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Entanglement swapping with atoms separated by long distance*

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This paper proposes a scheme for entanglement swapping based on cavity QED. The two atoms to be entanglement-swapped can be separated over long distance. The scheme is a non-post-selection one with the success probability of 1/2.

Keywords: entanglement, entanglement swapping, cavity QED
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1. Introduction

Entanglement is one of the most striking features of quantum mechanics. It is fundamental for the nonlocality of quantum mechanics,[1−3] quantum computing and information processing,[4,5] and much effort has been devoted, both theoretically and experimentally, in order to achieve a deeper understanding of this property. To set up entanglement between particles A and B, one may straightforwardly consider the method of collecting them from the same source or of having them interact each other and then obtaining the entangled state of A and B after an appropriate nontrivial time evolution. One can also obtain the entanglement of the two remote particles A and B without any interaction by the method of entanglement swapping.[6−8] In addition to the entanglement generation, entanglement purification is another application of entanglement swapping.[9] Entanglement swapping plays an essential role in the context of quantum information science, so the realization of entanglement swapping is very important.

The first entanglement swapping experiment was carried out by Pan *et al*, using two pairs of polarization entangled qubits encoded in photons around 800 nm created by two different parametric down conversion (PDC) events in the same nonlinear crystal.[10] Recently, the entanglement swapping has also been tested in photon number space.[11,12] The results of above works are postselective: that is to say, once a successful entanglement swapping is verified, the swapped entangled state is destroyed having already provided a sophisticated photodetector to distinguish the one-photon and two-photon events. Theoretically, Wang *et al*[13] proposed a simple and robust non-post-selection (after the swapping is done, the swapped entanglement is still there) experimental scheme for the entanglement swapping based on the spontaneous PDC process. Yang *et al*[14] proposed a scheme about entanglement swapping without joint measurement. The above-mentioned schemes can not be applied for long distance swapping. However, most applications in quantum communication require this process to happen over large distances. Very recently, Riedmatten *et al*[15] reported the experimental realization of entanglement swapping over long distance in optical fibres. Two photons separated by more than 2 km of optical fibres are entangled, although they never directly interacted. The approach is promising as they can use the existing network of optical fibres, and can push quantum teleportation and entanglement swapping experiments out of the laboratory.

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2. Generation of the entanglement

In this paper, we propose a scheme for realizing atoms entanglement swapping over long distance. Riedmatten *et al*[15] realized photons entanglement swapping over long distance. Cavity QED with long-lived states and high-Q cavities provides a promising tool for creating entanglement.[16] Our entanglement swapping scheme is based on Feng *et al*'s scheme for entangling distant atoms,[17] and we have improved the scheme. Now we introduce Feng's idea to generate entanglements. The basic model here is first built on four identical Λ-type three-level atoms, with the atoms 1, 2, 3 and 4 trapped in optical cavities A, B, C and D respectively, as shown in Fig.1. Suppose there are three spatially separate users Alice, Clare and Bob. Alice has atom 1 and cavity A, and Bob has atom 4 and cavity D. The other atoms and cavities belong to Clare. These three-level atoms have two ground states $|g\rangle$ and $|e\rangle$ (e.g., Zeeman sublevels) and an excited state $|i\rangle$. The transitions $|i\rangle \rightarrow |g\rangle$ and $|i\rangle \rightarrow |e\rangle$ are strongly coupled to left- and right-circularly polarized cavity modes respectively. The photons leaking out from the cavities A and B (C and D) first transmit a quarter wave plate (QWP), respectively, then are focused on a polarizing beam splitter (PBS), and finally detected by two photodetectors D_1 and D_2 (D_3 and D_4). We assume that the photons coming from cavities A and B (C and D) pass through equal distance to PBS, and these four cavities are one sided so that the only leakage of photons occurs through the sides of the cavities facing QWP.

Before the atom-field interaction, these four atoms are initially prepared in the state $|i\rangle$. Each excited atom has two possible transitions $|i\rangle \rightarrow |g\rangle$ or $|i\rangle \rightarrow |e\rangle$, corresponding to an emission of a left- or right-circularly polarized photon. If both photodetectors D_1 and D_2 (D_3 and D_4) click simultaneously after a finite waiting time, we detect two photons with different polarizations, so we can make sure that the two atoms 1,2 (3,4) are now in the maximally entangled state:[17]

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle), \tag{1}$$

or

$$|\psi\rangle_{34} = \frac{1}{\sqrt{2}}(|e_3\rangle|g_4\rangle + |g_3\rangle|e_4\rangle). \tag{2}$$

After we generate these two entangled states, the whole system is in the state (before swapping)

$$|\psi\rangle_{1234} = \frac{1}{2}(|e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle) \otimes (|e_3\rangle|g_4\rangle + |g_3\rangle|e_4\rangle). \tag{3}$$

From Eq.(3), we know that there is no correlation between atom 1 and atom 4 at this moment. Our purpose here is to realize the entanglement between atom 1 and atom 4 through entanglement swapping.

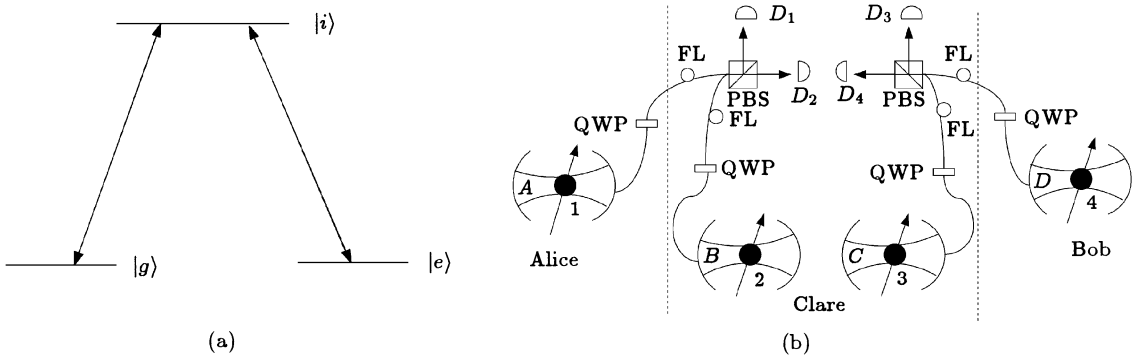


Fig.1. (a) The relevant atomic level structure. (b) Experimental setup. Four distant atoms 1, 2, 3 and 4 are trapped in cavities A, B, C and D respectively, and are initially driven to their excited states $|i\rangle$. The photons leaking out from the cavities A and B (C and D) are detected by photodectors D_1 and D_2 (D_3 and D_4) after transmitting two quarter wave plates (QWP) and a polarizing beam splitter (PBS). The FLs denote fibre loops, which are used to make the photons coming from cavities A and B (C and D) to pass through equal distances.

3. Realization of the entanglement swapping

To realize the entanglement swapping, Clare firstly turns off the traps of atom 2 and atom 3, then sends atom 2 and atom 3 simultaneously into a single-mode cavity R, which is off-resonance with the atoms transitions (see Fig.2). Let us first discuss the interaction between atom 2 (or atom 3) and the cavity R. The Hamiltonian under the rotating-wave approximation can be written as ($\hbar = 1$)

$$
\begin{aligned}
&H = H_0 + H_{\rm I},\\
&H_0 = \omega_{\rm f} a^+ a + \omega_a |i\rangle\langle i|,\\
&H_{\rm I} = g_1 |i\rangle\langle g| a + g_2 |i\rangle\langle e| a + {\rm H.C.},
\end{aligned} \tag{4}
$$

where a is the annihilation operator for the cavity mode R, and $g_1(g_2)$ is the atom-cavity coupling strength, and δ is the detuning between the atomic transition frequency ω_a and the cavity frequency $\omega_{\rm f}$.

In the case of large detuning, the upper atomic level can be eliminated adiabatically and the atom can be described as an effective two-level system.[18−21] In this case, the Hamiltonian can be reduced to

$$
\begin{aligned}
H_{\rm eff} =& - a^+ a \beta_1 |g\rangle\langle g| - a^+ a \beta_2 |e\rangle\langle e|\\
&- \lambda a^+ a (\sigma^+ + \sigma^-),
\end{aligned} \tag{5}
$$

where $\sigma^+ = |e\rangle\langle g|$, $\sigma^- = |g\rangle\langle e|$, $\lambda = g_1 g_2/\delta$ is the effective atom-field coupling constant, $\beta_1 = g_1^2/\delta$ and $\beta_2 = g_2^2/\delta$ are the Stark parameters for levels $|g\rangle$ and $|e\rangle$ respectively.

Let us now consider the situation, in which atom 2 and atom 3 simultaneously interact with cavity field R (different from the former four cavities). In addition, atom 2 and atom 3 interact each other via a dipole-dipole interaction. In this case, the effective Hamiltonian[18−22] is

$$
\begin{aligned}
H_{\rm eff} =& - a^+ a \beta_1 \sum_{j=2,3} |g_j\rangle\langle g_j| - a^+ a \beta_2 \sum_{j=2,3} |e_j\rangle\langle e_j|\\
&- \lambda a^+ a \sum_{j=2,3} (\sigma_j^+ + \sigma_j^-)\\
&+ G_a (\sigma_2^+ \sigma_3^- + \sigma_2^- \sigma_3^+),
\end{aligned} \tag{6}
$$

where $\sigma_j^+ = |e_j\rangle\langle g_j|$, $\sigma_j^- = |g_j\rangle\langle e_j|$ $(j = 2, 3)$. The term $G_a(\sigma_2^+\sigma_3^- + \sigma_2^-\sigma_3^+)$ describes the dipole-dipole interaction of the two atoms, where G_a is the coupling constant.

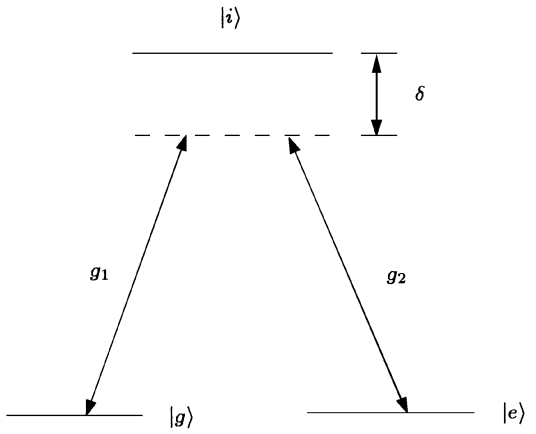


Fig.2. The level configuration of the atom. The $|i\rangle \to |g\rangle$ and $|i\rangle \to |e\rangle$ transitions are driven by the quantized cavity mode with the couplings g_1 and g_2 respectively. δ is the detuning between atomic transition frequency and cavity frequency.

If we assume that the cavity field R is initially in the vacuum state, the effective part of the Hamiltonian (6) is

$$
H'_{\rm eff} = G_a (\sigma_2^+ \sigma_3^- + \sigma_2^- \sigma_3^+), \tag{7}
$$

and the unitary time-evolution operator is given by

$$
U(t) = \exp(-{\rm i} H'_{\rm eff} t). \tag{8}
$$

Using

$$
\begin{aligned}
&(\sigma_2^+\sigma_3^- + \sigma_2^-\sigma_3^+)^{2\ell} = |e_2\rangle\langle e_2||g_3\rangle\langle g_3| + |g_2\rangle\langle g_2||e_3\rangle\langle e_3| \qquad (\ell \geq 1),\\
&(\sigma_2^+\sigma_3^- + \sigma_2^-\sigma_3^+)^{2\ell+1} = |e_2\rangle\langle g_2||g_3\rangle\langle e_3| + |g_2\rangle\langle e_2||e_3\rangle\langle g_3| \qquad (\ell \geq 0),
\end{aligned}
$$

we obtain

$$
\begin{aligned}
U(t) =& 1 - (1 - \cos(G_a t))[|e_2\rangle\langle e_2||g_3\rangle\langle g_3| + |g_2\rangle\langle g_2||e_3\rangle\langle e_3|]\\
&- {\rm i}\sin(G_a t)[|e_2\rangle\langle g_2||g_3\rangle\langle e_3| + |g_2\rangle\langle e_2||e_3\rangle\langle g_3|].
\end{aligned} \tag{9}
$$

From the time-evolution operator, we can know that the evolutions of the states of atom 2 and atom 3 in cavity R are as follows:

$$
|g_2 g_3\rangle \to |g_2 g_3\rangle, \tag{10}
$$

$$
|g_2 e_3\rangle \to \cos(G_a t)|g_2 e_3\rangle - {\rm i}\sin(G_a t)|e_2 g_3\rangle, \tag{11}
$$

$$
|e_2 g_3\rangle \to \cos(G_a t)|e_2 g_3\rangle - {\rm i}\sin(G_a t)|g_2 e_3\rangle, \tag{12}
$$

$$
|e_2 e_3\rangle \to |e_2 e_3\rangle. \tag{13}
$$

After an interaction time t, the state of the whole system evolves from Eq.(3) to

$$\begin{aligned}|\psi\rangle_{1234}(t) =&\frac{1}{2}\{|e_1\rangle|g_4\rangle[\cos(G_at)|g_2\rangle|e_3\rangle - \mathrm{i}\sin(G_at)|e_2\rangle|g_3\rangle]\\ &+ |e_1\rangle|e_4\rangle|g_2\rangle|g_3\rangle + |g_1\rangle|g_4\rangle|e_2\rangle|e_3\rangle\\ &+ |g_1\rangle|e_4\rangle[\cos(G_at)|e_2\rangle|g_3\rangle - \mathrm{i}\sin(G_at)|g_2\rangle|e_3\rangle]\}. \end{aligned} \tag{14}$$

With the choice of $G_a\tau = \pi/4$, we have

$$\begin{aligned}|\psi\rangle_{1234}\left(\tau = \frac{\pi}{4G_a}\right) =&\frac{1}{2}\left[\frac{1}{\sqrt{2}}(|e_1\rangle|g_4\rangle - \mathrm{i}|g_1\rangle|e_4\rangle)\right]|g_2\rangle|e_3\rangle + \frac{1}{2}|e_1\rangle|e_4\rangle|g_2\rangle|g_3\rangle\\ &+ \frac{1}{2}\left[\frac{1}{\sqrt{2}}(-\mathrm{i}|e_1\rangle|g_4\rangle + |g_1\rangle|e_4\rangle)\right]|e_2\rangle|g_3\rangle + \frac{1}{2}|g_1\rangle|g_4\rangle|e_2\rangle|e_3\rangle. \end{aligned} \tag{15}$$

If we detect atoms 2, 3 in the state $|ge\rangle$ or $|eg\rangle$, the state of atom 1,4 collapses into a maximally entangled state. If atoms 2, 3 are measured in the state $|ee\rangle$ or $|gg\rangle$, the entanglement swapping fails. Therefore, the present scheme is a probabilistic one with the probability of success being 50%. In our present scheme, the atoms 1 and 2 (3 and 4) can be separated by long distances while maintaining their quantum states, and the atoms 1 and 4 to be entanglement-swapped can also be separated by long distance. The scheme is based on photon, for which optical fibres provide a low-loss communication channel. The present scheme requires that two atoms (atoms 2 and 3) must be simultaneously sent through a cavity. But in a real experiment there are some errors. Assume that during the generation of a two-atom (atoms 2 and 3) maximally entangled state the atom initially in the state $|e\rangle$ enters the cavity $0.01t$ sooner than another atom, with t being the time of each atom staying in the cavity. Then, the two atoms are finally prepared in the state

$$\begin{aligned}|\psi\rangle =&\cos(0.99G_at)|e_2\rangle|g_3\rangle\\ &- \mathrm{i}\mathrm{e}^{-0.01\mathrm{i}G_at}\sin(0.99G_at)|g_2\rangle|e_3\rangle, \end{aligned} \tag{16}$$

with the common phase factor G_at being discarded. Choosing $t = \pi/(4G_a)$ we have

$$|\langle\psi_{\mathrm{EPR}}|\psi\rangle|^2 \simeq 0.99, \tag{17}$$

where $|\psi_{\mathrm{EPR}}\rangle = \frac{1}{\sqrt{2}}(|e_2\rangle|g_3\rangle - \mathrm{i}|g_2\rangle|e_3\rangle)$. In this case the operation is only slightly affected.

Next, we will discuss the experimental feasibility of our proposed scheme. The atom can be trapped and manipulated at the single-atom level within the current technology by the group led by Kimble.[16] Once the entanglements are generated, the traps are turned off quickly, then the two atoms (atom 2 and atom 3) crossing simultaneously a nonresonant cavity are entangled by coherent energy exchange.[23] In the experiments of Haroche's group,[23] the atomic states can be detected by the ionization detectors with high efficiency. Here the two atoms may have different velocities, but are sent at different times so that they can simultaneously cross the cavity axis. In addition, because the control of the velocity of an atom is very difficult, the interaction can be controlled by adjusting the atom-field detuning. Although it is not easy to realize the scheme experimentally, from the researches implemented by the Kimble group and Haroche group, the proposed scheme might be realizable.

4. Conclusion

In conclusion, we have proposed a scheme for realizing the entanglement swapping of atomic states, and the scheme is based on current cavity QED technique.[16,23] Our scheme is a non-post-selection one, and the two atoms to be entanglement-swapped in our scheme can be located far away. The present scheme may become an alternative to current methods to construct quantum networks for quantum communication.

References

[1] Bell J S 1965 *Physics* **1** 195

[2] Greenberger D M, Horne M A and Zeilinger A 1989 *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, ed M Kafatos (Dordrecht: Kluwer)
Greenberger D M, Horne M A, Shimony A and Zeilinger A 1990 *Am. J. Phys.* **58** 1131

[3] Mermin N D 1990 *Phys. Rev. Lett.* **65** 1838

[4] Nielsen M and Chuang I 2000 *Quantum Computation and Information* (Cambridge: Cambridge University Press)

[5] Jin X R and Zhang S 2005 *Chin. Phys.* **14** 1936
Zhang J 2004 *Chin. Phys.* **13** 1438
Zheng S B 2005 *Chin. Phys.* **14** 533
Zheng S B 2005 *Chin. Phys.* **14** 1825

[6] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[7] Zukowski M, Zeilinger A, Horne M A and Ekert A K 1993 *Phys. Rev. Lett.* **71** 4287

[8] Bose S, Vedral V and Knight P L 1998 *Phys. Rev.* A **57** 822

[9] Bose S, Vedral V and Knight P L 1999 *Phys. Rev.* A **60** 194

[10] Pan J W, Bouwmeester D, Weinfurter H and Zeilinger A 1998 *Phys. Rev. Lett.* **80** 3891

[11] Lombardi E, Sciarrino F, Popescu S and Martini F De 2002 *Phys. Rev. Lett.* **88** 070402

[12] Sciarrino F, Lombardi E, Milani G and Martini F De 2002 *Phys. Rev.* A **66** 024309

[13] Wang X B, Shi B S, Tomita A and Matsumoto K 2004 *Phys. Rev.* A **69** 014303

[14] Yang M, Song W and Cao Z L 2004 *Phys. Rev.* A **71** 034312

[15] Riedmatten H de, Marcikic I, Houwelingen J A W van, Tittel W, Zbinden H and Gisin N 2005 *Phys. Rev.* A **71** 050302(R)

[16] Ye J, Vernooy D W and Kimble H J 1999 *Phys. Rev. Lett.* **83** 4987
McKeever J and Kimble H J 2002 in *Laser Spectroscopy: XIV International Conference*, edited by Chu S and Kerman J (Singapore: World Scientific)
Hood C J, Lynn T W, Doherty A C, Parkins A S and Kimble H J 2000 *Science* **287** 1447

[17] Feng X L, Zhang Z M, Li X D, Gong S Q and Xu Z Z 2003 *Phys. Rev. Lett.* **90** 217902

[18] Agarwal G S 1970 *Phys. Rev.* A **1** 1445
Biswas A and Agarwal G S 2004 *Phys. Rev.* A **69** 062306

[19] Cirac J I and Sánchez-Soto L L 1991 *Phys. Rev.* A **44** 3317

[20] Alexanian M and Bose S K 1995 *Phys. Rev.* A **52** 2218

[21] Xu L and Zhang Z M 1994 *Z. Phys.* B **95** 507

[22] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392

[23] Osnaghi S, Bertet P, Auffeves A, Maioli P, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902
Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565