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Adaptive passive equivalence of uncertain Lü system*

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An adaptive passive strategy for controlling uncertain Lü system is proposed. Since the uncertain Lü system is minimum phase and the uncertain parameters are from a bounded compact set, the essential conditions are studied by which uncertain Lü system could be equivalent to a passive system, and the adaptive control law is given. Using passive theory, the uncertain Lü system could be globally asymptotically stabilized at different equilibria by the smooth state feedback.

Keywords: Lü system, passive control, uncertain parameters
PACC: 0545

1. Introduction

In 1963, Lorenz first found a chaotic attractor in a simple three-dimensional autonomous system.[1] Then Chen and Ueta found a similar but non-equivalent chaotic attractor,[2] which is known to be the counterpart of the Lorenz system. Recently the Lü system is reported which bridges the gap between the Lorenz system and Chen system.[3] In a recent work of Lü *et al*,[4,5] the compound structure of Lü chaotic attractor was analysed, and its forming mechanism was explored. Because of the characteristic of the Lü system, the control of Lü system has been paid much more attention. Using the backstepping design, some researchers designed an observer to identify the unknown parameters, and stabilized the chaotic motions to stable states.[6−8] However, these methods only control Lü systems with only one unknown parameter. If all parameters are unknown, the control method is invalid. Control chaos of Chen chaotic system whose parameters were all known was discussed using the passive control theory put forward by Qi *et al*, and the Lorenz–Chen chaotic system could be globally asymptotically stabilized at different equilibria via smooth state feedback.[9] With the help of passive control theory, how to control uncertain Lü system stabilized at different equilibria is a problem which we will study.

In this paper, a new chaos control method is proposed to control chaos in Lü system. Firstly, we will give the definition of passive system and introduce some theories of passive equivalence control. Then, we will study the essential conditions, under which uncertain Lü system could be equivalent to passive system. Finally, our conclusion is given.

2. Passive system

Consider a continuous chaotic system Σ given by

$$\begin{aligned} \dot{x} &= f(x) + g(x) \cdot u, \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R^m$ is the input, $y \in R^p$ is the output, f and g are smooth vector fields. Function h is smooth mapping. We suppose that the vector field f has at least one equilibrium. Without loss of generality, we can assume that the equilibrium is $x = 0$. If the equilibrium isn't $x = 0$, through coordinate shift we can transform it to $x = 0$.

Theorem 1[10] A system Σ of the form (1) is said to be passive if function $f(x)$ and $g(x)$ exist and there is a real-valued constant β, such that for any $\forall t \geq 0$

$$\int_0^t u^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau \geq \beta, \tag{2}$$

or if there is a constant $\rho > 0$ and a real-valued constant β, such that for any $\forall t \geq 0$

$$\int_0^t u^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau + \beta \geq \int_0^t \rho \cdot y^{\mathrm{T}}(\tau) y(\tau) \mathrm{d}\tau. \tag{3}$$

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In other words, a system Σ of the form (1) is said to be passive if there is a nonnegative function $V(x) : X \to R, V(0) = 0$, named storage function, such that for any $\forall x \in X$, $\forall t \geq 0$

$$V(x) - V(x_0) \leq \int_0^t y^{\mathrm{T}}(s)u(s)\mathrm{d}s. \tag{4}$$

A passive system with storage function $V(x)$ is said to be strictly passive if there exists a positive definite function $S(x)$, such that for any $\forall x \in X$, $\forall t \geq 0$

$$V(x) - V(x_0) = \int_0^t y^{\mathrm{T}}(s)u(s)\mathrm{d}s - \int_0^t S(x(s))\mathrm{d}s. \tag{5}$$

Theorem 2[10] Suppose system Σ is passive and locally zero-state detectable with positive definite storage function $V(x)$. Let φ be any smooth function such that $y^{\mathrm{T}}\varphi(y) > 0$ and $\varphi(0) = 0$ for each nonzero y. The control law

$$u(t) = -\varphi(y)$$

asymptotically stabilizes at the equilibrium point $x = 0$.

When f and g in system (1) are also functions of some vector of unknown parameters θ, it is possible to build some adaptive passivity approaches. Consider the system (1) with the explicit parameter dependence

$$\begin{aligned} \dot{x} &= f(x, \theta) + g(x, \theta) \cdot u, \\ y &= h(x, \theta), \end{aligned} \tag{6}$$

where θ is a vector of the unknown parameter.

For system (6), the adaptive passivity approach of nonlinear dynamical system has been discussed by some researchers.[11,12] Based on the adaptive passivity approach, we will discuss the passive control method of uncertain Lü system.

3. Adaptive passive control of Lü system

Lü system can be described as follows:

$$\begin{aligned} \dot{x}_1 &= -ax_1 + ax_3, \\ \dot{x}_2 &= x_1x_3 - bx_2, \\ \dot{x}_3 &= -x_1x_2 + cx_3, \end{aligned} \tag{7}$$

where uncertain parameters $\theta = [\, a \;\; b \;\; c \,]^{\mathrm{T}}$.

Add the controller u to the third equation of Eq.(6). Suppose $z_1 = x_1$, $z_2 = x_2$, $y = x_3$, then the system can be expressed by the normal form

$$\begin{aligned} \dot{z}_1 &= -az_1 + ay, \\ \dot{z}_2 &= -bz_2 + z_1y, \\ \dot{y} &= cy - z_1z_2 + u. \end{aligned} \tag{8}$$

Equation (8) can be described as the following form

$$\begin{aligned} \dot{z} &= f_0(z, \theta) + p(z, y, \theta)y, \\ \dot{y} &= b(z, y, \theta) + a(z, y, \theta)u, \end{aligned} \tag{9}$$

where $f_0(z, \theta) = [-az_1 \;\; -bz_2]^{\mathrm{T}}$, $p(z, y, \theta) = [a \;\; z_1]^{\mathrm{T}}$, $a(z, y, \theta) = 1$, $b(z, y, \theta) = cy - z_1z_2$.

Theorem 3 For Lü system (9), if there is the loop control

$$\begin{aligned} u(z, y, \theta) &= -b(z, y, \theta) - \alpha y + v - p(z, y, \theta)^{\mathrm{T}} [z_1 \; z_2]^{\mathrm{T}}, \\ \dot{\theta}^{\mathrm{T}} &= -\frac{\partial[-b(z, y, \theta)y - p(z, y, \theta)^{\mathrm{T}} [z_1 \; z_2]^{\mathrm{T}} y]}{\partial\theta}\Gamma, \end{aligned} \tag{10}$$

with Γ a positive-definite matrix, α a positive constant and v an external signal, the Lü system can be adaptively equivalent to a passive system.

Proof Choose a store function

$$\begin{aligned} V(z, y, \theta, \theta^*) =& W(z, \theta^*) + \frac{1}{2}y^{\mathrm{T}}y \\ &+ \frac{1}{2}(\theta - \theta^*)^{\mathrm{T}}\Gamma^{-1}(\theta - \theta^*), \end{aligned} \tag{11}$$

where $W(z, \theta)$ is the Lyapunov function of $f_0(z, \theta)$, and $W(0) = 0$.

$$W(z, \theta) = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2, \tag{12}$$

we have

$$\begin{aligned} \dot{W}(z, \theta) = \frac{\mathrm{d}W(z, \theta)}{\mathrm{d}t} &= [z_1 \quad z_2] \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} \\ &= [z_1 \;\; z_2] \begin{bmatrix} -az_1 \\ -bz_2 \end{bmatrix}. \end{aligned} \tag{13}$$

Because $\dfrac{\mathrm{d}W(z, \theta)}{\mathrm{d}t} \leq 0$, it can be concluded that $W(z, \theta)$ is the Lyapunov function of $f_0(z, \theta)$ and that $f_0(z, \theta)$ is globally asymptotically stable.

According to the above analysis, zero dynamical character of Lü system is Lyapunov stable and Lü system is minimum phase.[13] From Eq.(10) and Eq.(11),

$$\dot{V}(z,y,\theta,\theta^*) = \frac{\partial}{\partial z}W(z,\theta^*)\cdot f_0(z,\theta^*) + \frac{\partial}{\partial z}W(z,\theta^*)\cdot p^{\mathrm{T}}(z,y,\theta^*)\cdot y + [b(z,y,\theta^*) + a(z,y,\theta^*)u]y + \left[-b(z,y,\theta^*) - p^{\mathrm{T}}(z,y,\theta^*)\,[z_1\ z_2]^{\mathrm{T}}\right](\theta-\theta^*). \tag{14}$$

Because the equivalent system is minimum phase and has KYP property,[9] we have $\dfrac{\partial}{\partial}W(z,\theta)\cdot f_0(z,\theta)\le 0$, and then

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le \frac{\partial}{\partial z}W(z,\theta^*)\cdot p^{\mathrm{T}}(z,y,\theta^*)\cdot y + [b(z,y,\theta^*) + a(z,y,\theta^*)u]y + \left[-b(z,y,\theta^*) - p^{\mathrm{T}}(z,y,\theta^*)\,[z_1\ z_2]^{\mathrm{T}}\right](\theta-\theta^*). \tag{15}$$

Choose

$$u(z,y,\theta) = -b(z,y,\theta) - \alpha y + v - p(z,y,\theta)^{\mathrm{T}}\,[z_1\ z_2]^{\mathrm{T}},$$
$$\dot{\theta}^{\mathrm{T}} = -\frac{\partial[-b(z,y,\theta)y - p(z,y,\theta)^{\mathrm{T}}\,[z_1\ z_2]^{\mathrm{T}}\,y]}{\partial\theta}\Gamma, \tag{16}$$

where α is a positive real constant, v is the external input signal. That is,

$$u = -(c+\alpha)y + v - az_1. \tag{17}$$

Enter Eq.(17) into Eq.(15),

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le -\alpha y^2 + vy - (cy + az_1)(\theta-\theta^*). \tag{18}$$

If there is $(cy+az_1)(\theta-\theta^*)\ge 0$ and let $\beta = V(z_0,y_0)$, we have

$$V(z,y) + \int_0^t \alpha y^2(s)\mathrm{d}s \le \int_0^t v(s)y(s)\mathrm{d}s + \beta. \tag{19}$$

Because storage function $V(z,y)$ is positive definite, $V(z,y)\ge 0$, and thus we have

$$\int_0^t \alpha y^2(s)\mathrm{d}s \le \int_0^t v(s)y(s)\mathrm{d}s + \beta. \tag{20}$$

So the equivalent system is a passive system. Putting the control u into system (7), we have

$$\dot{x}_1 = -ax_1 + ax_3, \quad \dot{x}_2 = x_1x_3 - bx_2,$$
$$\dot{x}_3 = -x_1x_2 - \alpha x_3 - ax_1 + v; \tag{21}$$

and

$$\dot{a} = x_1x_3, \quad \dot{b} = 0, \quad \dot{c} = x_3^2. \tag{22}$$

Equations (21) and (22) constitute the adaptive passivity method of the uncertain Lü system.

4. Simulation and results

Suppose the initial value of Lü system be (6, 6, 6) and the initial value of the unknown parameter be (10, 10, 10). Figure 1 plots the curves of the state variables and parameters estimated under passive control at the equilibria (0, 0, 0) when $\alpha = 1, v = 0$. From these figures, we can find that it takes only very short time for the system to be stabilized at the equilibrium (0, 0, 0).

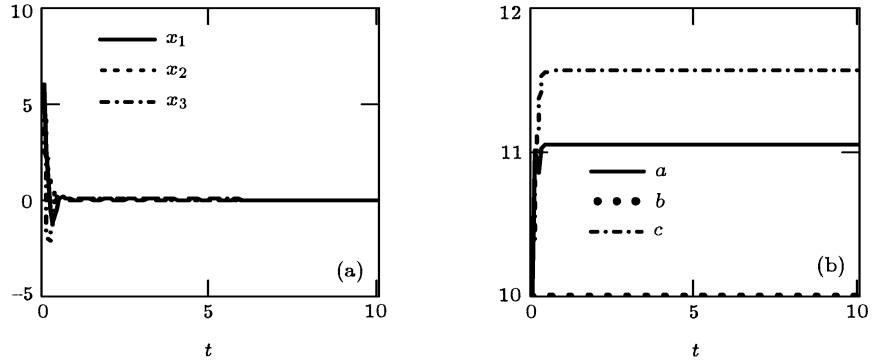


Fig.1. The state variables and parameters estimate under passive control at the equilibrium (0, 0, 0).

We will study how to design the controller to stabilize Lü system at any equilibrium (x_1^*, x_2^*, x_3^*). Let $\dot{x} = 0$ and substituting (x_1^*, x_2^*, x_3^*) into (21),

$$x_1^* = x_3^*, x_2^* = \frac{1}{b}(x_3^*)^2, v = x_1^* x_2^* + \alpha x_3^* + a x_1^*.$$

Choose the new equilibrium (2, 0.4, 2) and put v into the system. Figure 2 plots the curves of the state variables and parameters estimated under passive control at the equilibrium (2, 0.4, 2) when $\alpha = 1, v = x_1^* x_2^* + \alpha x_3^* + a x_1^*$. From Fig.2, we can find that it takes only very short time for the system to be quickly stabilized at the equilibrium (2, 0.4, 2). Therefore we can stabilize the system at any equilibrium (x_1^*, x_2^*, x_3^*) through adjusting the external input v.

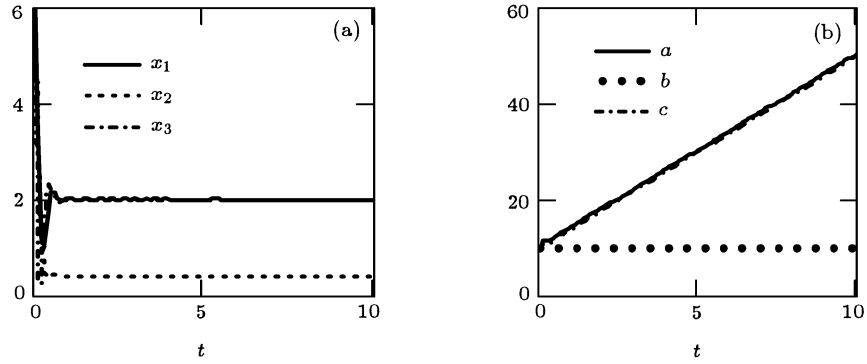


Fig.2. The state variables and parameters estimated under passive control at equilibrium (2, 0.4, 2).

The equilibria of Lü system also can be switched through varying the controller u. Figure 3 shows that the equilibrium (0, 0, 0) is switched to the equilibrium (2, 0.4, 2) of Lü system when t=10s.

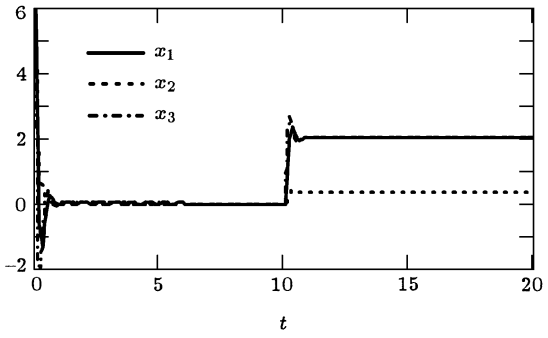


Fig.3. Switching the equilibrium (0, 0, 0) to the equilibrium (2, 0.4, 2).

Clearly, the results in both of the above figures demonstrated that the closed-loop system is stable.

5. Conclusion

In the nonlinear control field, many researches are beginning to pay attention to the passive nonlinear system theory. Lü system can be controlled with passive control theory. This control method combines chaotic systems with passive systems. Because the uncertain Lü system is minimum phase and the uncertain parameters are from a bounded compact set, the uncertain Lü system can be globally asymptotically stabilized at the equilibria by state feedback.

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