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Pole placement method of controlling chaos in DC–DC buck converters*

Zou Yan-Li(邹艳丽)[a], Luo Xiao-Shu(罗晓曙)[a]†, and Chen Guan-Rong(陈关荣)[b]

[a] *College of Physics and Electronic Technology, Guangxi Normal University, Guilin* 541004, *China*

[b] *Department of Electronic Engineering, City University of Hong Kong, China*

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Based on the mechanism for the generation of chaos in a buck converter, a pole placement method is proposed and applied to controlling the chaos in a circuit. The control circuit is designed and tested. Numerical calculation and circuit implementation demonstrate the validity of this chaos control method.

Keywords: buck converters, chaos, chaos control, pole placement method
PACC: 0545

1. Introduction

DC–DC converters have been extensively used in industrial electronics such as communications, military equipment, computers, etc., since they have desirable characteristics of high work frequency, high efficiency, high power density and high power factor. In recent years, discrete-time iterative-map approach,[1] piecewise-linear modelling approach[2] and the averaging approach[3] have been adopted as modelling strategies for DC–DC switching converters. Meanwhile, through nonlinear dynamic theory, numerical calculations and circuit experiments, Hopf bifurcation,[4,5] border-collision bifurcation,[6] Flip bifurcation,[7] quasi-periodicity[8] and chaos[9−11] have been discovered from such converters as those mentioned in Ref.[12]. Due to the broad spectrum of applications of buck and boost converters controlled by voltage or current and due to the fact that when a circuit works in a chaos state, it can produce high amplitude noises and unstable behaviours, it is necessary to study how to control the chaos in this kind of circuit.

In Ref.[13], we studied the mechanism for the generation of chaos in buck converters[14] through three kinds of typical switching logical diagrams. We find that when the switch is on, the fast charge of capacitor is the main source responsible for the generation of the chaos in the circuit. Based on the above observation, in this paper the pole placement method is used to control the chaos in the system. Theoretical analysis and numerical simulation both show that the chaos in the circuit can be successfully controlled and periodic orbits can also be generated by appropriately assigning the poles of the system transfer function, and this control method can be realized by choosing appropriate parameter values of the feedback gain matrix. The control circuit is also designed and tested. Both the numerical simulation and circuit implementation demonstrate the validity of this chaos control method.

2. Circuit model and analysis

Figure 1 shows the block diagram of a voltage-controlled buck converter. For convenience in obtaining the circuit state equations, we assume that all the components in the circuit are idealized. The comparator A_2 has an infinite gain, the switches S and S′ have zero on, and infinite off resistances, and they can switch on or off instantly without time delay. In the time interval when the switch S is on and the switch S′ is off, the circuit input provides energy for the load as well as the inductor. In the time interval when the switch S is off and the switch S′ is on, the inductor current flows through S′ and transfers part of its stored energy to the load. The switching period of the

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†Corresponding author: lxs@mailbox.gxnu.edu.cn

switch S is often fixed ($T = t_{\rm on} + t_{\rm off}$), and the average output voltage can be adjusted by changing the duty ratio d $\left(d = \dfrac{t_{\rm on}}{T}\right)$.

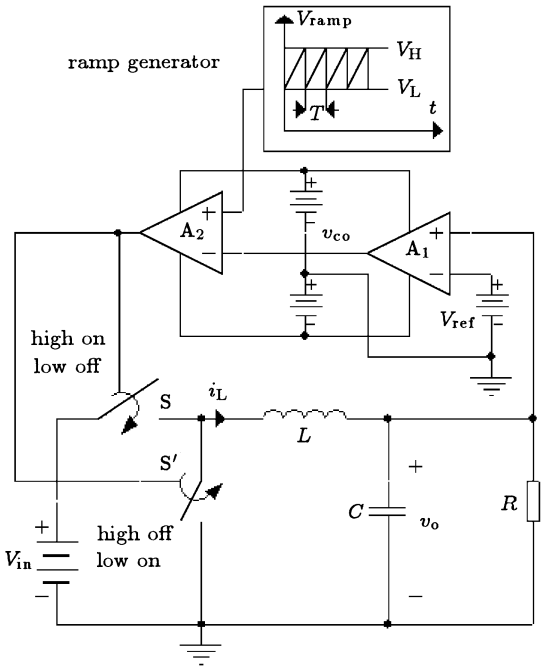


Fig.1. Block diagram of the buck converter.

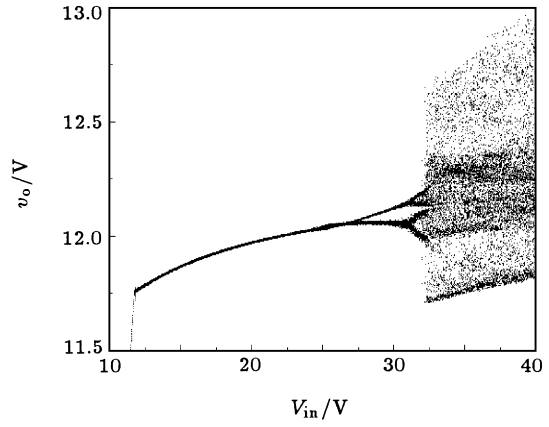


Fig.2. Bifurcation diagram of $v_{\rm o}$ with $V_{\rm in}$ being the bifurcation parameter.

Considering that the linear amplifier A_1 has a gain a, we can write

$$v_{\rm co}(t) = a(v_{\rm o}(t) - v_{\rm ref}). \tag{1}$$

Then, both $v_{\rm co}(t)$ and $v_{\rm ramp}$, the voltage of the ramp, are applied to the comparator, and when $V_{\rm ramp} > v_{\rm co}$, the output voltage of comparator A_2 is at a high level, with the switch S being on and S′ being off; when $V_{\rm ramp} < v_{\rm co}$, the output voltage of comparator A_2 is at a low level, with the switch S being off and S' being on.

Under the above-described ideal conditions, the state equations of the buck converter are given by

$$\begin{cases} \dot{v}_o = -\dfrac{1}{RC}v_o + \dfrac{1}{C}i_{\rm L}, \\ \dot{i}_{\rm L} = -\dfrac{1}{L}v_o + V_{\rm in}/L \cdot g(S), \end{cases} \tag{2}$$

where

$$g(S) = \begin{cases} 0, \text{ when the switch S is off,} \\ 1, \text{ when the switch S is on.} \end{cases} \tag{3}$$

As indicated in Ref.[14], the theoretical analysis and numerical simulations are performed with the following parameter values:

$$\begin{aligned} &L = 20\text{mH}, \quad C = 47\mu\text{F}, \quad R = 22\Omega, \\ &a = 8.4, \quad V_{\rm ref} = 11.3\text{V}, \quad V_{\rm L} = 3.8\text{V}, \\ &V_{\rm H} = 8.2\text{V}, \quad T = 400\mu\text{s}. \end{aligned} \tag{4}$$

Figure 2 shows a bifurcation diagram of $v_{\rm o}$, with $V_{\rm in}$ taken as the bifurcation parameter, which is obtained through the stroboscope sampling method. It can be seen from Fig.2 that the double-period bifurcation and then chaos occur as $V_{\rm in}$ increases.

3. Pole placement method of chaotic control in the circuit

From the analysis in Ref.[13], we know that the generation of the chaos in buck converters is attributed to the very large charging current and the very fast rising rate of the output voltage when the switch S is on. When the input is a constant voltage, the rising rate of $v_{\rm o}$ is determined by the locations of the poles of the system transfer function. If the system works in an underdamped state, the voltage of capacity C rises fast and has overshoot. On the contrary, if the system works in an overdamped state, the voltage of capacity C rises slowly and has no overshoot.[15] So, changing the locations of system poles can change the rising rate of $v_{\rm o}$, thereby chaos can be controlled. Therefore, the pole placement method is a natural option for chaos control in a circuit.

In the following, we discuss the controllability and the pole placement method of the circuit shown in Fig.1. Derived from Eq.(2), the controllability matrix

of the system is given by

$$Q_k = \begin{bmatrix} 0 & 1/LC \\ 1/L & 0 \end{bmatrix} \tag{5}$$

with rank (Q_k)=2. So, the system is completely controllable, and the system's poles can be relocated at any location as desired by a suitable design of state feedback. From an engineering point of view, the feedback state variables should be measurable in practice, and the realization of the feedback should be relatively easy and cost-effective. If the current of the inductance is chosen as the feedback variable, since none of the two ends of inductance is connected with the ground, the realization of feedback variable i_{L} is not easy physically. Therefore, we choose the new state variables as follows:

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v_o \\ \dot{v}_o \end{bmatrix}. \tag{6}$$

To obtain a state space description of the system with these new state variables x_1 and x_2, we first change the state equation of the system into the controllable canonical form, by taking the following nonsingular transformation on state variables v_{o} and i_{L} in Eq.(2)

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} LC & 0 \\ -\dfrac{L}{R} & L \end{bmatrix} \begin{bmatrix} v_o \\ i_{\mathrm{L}} \end{bmatrix}. \tag{7}$$

The canonical form is obtained as follows:

$$\dot{\hat{X}} = \hat{A}\hat{X} + \hat{B}V_{\mathrm{in}}; \quad y = \hat{C}\hat{X}, \tag{8}$$

where

$$\hat{X} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}, \hat{A} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC} & -\dfrac{1}{RC} \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \hat{C} = \begin{bmatrix} \dfrac{1}{LC} & 0 \end{bmatrix}, \quad y = v_o. \tag{9}$$

Then, we use the following linear transformation:

$$X = \frac{1}{LC}\hat{X} \tag{10}$$

to obtain

$$\begin{aligned} \dot{X} =& \frac{1}{LC}\dot{\hat{X}} \\ =& \frac{1}{LC}(\hat{A}\hat{X} + \hat{B}V_{\mathrm{in}}) \\ =& \frac{1}{LC}(\hat{A} \times LC \cdot X) + \frac{1}{LC}\hat{B}V_{\mathrm{in}} \\ =& \hat{A}X + \frac{1}{LC}\hat{B}V_{\mathrm{in}}, \end{aligned} \tag{11}$$

namely,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{LC} \end{bmatrix} V_{\mathrm{in}}. \tag{12}$$

Let the state feedback matrix be $K = [\, k_1 \;\; k_2]$, then the closed-loop controlled system will become

$$\dot{X} = (A - BK)X + BV_{\mathrm{in}}, \tag{13}$$

in which

$$A = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC} & -\dfrac{1}{RC} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \dfrac{1}{LC} \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{14}$$

The poles of the controlled system transfer function are the characteristic roots of matrix $(A - BK)$. It follows from Eq.(14) that

$$A - BK = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC}(k_1 + 1) & -\dfrac{1}{RC} - \dfrac{1}{LC}k_2 \end{bmatrix}. \tag{15}$$

When the parameter values in Eq.(15) are equal to those in Eq.(4), the characteristic cubic equation of the controlled system is given by

$$|sI - (A - BK)| = 0, \tag{16}$$

that is,

$$\begin{aligned} &s^2 + \left(967.117 + 1063.83 \times 10^3 \times k_2\right) s \\ &+ 1063.83 \times 10^3 \times (k_1 + 1) = 0. \end{aligned} \tag{17}$$

Before control is applied, k_1=0, k_2=0, and the buck converter has a pair of conjugate poles, $s_{1,2} = -483.5 \pm 911.06$i, and works in an underdamped state. When the switch S is on (this situation is equivalent to a step input), the output voltage v_{o} of the system rises fast and has a large overshoot. According to Eq.(17), by choosing the values of k_1 and k_2 appropriately, the poles of the system transfer function can be relocated on the negative real axis, where the system works in an overdamped state and the output voltage v_{o} of the system rises slowly, with no overshoot, so chaos in the system is suppressed effectively.

4. Numerical simulation results

The above theoretical analysis is verified and illustrated via numerical simulation in this section. The proposed control method is applied to the buck converter in simulation using the fourth-order Runge-Kutta method. In the calculations, the input voltage $V_{\rm in}$=35V, the circuit works in a chaos state.[13] The control results are shown in Fig.3. In Figs.3(a) and (b), the feedback matrix is K=[0.001 0.0056], and the poles of the controlled system are s_1=–6767.20 and s_2=–157.36. The controlled system works in a period-1 state. In Figs.3(c) and (d), K=[0.1 0.0039], and the poles of the controlled system are s_1=–4876.06 and s_2=–239.99. The controlled system works in a period-2 state. Obviously, in the above two situations, the circuit works in an overdamped state.

As can be seen from Fig.3, the buck converter can be stabilized to different periodic orbits via the appropriate choice of the control gain parameters k_1 and k_2.

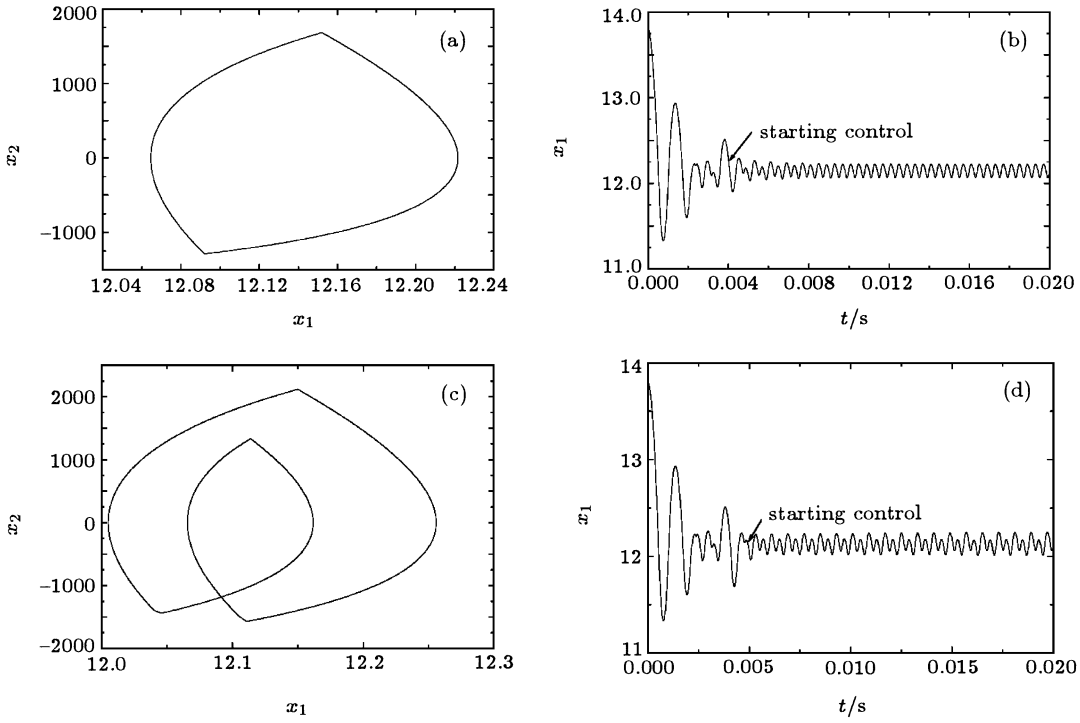


Fig.3. Numerical results: (a) phase diagram of $x_1 - x_2$ with k_1=0.001 and k_2=0.0056; (b) time evaluation of x_1 with k_1=0.001 and k_2=0.0056; (c) phase diagram of $x_1 - x_2$ with k_1=0.1 and k_2=0.0039; (d) time evaluation of x_1 withk_1=0.1 and k_2=0.0039.

5. Circuit design and implementation

To further verify the physical feasibility of the proposed chaos control method, we design and implement the control circuit, based on the Pspice software as shown in Fig.4. In this diagram, S115 is a voltage control switch used as a substitute for the switch S in Fig.1, E1 is a voltage control source, with input control voltage being the capacity branch circuit voltage $v_{\rm o}$, and output voltage being $k_{\rm v}v_{\rm o}$, where k_v is the ride gain of E_1, its output ports connect with the source $V_{\rm in}$ in series. When the ride gain $k_{\rm v}$ of E_1 takes a negative value, this branch circuit simulates the negative feedback of state variable x_1 (i.e. $v_{\rm o}$) and the feedback coefficient $k_1 = -k_{\rm v}$. In the diagram, H_1 is a current control voltage source, $R_{\rm S}$ is a $1-\Omega$ sampling resistance, connecting with the capacity C in series, and moreover the input voltage of H_1 is equal to the voltage of $R_{\rm s}$ (equal to $C\dot{v}_{\rm o}$ in value), the input current of H_1 is $k_{\rm a}C\dot{v}_{\rm o}$, the output voltage of H_1 is $k_{\rm b}k_{\rm a}C\dot{v}_{\rm o} = k_{\rm c}C\dot{v}_{\rm o}$, where $k_{\rm c}$ is the ride gain of H_1, and the output ports of H_1 also connect with the source $V_{\rm in}$ in series. When $k_{\rm c}$ takes a negative value, this branch circuit simulates the negative feedback of state variable x_2 (i.e. $\dot{v}_{\rm o}$) and the feedback coefficient $k_2 = -k_{\rm c}C$. In the diagram, U_1 is an on-delay switch, U_2 is an off-delay switch. When U_1 is

close but U_2 is open, the state feedback control based on the pole placement method is applied. The results are shown in Fig.5 and Fig.6, respectively. Figure 5 is the chaotic attractor of the buck converter before being controlled. Figure 6 is the time evaluation of v_o with k_v=–0.001 and k_c=–106.383. The circuit implementation shows that this chaos control method is practical and also quite effective.

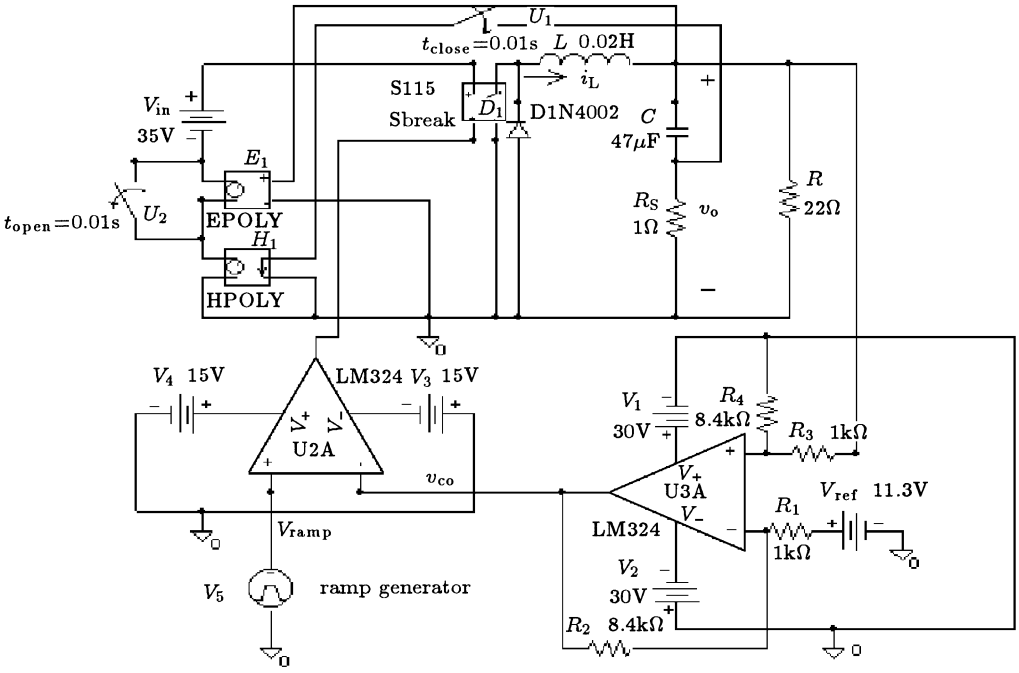


Fig.4. Chaos control circuit of buck converter based pole placement feedback.

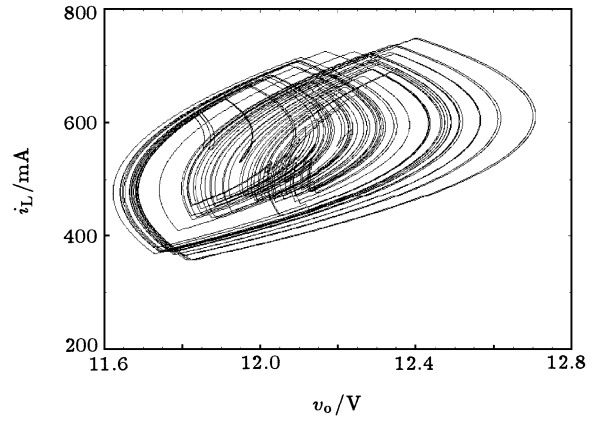


Fig.5. The chaotic attractor of buck converter before being controlled.

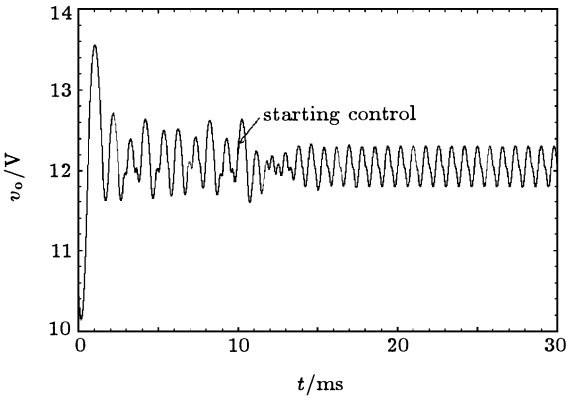


Fig.6. The time evaluation of v_o with k_v=–0.001 and k_c=–106.383.

6. Conclusions

In this paper, in view of the mechanism for the generation of the chaos in buck converters, a chaos control method based on pole placement feedback is presented. To facilitate engineering applications, feasible state variables are chosen for feedback to realize the chaos control. One prominent advantage of using the pole placement method is that the buck converter can be stabilized to various periodic orbits via appropriately choosing the control gains. Both numerical simulation and circuit implementation have demonstrated the validity and effectiveness of the proposed chaos control method.

References

[1] Hamill D C and Jefferies D J 1988 *IEEE Trans. Circ. Syst.* I **35** 1059

[2] Chakrabarty K, Podder G and Banerjee S 1995 *IEEE Trans. Power Electron* **11** 439

[3] Middlebrook R D and Cuk S 1976 *IEEE Power Electron. Spec. Conf. Rec.* p8

[4] Tse C K, Lai Y M and Lu H H C 2000 *IEEE Trans. Circ.Syst.* I **47** 448

[5] Ren H P and Liu D 2005 *Chin. Phys.* **14** 1352

[6] Yuan G H, Banerjee S, Ott E and York J A 1998 *IEEE Trans. Circ. Syst.* I **45** 707

[7] Tse C K 1994 *IEEE Trans. Circ. Syst.* I **41** 16

[8] Chan W C Y and Tse C K 1997 *IEEE Trans. Circ. Syst.* I **44** 1129

[9] di Bernardo M, Garofalo F, Glielmo L and Vasca F 1998 *IEEE Trans. Circ. Syst.* I **45** 133

[10] Li M, Ma X K, Dai D and Zhang H 2005 *Acta Phys. Sin.* **54** 1084 (in Chinese)

[11] Luo X S, Quan H J, Wang B H, Zou Y L, Chen G R, Fang J Q and Jiang P Q 2003 *Acta Phys. Sin.* **52** 12 (in Chinese)

[12] Tse C K 2003 *Complex Behavior of Switching Power Converters* (Boca Raton, FL: CRC Press)

[13] Zou Y L, Luo X S, Fang J Q and Wang B H 2003 *Acta Phys. Sin.* **52** 2978 (in Chinese)

[14] Fossas E and Olivar G 1996 *IEEE Trans. Circ. Syst.* I **43** 13

[15] Dorf R C and Bishop R H 1998 *Modern Control Systems* (New York: Addison Wesley) p199