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Analysis of spectrum characteristics of optical scintillation in stack gas flow

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Based on the analysis of spectrum characteristics of intensity fluctuations while light beams pass through stack gas flow in an industrial setting, this paper puts emphasis upon discussing the spectrum of optical intensity fluctuations by the variety of particle concentration in stack gas flow. This paper also gives the primary theoretical explanation of the measurement results in the stack of coal-fired utility boilers. Meanwhile, the cross-correlation formula is given as the theoretical basis of velocity measurement by using particle concentration scintillation.

Keywords: stack gas flow, optical scintillation, particle concentration, spectrum
PACC: 4220E, 8670L

1. Introduction

Particulate matter (PM) is one of the most important forms of air pollution. The stack gas emitted by coal-fired power plants, cement factories, metal smelt manufacturers and so on is the main source of particulate matter and pollution in the atmosphere. This situation will continue for a long time. Online continuous measurement needs to be performed at these enterprises in order to control and deal with pollution. We can use optical scintillation caused by a light beam passing through the stack gas flow to measure flow velocity and particle concentration. This method has many advantages, but an integrated theory about the method is not found yet.

First of all, when a light beam passes through stack gas flow, particles move in or out the view of sight in random. This will induce optical intensity fluctuations. The scintillation made by particle statistical fluctuations only can be observed when the view of sight is small, the optical path is short, the particle diameter is large and the concentration is low. For example, Yuan *et al*[1] used laser and fibre to observe the scintillation caused by the random distribution of particle in the pipe with 75mm diameter. They use the cross-correlation method to measure stack gas flow velocity. The fibre diameter of the two receiver fibres was 200 μm and the distance was 0.6 mm. Chen *et al*[2] in Tsinghua university gave the theoretical relationship between particle concentration and standard deviation of relative optical intensity of scattering light. It showed that the standard deviation of relative optical intensity of scattering light is in inverse proportion to concentration and volume. Moreover, the relationship applied to measure concentration can only be used at the angle of non-transmission light. So the scintillation caused by particle spatial random distribution along the direction of light beam propagation is not suitable to measure particle concentration.

When the optical signal is received by a point detector, the correlation time of scintillation is equal to the time which particles move over the scale of its self-size.[3] So the characteristic frequency is

$$f_1 \approx \frac{v}{d}, \tag{1}$$

where v is the average flow velocity, d is the particle diameter. Let $v = 10$ m/s, $d = 10$ μm, thus $f_1 = 10^6$ Hz. So the frequency is very high. While the diameter of the receiver aperture is $D_{\rm r}$, the correlation time in magnitude is approximately equal to the time which particle move over the aperture.[3] So the characteristic frequency is

$$f_2 \approx \frac{v}{D_{\rm r}}. \tag{2}$$

If $D_{\rm r} < 1$ mm, then f_2 is larger than 10^4 Hz. Lacaze[4]

analysed the spectrum of scattering light and got the similar results.

Based on the characteristics of the scintillation caused by particle movement in random, the scintillation signal caused by the statistical fluctuations of particle spatial distribution cannot be received commonly when the larger size of transmitter and receiver aperture is used to measure optical scintillation in the large stack of factory.

Secondly, in high temperature stack gas flow, the turbulence will induce the refractive index fluctuations in the temporal and spatial domain. When the light beam passes through the flow, scintillation will come into being and the received optical intensity will change with time. Supposing that the turbulence is homogeneous isotropic, the theory about scintillation intensity, scintillation spectrum and the correlation is consummate except for some problems about probability distribution. The characteristic frequency of scintillation is

$$f_3 = \frac{\upsilon}{(2\pi\lambda L)^{1/2}}\,. \tag{3}$$

Where λ is the wave length, L is the optical pathlength, $(\lambda L)^{1/2}$ is the Fresnel length. For the stack whose diameter is $1-10$ m, f_3 is in the range of hundreds and thousands Hz. While the transmitter aperture size or the receiver aperture size is larger (for incoherent light), for example, near to cm in magnitude, the characteristic frequency of scintillation will decline because of 'aperture average'. It is

$$f_4 \approx \frac{\upsilon}{D_{\mathrm{r}}}\,. \tag{4}$$

The cross-correlation of the amplitude of optical field was given by Wang *et al*:[5]

$$C_\chi(\rho,\tau) = 0.132\pi^2k^2\int_0^L \mathrm{d}z\int_0^\infty C_n^2(z)\kappa^{-8/3}\sin^2\Big[\frac{\kappa^2 z(L-z)}{2kL}\Big] \times J_0\Big[\kappa\Big|\rho\frac{z}{L}-\upsilon\tau\Big|\Big]\cdot\left\{\frac{2J_1\left(\frac{\kappa D_{\mathrm{r}}z}{2L}\right)}{\frac{\kappa D_{\mathrm{r}}z}{2L}}\right\}^2\left\{\frac{2J_1\left[\frac{\kappa D_{\mathrm{t}}}{2}\left(1-\frac{z}{L}\right)\right]}{\frac{\kappa D_{\mathrm{t}}}{2}\left(1-\frac{z}{L}\right)}\right\}^2\mathrm{d}\kappa\,, \tag{5}$$

where $k=2\pi/\lambda$, ρ is the distance of the two detectors, $C_n^2(z)$ is the refractive index structure parameter of the flow medium, L is the optical pathlength, z is the axis direction coordinate of light beam, υ is the flow velocity, J_0, J_1 is zero-order and first-order Bessel function respectively, D_{t}, D_{r} is the diameter of the transmitter and receiver aperture respectively, κ is the wave number of turbulence. The formula may be considered as the basic theory of velocity measurement by using correlation method. Suppose $\rho=0$, we can get the time-correlation curve. Then scintillation spectrum can be obtained by Fourier transform.

The fluctuations of particle concentration field also can cause optical scintillation. The scintillation intensity shows good linearity with particle concentration. This can be used to measure concentration. This paper will research into spectrum characteristics by theoretical analysis and experimental measurement. Then it tries to give the formula of cross-correlation as the theoretical basis of particle concentration measurement using scintillation, and the theoretical basis of flow velocity measurement using low frequency of optical scintillation.

2. The measurement results

The measurement was taken in the stack of No.3 boiler in Anhui Tongling power plant. The width of the stack is 4.8 m. The measure point is about 10 m away from the fan and electrostatic dust catcher is at work. The velocity of the stack gas flow is about 15 m/s and the temperature is about 120 °C. The measurement was taken at 17/04/2005 and 01/09/2005 respectively. The light source is LED whose wave length is 630 nm. The processed source is a much more ideal Gauss spherical wave. Both the transmitter aperture and receiver aperture diameter are 18 mm. The signals received by silicon photoelectric diode were magnified, processed in logarithm and filtered by bandpass filter. Then the analogous signals were converted

into digital signals by AD card. The samples was recorded by computer, and then the spectrum was analysed. The frequency band of the band-pass filter was 70–2800 Hz at April 17 while the band was 0.05–250 Hz at September 2. Figures 1 and 2 show the representative spectrum in part of low frequency and in part of high frequency respectively.

Figure 1 shows that the power ratio of the spectrum in the range of 1 and 250 Hz is $-5/3$ and it shows good agreement with turbulent spectra. Perhaps because the intensity fluctuations is very strong, the influence of refractive index scintillation is too weak to be observed. The flow velocity of the stack gas is 15 m/s and the outer-scale of turbulence is about 10 m, so the power ratio of the spectrum in inertial range is $-5/3$. The spectrum near to 0.04Hz reflects another physical process, that is, the great fluctuations in low frequency is caused by production processing. The concentration increases periodically because the pole of the electrostatic dust catcher is quivered every now and then. The increase induces measurement error. But the error can be removed by making use of its spectrum characteristics.

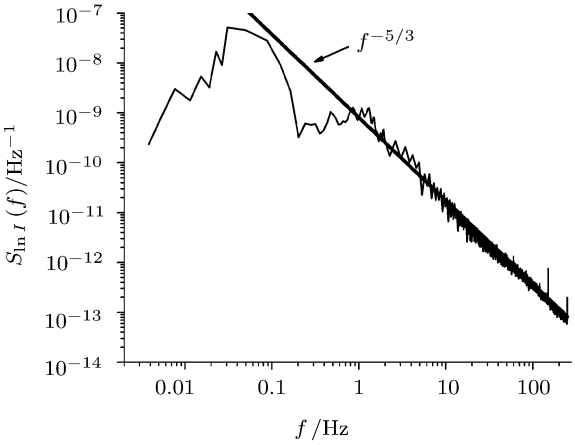


Fig.1. The low frequency part of optical scintillation spectrum of stack gas flow. The time of measurement is 04/17/2005. The site is Anhui Tonglin power plant. The standard deviation of log-intensity fluctuations $\sigma_{\ln I} = 0.013$. The particle concentration is 181 mg/m^3. The optical pathlength is 4.8 m.

In Fig.2, the power ratio of the spectrum in the range of 150 and 2000 Hz is $-8/3$. The limitation of lower frequency is less than v/D_r. The ratio in part of low frequency approaches to $-5/3$. This perhaps has to do with the influence of scintillation in part of low frequency caused by concentration fluctuations. Actually, although the characteristic frequency caused by concentration fluctuations is different from that caused by refractive index fluctuations, the overlap of the two spectrums must have influence on the cross-correlation and concentration measurement. This problem must be overcome when the sensor is designed. The optical scintillation in part of low frequency, that is, the scintillation caused by the fluctuations in concentration field will be analysed in what follows.

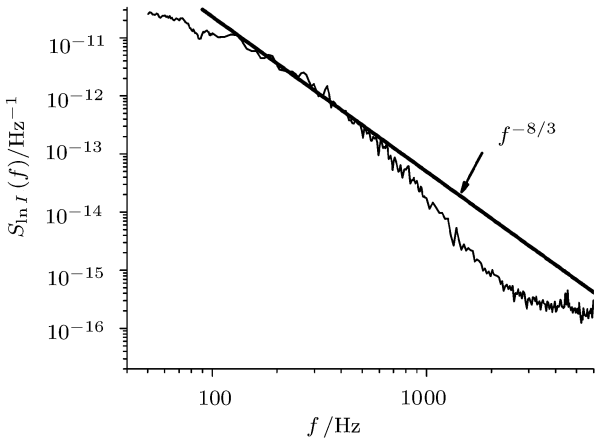


Fig.2. The high frequency part of optical scintillation spectrum of stack gas flow. The time of measurement is 07/07/2005. The site is Anhui Tonglin power plant. The standard deviation of log-intensity fluctuations $\sigma_{\ln I} = 0.004$. The optical pathlength is 4.8 m.

3. The cross-correlation function of scintillation caused by extinction coefficient fluctuation

The signal is received in manner of Fig.3. The transmitter beam is a divergent spherical wave. The axes of light beam is z-axis and the origin is at $z = 0$. The direction of stack gas flow is y-axis and the average velocity is v. The diameter of transmitter aperture is D_t and the diameter of the two receivers is D_r. The distance of the two receivers is ρ.

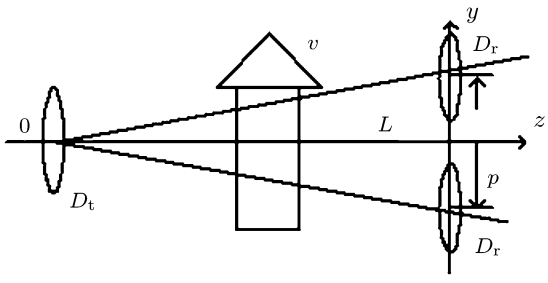


Fig.3. The layout of scintillation measurement.

Let the extinction coefficient of stack gas flow be $\alpha(r,t)$, it can be written as

$$\alpha(r,t) = \langle\alpha(r,t)\rangle + \alpha'(r,t)\,, \tag{6}$$

where $\alpha'(r,t)$ is the perturbation part. Here we discuss a system with one point source transmitter and two point receivers. Then the intensity is

$$I(t) = I_0\exp\Big[-\int_0^L \alpha(r,t)\mathrm{d}z\Big]\,. \tag{7}$$

For $\int_0^L \alpha'(r,t)\mathrm{d}z \ll 1$, the formula (7) can be written as

$$\begin{aligned} I(t) &= I_0\exp[-\langle\alpha(r,t)\rangle L]\Big[1-\int_0^L \alpha'(r,t)\mathrm{d}z\Big] \\ &= \langle I\rangle\Big[1-\int_0^L \alpha'(r,t)\mathrm{d}z\Big], \end{aligned} \tag{8}$$

or

$$\ln I(t) = \ln\langle I\rangle - \int_0^L \alpha'(r,t)\mathrm{d}z\,. \tag{9}$$

So

$$\begin{aligned} C_I(r,\rho,t,\tau) = \langle I\rangle^2 \Big\langle \int_0^L \alpha'(r_1,t-\tau)\mathrm{d}z_1 \\ \times \int_0^L \alpha'(r_2,t)\mathrm{d}z_2\Big\rangle, \end{aligned} \tag{10}$$

or

$$\begin{aligned} C_{\ln I}(r,\rho,t,\tau) = \Big\langle \int_0^L \alpha'(r_1,t-\tau)\mathrm{d}z_1 \\ \times \int_0^L \alpha'(r_2,t)\mathrm{d}z_2\Big\rangle, \end{aligned} \tag{11}$$

where τ is time delay and the angle brackets denote an ensemble average. For homogeneous isotropic time stationary turbulence, the correlation function is only relative to the distance of the two receivers and the time delay. So the former formula can be written as

$$C_{\ln I}(\rho,\tau) = \int_0^L\int_0^L R_\alpha(r_1-r_2,\tau)\mathrm{d}z_1\mathrm{d}z_2\,, \tag{12}$$

where $R_\alpha(r_1-r_2,\tau)$ is the correlation function of extinction coefficient. If the turbulence is frozen and the geometry accords with Fig.3, we obtain

$$R_\alpha(r_1-r_2,\tau) = R_\alpha\Big(0,\frac{(z_1+z_2)\rho}{2L}-\upsilon\tau,z_1-z_2\Big)\,. \tag{13}$$

Let $\xi = z_1 - z_2$, $z = z_2$, then

$$C_{\ln I}(\rho,\tau) = 2\int_0^L\int_0^z R_\alpha\Big[(z+\xi/2)\frac{\rho}{L}-\upsilon\tau,\xi\Big]\mathrm{d}z\mathrm{d}\xi, \tag{14}$$

where

$$\begin{aligned} R_\alpha(y,\xi) = \iiint_0^\infty \cos(\kappa_2 y+\kappa_3\xi) \\ \times\phi_\alpha(\kappa)\mathrm{d}\kappa_1\mathrm{d}\kappa_2\mathrm{d}\kappa_3\,. \end{aligned} \tag{15}$$

Here $y = z+\xi/2-\upsilon\tau$, $\varphi_\alpha(\kappa)$ is three-dimensional power spectrum of extinction coefficient fluctuations and $\kappa^2 = \kappa_1^2+\kappa_2^2+\kappa_3^2$.

Considering the aperture of transmitter and receiver, we can read in the Fourier transform of pupil function similarly as formula (5). Then the cross-correlation will be got.

When the source is collimated wave, following as Kampe de Fériet,[6] we can write the formula (14) as

$$C_{\ln I}(\rho,\tau) = 2\int_0^L (L-z)R_\alpha(\rho/L-\upsilon\tau,z)\mathrm{d}z\,. \tag{16}$$

Similarly, the correlation function of extinction coefficient can be predigested as

$$\begin{aligned} R_\alpha\Big(\frac{\rho}{L}-\upsilon\tau,\xi\Big) = \iiint_0^\infty \cos[\kappa_2(\rho/L-\upsilon\tau) \\ +\kappa_3\xi]\phi_\alpha(\kappa)\mathrm{d}\kappa_1\mathrm{d}\kappa_2\mathrm{d}\kappa_3. \end{aligned} \tag{17}$$

In order to calculate the scintillation spectrum, we let $\rho = 0$. And then, from Eq.(14), we obtain

$$C_{\ln I}(\tau) = 2\int_0^L\int_0^z R_\alpha(-\upsilon\tau,\xi)\mathrm{d}z\mathrm{d}\xi\,. \tag{18}$$

Similarly, Eq.(14) becomes

$$C_{\ln I}(\tau) = 2\int_0^L (L-z)R_\alpha(-\upsilon\tau,z)\mathrm{d}z\,. \tag{19}$$

And

$$\begin{aligned} R_\alpha(-\upsilon\tau,z) = \iiint_0^\infty \cos(\kappa_2\upsilon\tau+\kappa_3 z) \\ \times\phi_\alpha(\kappa)\mathrm{d}\kappa_1 d\kappa_2\mathrm{d}\kappa_3\,. \end{aligned} \tag{20}$$

4. Optical scintillation spectrums in part of low frequency

The relationship between extinction coefficient and particle concentration is

$$\alpha = K_m m\,, \tag{21}$$

where K_m is the relative extinction coefficient and it has concerned with the particle scale distribution and

refractive index, m is particle concentration. Because of turbulence, in the optical path, we have

$$m = m_0 + m' . \tag{22}$$

where m_0 is average concentration of particles. So the extinction coefficient fluctuations is proportional to the particle concentration.

$$\alpha' = K_m m' , \tag{23}$$

Suppose that particle concentration is a conservative and passive scalar quantity, for sufficiently developed turbulence, the square mean of the extinction coefficient difference of the two points s_1, s_2 in turbulent field is

$$D_\alpha^2 = \langle [\alpha(s_1) - \alpha(s_2)]^2 \rangle = C_\alpha^2 l^{2/3} . \tag{24}$$

Where C_α^2 is the structure function of extinction coefficient and l is the distance of the two points, $l_0 \ll l \ll L_0$, here l_0 and L_0 is the inner-scale and outer-scale of turbulence respectively.

Let

$$\langle \alpha'^2 \rangle = \sigma_\alpha^2 = C_\alpha^2 L_0^{2/3}/2 .$$

So, from the formula (19), we obtain

$$R_\alpha(-\upsilon\tau, z) = C_\alpha^2 (L_0^{2/3} - l^{2/3})/2 , \tag{25}$$

where $l = \sqrt{(\upsilon\tau)^2 + z^2}$. If $l > L_0$, $R_\alpha = 0$.

Substituting Eq.(25) into Eq.(19), we can get the auto-correlation function. Then the scintillation power spectrum will be obtained by Fourier transformation. In order to get the scintillation power spectrum in part of low frequency intuitively, we construct the time structure function of scintillation as follows:

$$\begin{aligned} D_{\ln I}(\tau) &= \langle [I(\tau) - I(0)]^2 \rangle / \langle I \rangle^2 \\ &= 2[C_{\ln I}(0) - C_{\ln I}(\tau)] . \end{aligned} \tag{26}$$

From Eqs.(19), (25) and (26), we have

$$\begin{aligned} D_{\ln I}(\tau) = 2 \int_0^L & (L - z) C_\alpha^2 \\ & \times \{ [(\upsilon\tau)^2 + z^2]^{1/3} - z^{2/3} \} \mathrm{d}z . \end{aligned} \tag{27}$$

In the former formula, if

$$\sqrt{(\upsilon\tau)^2 + z^2} > L_0,$$

we still let

$$\sqrt{(\upsilon\tau)^2 + z^2} = L_0.$$

When $\upsilon\tau \ll L$, we obtain $D_{\ln I}(\tau) \approx \tau^p$, $p \to 2$. Here the power ratio of the spectrum tends to -3.[7] It is much steeper than $-5/3$. Only when $\upsilon\tau \ll L$, $p \to 2/3$, the power ratio of the scintillation spectrum is $-5/3$. Because the integral along the path smoothes the small-scale fluctuations of extinction coefficient, the power ratio becomes steeper. This is an inevitable result when homogeneous isotropic turbulence averages along the path and it is different from the measurement results in Fig.1.

In practice, the turbulence in stack is different from that caused by instability of average flow. The optical path is in backward position of the fan. So the fluctuations in concentration field will be disturbed by the fan. Perhaps the cascade process of turbulent eddies is only in the flow direction. But the optical path is perpendicular to the direction. So we can consider that the fluctuations of extinction coefficient at every point in z-axis is irrelevant. And then

$$\begin{aligned} D_{\ln I}(\tau) &= 2 \int_0^L (L - z) C_\alpha^2 (\upsilon\tau)^{2/3} \mathrm{d}z \\ &= C_\alpha^2 L^2 \upsilon^{2/3} \tau^{2/3} . \end{aligned} \tag{28}$$

The power ratio of the spectrum whose time structure function is satisfied with the formula (28) is $-5/3$:

$$\begin{aligned} & \Phi_{\ln I}(\kappa) = 0.24 C_\alpha^2 L^2 \kappa^{-5/3} , \\ & \kappa = 2\pi f/\upsilon , \quad 2\pi/L_0 \ll \kappa \ll 2\pi/l_0 . \end{aligned} \tag{29}$$

Then the result is in agreement with Fig.1. From Eq.(23), the upper formula can be written as

$$\Phi_{\ln I}(\kappa) = 0.24 K_m C_m^2 L^2 \kappa^{-5/3}, \tag{30}$$

where C_m^2 is the structure function of concentration field.

From Eq.(28), a simple relationship between mean square of the optical intensity and mean square of concentration fluctuations can be obtained:

$$\sigma_{\ln I}^2 = K_m^2 L^2 \sigma_m^2 . \tag{31}$$

If $\sigma_m = a m_0$, where a is defined as the index of fluctuations depth (obviously $a < 1$), then

$$\sigma_{\ln I} = a K_m L m_0 . \tag{32}$$

For a certain kind of fuel, K_m is a approximately constant. Suppose that the index a is only relative to the scale of the stack and the position of the optical path while it is irrelative with the velocity of stack gas flow and the average concentration of particle, the formula (32) can be considered as the basic theory of measurement. The measurement results in Fig.4 also

show that the linear relation between the standard deviation of the scintillation and average concentration is not changed obviously even if the particle concentration decreases to a half after the coal-fired utility boiler is repaired. The measurement was taken on September 1st.

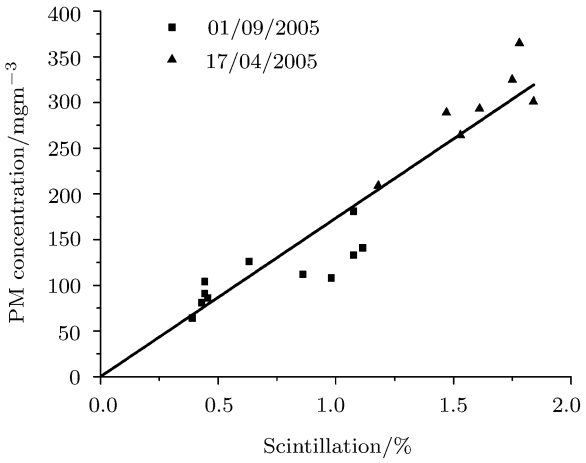


Fig.4. The measurement results of particle concentration and optical scintillation. The concentration was measured by weighting. The sample time was ten minutes at April 17th while the sample time was four minutes at September 1st.

5. Discussions

There are four main reasons of the optical intensity fluctuations caused by light beam passing through stack gas flow. First, there are the fluctuations in temperature field. The frequency of this kind of scintillation caused by turbulence is higher and it reaches hundreds or thousands Hz. The scintillation submits to the theory of optical wave propagation in a turbulent medium. The cross-correlation of this kind of scintillation can be used to measure the velocity of stack gas flow. Second, there are the fluctuations in particle concentration field. The frequency of this kind of optical scintillation is lower. It is less than hundreds Hz. The power ratio of the spectrum is in good agreement with $-5/3$. The intensity scintillation shows stable linearity with particle concentration. So it can be used to measure the concentration. Third, there is the variety of production processing. For example, because the pole of electrostatic dust catcher is shaking, strong fluctuations occur near 0.04 Hz. This kind of variety may influence the precision of concentration measurement. Fourth, there is the spatial random distribution of particle. Because the view of sight of light beam is large, the scintillation intensity is too small to be measured.

The theory of homogeneous isotropic turbulence cannot explain the scintillation characteristics whose power ratio is -5/3 in part of low frequency. The turbulence model constructed by this paper give a possible explanation. It is necessary to give further study on the turbulence characteristics in stack. And also much more effort should be devoted to studying the reason why the turbulence is produced. Especially, the theoretic formula of the fluctuations depth index a should be obtained. All of those are helpful to improve the precision of concentration measurement.

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