive waves of waveform breaking at different distance during linear propagation of the exponential pulse with different nonlinear chirp described by Eq.(14), as shown as the solid curve in Fig.6(a). An asymmetrical structure with dispersive waves of waveform breaking is observed for an asymmetrical nonlinear chirp described by

$$T \leq -2.1875,\ \Phi = 3; T \geq 2.1875,\ \Phi = 2.5; |T| < 2.1875,\ \Phi = -CT^2/2, \tag{15}$$

the front peak is lower than the back one, as shown as solid curve in Fig.6 (b). The temporal waveform is split into another asymmetrical structure with dispersive waves for an asymmetrical nonlinear chirp described by

$$T \leq -2.1875,\ \Phi = 2.5; T \geq 2.1875,\ \Phi = 3; |T| < 2.1875,\ \Phi = -CT^2/2, \tag{16}$$

the front peak is higher than the back one, as shown as solid curve in Fig.6(c). The larger the value of $|C|$ is, the longer the distance propagates, the stronger waveform breaking and dispersive waves are.

Figure 7 shows that the frequency spectra of exponential pulse vary with different nonlinear chirp described by Eqs.(14), (15) and (16) for $C = -1$. In the figure the x-coordinate is the frequency which normalized to $1/T_0$, and the y-coordinate is intensity of frequency spectra. It can be found from the numerical results that the nonlinear chirp changes the frequency spectra such as spectra breaking with oscillatory structures near the edges (Fig.7), pedestal, or other distortion. The frequency spectra of the pulse change strongly with the increase of $|C|$.

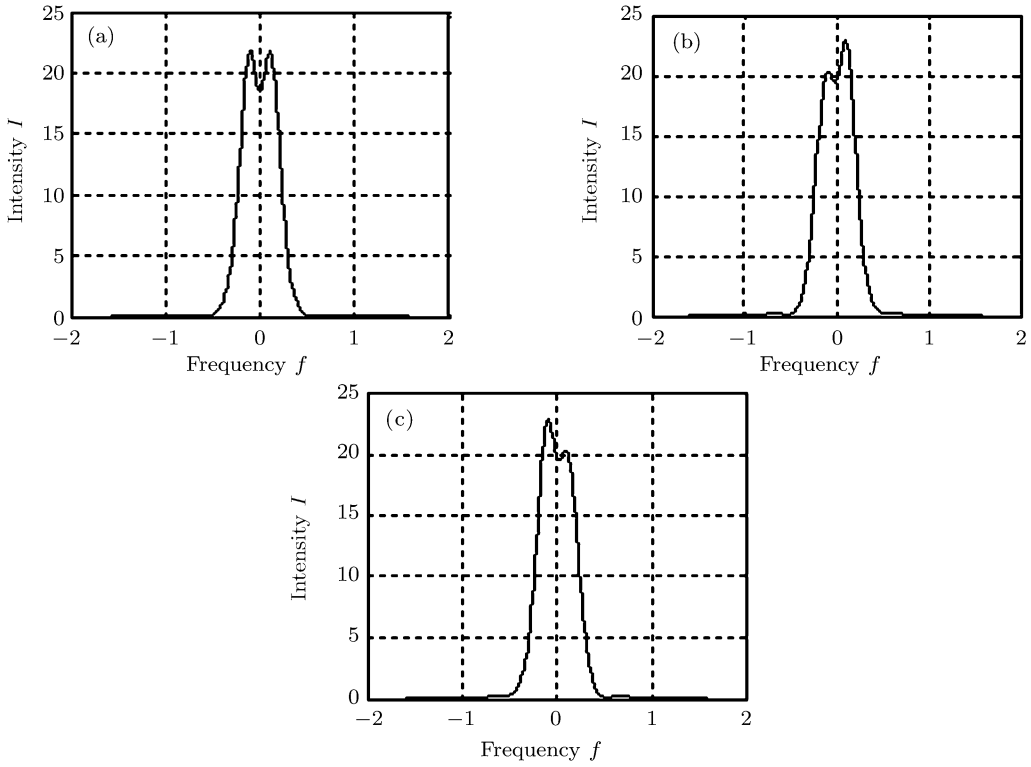


Fig.7. Spectra of exponential pulse with nonlinear chirp described by Eqs.(14)(a), (15)(b) and (16)(c) during propagation for $C = -1$.

The temporal waveform breaking of exponential pulse in linear propagation is induced by frequency spectra changing. Although the frequency spectra of the pulses remains unchanged in profile during propagation from the numerical results, it is changed by the nonlinear chirp when the pulses input into the fibre.

It can be found from numerical results that the Gaussian pulse with nonlinear chirp described by Eqs.(14), (15) and (16) remains almost unchanged in profile during propagation for $-1.7 < |C| < 2.3$ because the temporal waveform of the input pulse decreases more quickly to zero than that of the exponential pulse near $|T| = 2.1875$ and the effect of nonlinear chirp on frequency spectra is much less than that of exponential pulse. Waveform breaking of Gaussian pulse with the nonlinear chirp is obviously observed for $C \leq -1.7$ or $C \geq 2.3$ because the effect of nonlinear chirp on frequency spectra increases with the increase of $|C|$.

5. Conclusion

The expressions of the spectral width and time-bandwidth product of the exponential optical pulse with the frequency chirp are given by use of the numerical analysis method. The linear propagation characteristics of the exponential optical pulse with initial linear and nonlinear frequency chirp are numerically studied in a single mode fibre for $\beta_2 < 0$. It can be found that the temporal FWHM and time-bandwidth product of exponential pulse monotonically increase with the increase of propagation distance and decrease with the increase of linear chirp C for $C < 0.5$, go through an initial decreasing stage, then increase with the increase of propagation distance and linear chirp C for $C \geq 0.5$. The broadening of pulses with negative chirp is faster than that with positive chirp. The exponential pulse with linear chirp gradually evolves into near Gaussian pulse. The effect of nonlinear chirp on waveform of the pulse is much greater than that of linear chirp. The temporal waveform breaking of the pulses with nonlinear chirp is firstly observed in linear propagation. The larger the value of $|C|$ is, the stronger waveform breaking and dispersive waves are.

Although pulse broadening, waveform breaking and dispersive waves induced by chirp can lead to errors in an optical communication system, the exponential pulse with initial linear chirp evolving into Gaussian pulse and waveform breaking induced by nonlinear chirp give us new methods on research such as waveform converting.

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References

[1] Agrawal G P 2002 *Nonlinear Fiber Optics & Applications of Nonlinear Fiber Optics* (Jia D F, Yu Z H, Tan B and Hu Z Y transl. Beijing: Publishing House of Electronics Industry) p1–552 （in Chinese）

[2] Rao M, Sun X H and Zhang M D 2003 *Chin. Phys.* **12** 0502

[3] Pan L Z and Lü B D 2004 *Chin. Phys.* **13** 0637

[4] Wang J and Wang Z L 2004 *Chin. Phys.* **13** 0877

[5] Bakhshi B and Andrekson P A 2000 *Electron. Lett.* **36** 325

[6] Klaus M and Shaw J K 2001 *Opt. Commun.* **197** 491

[7] Ganapathy R, Kuriakose V C and Porsezian K 2001 *Opt. Commun.* **194** 299

[8] Desaix M, Helczynski L, Anderson D and Lisak M 2002 *Phys. Rev.* E **65** 056602-1

[9] Li Z H, Li L, Tian H P, Zhou G S and Spatschek K H 2002 *Phys. Rev. Lett.* **89** 263901-1

[10] Li W, Liu J S , Zeng Z N and Zhang Z Q 2003 *Chin. J. Lasers* **30** 509 (in Chinese)

[11] Han W, Wu J H and Wen S C, 2004 *J. Optoelectron. Laser* **15** 1452 (in Chinese)

[12] Bu Y ,Zhang X Y and Wang X Z 2004 *J. Optoelectron. Laser* **15** 683 (in Chinese)

[13] Du J X, Lou Q H, Chen L, Kong L F and Zhou J 2005 *Chin. J. Quantum Electron.* **22** 453 (in Chinese)

[14] Chen H, Chu X L and Zhang B 2005 *Laser J.* **26** 5 (in Chinese)

[15] Song Y R, Zhang Z G and Wang Q Y 2003 *Acta Phys. Sin.* **52** 0581 (in Chinese)

[16] Shen J L, Zhang C L, Hu Y and Jamison S P 2004 *Acta Phys. Sin.* **53** 2212 (in Chinese)

[17] Chu X L, Zhang B, Cai B W, Wei X F, Zhu Q H, Huang X J, Yuan X D, Zeng X M, Liu L Q, Wang X, Wang X D, Zhou K N and Guo Y 2005 *Acta Phys. Sin.* **54** 4696 (in Chinese)

[18] Liu S L and Zheng H J 2006 *Chin. J. Lasers* **33** 199 (in Chinese)

[19] Pei L, Jian S S, Yan F P, Ning T G, Jian W and Li T J 2003 *Acta Phys. Sin.* **52** 615 (in Chinese)

[20] Ding Y K, Li Z Y, Ni W J, Ge C F and Li S C 2004 *Acta Phys. Sin.* **53** 1384 (in Chinese)

[21] MacLeod A M, Yan X, Gillespie W A, Knippels G M H, Oepts D, van der Meer A F G, Rella C W, Smith T I and Schwettman H A 2000 *Phys. Rev.* E **62** 4216

[22] S L Shapiro 1977 *Ultrafast Laser Pulses—Picosecond Techniques and Applications* (Beijing: Zhu S Q, Science Press, transl. 1987) pp1–488 (in Chinese)

[23] Zhu J P 2003 *The Basic of Photoelectron Technology* (Beijing: Science Press) pp1–313 (in Chinese)