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The Hawking radiation of the charged particle via tunnelling from the axisymmetric Sen black hole*

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Extending Parikh's semi-classical quantum tunnelling model, this paper has studied the Hawking radiation of the charged particle via tunnelling from the horizon of the axisymmetric Sen black hole. Different from the uncharged massless particle, the geodesics of the charged massive particle tunnelling from the horizon is not light-like. The derived result supports Parikh's opinion and provides a correct modification to Hawking strictly thermal spectrum developed by the fixed background space-time and not considering the energy conservation and the self-gravitation interaction.

Keywords: Sen black hole, charged particle, tunnelling rate, Bekenstein–Hawking entropy
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1. Introduction

In 1974, Stephen Hawking proved that black holes can radiate thermally, and the temperature is the true quantum temperature.[1] During the whole evaporation, the horizon of the black hole will shrink with the loss of energy until the black hole evaporates away completely, which results in the paradox of the information lost[2] (namely, the pure quantum state will be reduced to the thermal mixture state) which disobeys the underlying unitary theory in quantum mechanics. The created mechanism of the Hawking radiation can be explained by the tunnelling effect in quantum mechanics or the vacuum fluctuation in the quantum field theory (namely, a pair of virtual particles is created just inside the horizon of black holes, and the positive particle materializes as a real particle via tunnelling out from the horizon to go infinity, however the negative anti-particle is absorbed by black holes. In other words, when a pair of virtual particle is created just outside the horizon of black holes, the negative anti-particle tunnels into the horizon, leaves the positive particle and escapes from the horizon to the infinity to form the Hawking radiation. The above picture has a tunnelling process, so only after finding the true potential barrier can we have a true tunnelling picture and then obtain the true radiation spectrum of the black hole.

In fact, the background spacetime of black holes varies with the loss of energy due to the radiation of black holes. But until now, most of the methods for deriving Hawking radiation spectrum are based on a fixed background, and do not consider the fluctuation for the geometry of the space-time.[3−9] Recently, Parikh and Wilczek have adopted the semi-classical tunnelling picture to study the Hawking radiation of the spherically symmetric Schwarzschild and Reissner–Nordstrom black hole[10,11], the tunnelling rate is obtained by considering the energy conservation and an unfixed background spacetime. The result makes a correction to the Hawking precisely thermal spectrum and proves that the radiation spectrum is not strictly thermal. In their work, the Painlevé coordinate transformation to eliminate the coordinate singularity is adopted. Following that, the method is successfully applied to all kinds of spherically symmetric black holes, and the correctness gets a further validation.[12,13] In 2005, Zhang and Yang extended Parikh's semi-classical quantum tunnelling model to discuss the tunnelling radiation characteristics of the axi-symmetric black hole and got satisfying results.[14−17] All of the above researches indicate that the true Hawking radiation spectrum is not precisely thermal when the self-gravitation action is taken into

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account, which extends our understanding on quantum tunnelling effect of black holes.

However, up to now the tunnelling radiation characteristics are still limited to study of the uncharged particle. But in fact, a general black hole should be charged and the corresponding tunnelling particle should be also charged, so the geodesics of the charged particle tunnelling from the black hole is not light-like. This paper applies Parikh's semi-classical quantum tunnelling model to study the tunnelling characteristics of the charged particle from the stationary axi-symmetric Sen black hole. The derived result supports Parikh's opinion and points out that the tunnelling rate is related to the change of Bekenstein–Hawking entropy, and the factual radiation spectrum from black holes is not strictly thermal, but is consistent with the underlying unitary theory.

2. Painlevé coordinate transformation and the geodesics of the charged particle

According to Ref.[18], the line element of the stationary axi-symmetric Sen black hole is given by

$$\begin{aligned}\mathrm{d}s^2 =& -\frac{\Sigma - a^2\sin^2\theta}{\Delta}\mathrm{d}t_{\mathrm{S}}^2 + \frac{\Delta}{\Sigma}\mathrm{d}r^2 + \Delta\mathrm{d}\theta^2 \\ &+ \frac{\sin^2\theta}{\Delta}[(r^2+a^2-2\beta r)^2 - \Sigma a^2\sin^2\theta]\mathrm{d}\varphi^2 \\ &- \frac{2a\sin^2\theta}{\Delta}[(r^2+a^2-2\beta r) - \Sigma]\mathrm{d}t_{\mathrm{S}}\mathrm{d}\varphi, \end{aligned} \tag{1}$$

the electromagnetic potential is

$$A_\mu = \frac{mrsh\alpha}{\sqrt{2}\Delta}\left(\mathrm{d}t_{\mathrm{S}} - a\sin^2\theta\mathrm{d}\varphi\right), \tag{2}$$

where

$$\begin{aligned}&\Sigma = r^2 - 2mr + a^2, \\ &\Delta = r^2 - 2\beta r + a^2\cos^2\theta, \\ &\beta = -msh^2\frac{\alpha}{2}, \end{aligned} \tag{3}$$

and t_{S} is the time coordinate of the Sen black hole, and the metric describes the black hole with mass M, charge Q, angular momentum J and magnetic monopoles μ which are given by the parameters m, a and α as following

$$\begin{aligned}&M = \frac{m}{2}\left(1 + ch\alpha\right), \\ &Q = \frac{m}{\sqrt{2}}sh\alpha, \\ &J = \frac{ma}{2}\left(1 + ch\alpha\right), \\ &\mu = \frac{1}{\sqrt{2}}mash\alpha. \end{aligned} \tag{4}$$

For the sake of convenience to analyse the tunnelling behaviour across the horizon of the black hole, replacing the parameters m, α, β, μ with M, Q, J, we have

$$\begin{aligned}&m = M - \frac{Q^2}{2M}, \quad \beta = -\frac{Q^2}{2M}, \\ &a = \frac{J}{M}, \quad \mu = aQ. \end{aligned} \tag{5}$$

According to the null surface equation $g^{\mu\nu}\partial_\mu f\partial_\nu f = 0$, the event horizon (EH) r_+ and the inner horizon (IH) r_- of the Sen black hole can be written as

$$\begin{aligned}r_\pm =& m \pm \sqrt{m^2 - a^2} \\ =& M - \frac{Q^2}{2M} \pm \sqrt{\left(M - \frac{Q^2}{2M}\right)^2 - a^2}. \end{aligned} \tag{6}$$

In the presence of the rotation for the black hole, the infinite red-shift surface $r_\pm^s = m \pm \sqrt{m^2 - a^2\sin^2\theta}$ and the horizons given by Eq.(6) are not coincident with each other, so the geometrical optical limit can not be applied here. For another, the existence of the rotation makes the dragging effect of the coordinate system find in the spacetime. So a rationally physical picture should be described in the dragging coordinate, and then substituting the dragging coordinate transformation $\frac{\mathrm{d}\varphi}{\mathrm{d}t_{\mathrm{S}}} = -\frac{g_{03}}{g_{33}} = \Omega$ into Eqs.(1) (2), we can express the new line element of the Sen black hole in the dragging coordinate system as

$$\mathrm{d}\hat{s}^2 = \hat{g}_{00}\mathrm{d}t_{\mathrm{S}}^2 + \frac{\Delta}{\Sigma}\mathrm{d}r^2 + \Delta\mathrm{d}\theta^2, \tag{7}$$

where $\hat{g}_{00} = -\dfrac{\Sigma\Delta}{\left(r^2 - 2\beta r + a^2\right)^2 - \Sigma a^2\sin^2\theta}$, and the non-null component of the corresponding electromagnetic potential is

$$A_{\mathrm{t}} = \frac{mrsh\alpha}{\sqrt{2}}\frac{r^2 - 2\beta r + a^2}{\left(r^2 - 2\beta r + a^2\right)^2 - \Sigma a^2\sin^2\theta}. \tag{8}$$

Thus the infinite red-shift surface and the horizons of the black hole are coincident with each other in the dragging coordinate system, but there still exists a coordinate singularity at the horizon of the black hole, which brings us inconvenience to research into the tunnelling behaviour across the horizon of the black hole. So the new coordinate transformation is needed. Parikh applied the Painlevé coordinate $\mathrm{d}t_{\mathrm{S}} = \mathrm{d}t + f'(r)\,\mathrm{d}r$ to eliminate the coordinate singularity and studied the Hawking radiation of the spherically symmetric black hole via tunnelling. In this paper, the spacetime we study is axi-symmetrical, so we should go on performing the general Painlevé coordinate transformation as[19]

$$\mathrm{d}t_{\mathrm{S}} = \mathrm{d}t + F(r,\theta)\,\mathrm{d}r + G(r,\theta)\,\mathrm{d}\theta, \tag{9}$$

where $F(r,\theta)$ and $G(r,\theta)$ are two functions of r and θ, and the integrability condition of Eq.(9) satisfies $\partial_\theta F(r,\theta) = \partial_r G(r,\theta)$. Substituting Eq.(9) into Eq.(7) and ordering the derived constant-time slice of the spacetime flat Euclidean in radial, we get the line element of the black hole in the general Painlevé coordinate system

$$\begin{aligned}\mathrm{d}\hat{s}^2 =& \hat{g}_{00}\mathrm{d}t^2 \pm 2\sqrt{\hat{g}_{00}\left(1-\Delta\Sigma^{-1}\right)}\mathrm{d}t\mathrm{d}r + \mathrm{d}r^2 \\ &+ \left[\hat{g}_{00}G^2(r,\theta) + \Delta\right]\mathrm{d}\theta^2 + 2\hat{g}_{00}G(r,\theta)\,\mathrm{d}t\mathrm{d}\theta \\ &+ 2\sqrt{\hat{g}_{00}\left(1-\Delta\Sigma^{-1}\right)}G(r,\theta)\,\mathrm{d}r\mathrm{d}\theta,\end{aligned} \tag{10}$$

and under this transformation, the form of the electromagnetic vector potential remains invariant (modulo a gauge transformation), where the positive sign(+) denotes the space-time of the outgoing particle, and the negative sign(–) represents the line element of the ingoing particle. Substituting Eq.(10) into the Landau's condition of the coordinate clock synchronization, we can also obtain $\partial_\theta F(r,\theta) = \partial_r G(r,\theta)$. Thus the Painlevé-Sen line element satisfies the Landau's condition of the coordinate clock synchronization, apart from this, there are many other superior features such as: the infinite red-shift surface and the horizons of the black hole are coincident with each other; the metric is regular at the horizons of the black hole; the measure on the surfaces of constant-time slices is the same as that of flat spacetime. All of these characters are advantageous for us to study the Hawking radiation of the charged particle via tunnelling.

The null radial geodesics followed by the uncharged and massless particle can be obtained from Eq.(10) as follows:

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = -\sqrt{\hat{g}_{00}\left(1-\Delta\Sigma^{-1}\right)} \pm \sqrt{\hat{g}_{00}\Delta\Sigma^{-1}}, \tag{11}$$

where the plus (minus) sign in Eq.(11) denotes the outgoing (ingoing) geodesics. But in fact, the Sen black hole is charged and so the tunnelling particle should also be charged and massive. So Eq.(11) can not be applied here and we have to find out the geodesics corresponding to the outgoing charged particle. According to Ref.[20] and the de Broglie hypothesis, we can derive that the phase velocity v_{p} and the group velocity v_{g} under the condition of the relativity satisfy the relationship

$$v_{\mathrm{p}} = \frac{1}{2}v_{\mathrm{g}}. \tag{12}$$

As in quantum field theory, the Hawking quantum tunnelling radiation may be explained as that a pair of virtual particle creates spontaneously just outside the horizon, the negative anti-particle is absorbed by the black hole (the negative orbit only exists inside the horizon), while the positive particle materializes a real particle to go to infinity and then form the Hawking radiation; it is an instantaneous process and the metric in the line element (10) satisfies Landau's condition of the coordinate clock synchronization. Thus the coordinate time difference of two events, which take place simultaneously in different places, is

$$\mathrm{d}t = -\frac{\hat{g}_{01}}{\hat{g}_{00}}\mathrm{d}r_{\mathrm{c}}, \quad (\mathrm{d}\theta = \mathrm{d}\varphi = 0), \tag{13}$$

where r_{c} is the location of the outgoing charged particle, so the geodesics (the phase velocity) of the charged particle can be written as

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = v_{\mathrm{p}} = \frac{1}{2}v_{\mathrm{g}} = -\frac{1}{2}\frac{\hat{g}_{00}}{\hat{g}_{01}} = \pm\frac{1}{2}\sqrt{\frac{\Sigma\Delta}{\left[\left(r^2-2\beta r+a^2\right)^2 - \Sigma a^2\sin^2\theta\right]\left(\Delta\Sigma^{-1}-1\right)}}, \tag{14}$$

where the plus and minus signs denote the outgoing and ingoing geodesics of the charged particle respectively.

3. The Hawking radiation of the charged particle via tunnelling from the horizon of the black hole

In this section, we will research into the tunnelling radiation of the charged particle from the horizon of the black hole. Fixing the total ADM mass, angular momentum, electric charge of the hole-particle system and allowing those of the black hole fluctuate, when a particle with energy ω and charge q tunnels out from the horizon of the black hole, we find that the mass M and the charge Q of the black hole will be replaced by $M-\omega$ and $Q-q$ respectively in Eqs.(6), (8), (10), (14) after taking energy conservation, angular momentum conservation, electric charge conservation into account. Applying the WKB approximation, we find that the relationship between the tunnelling rate and the imaginary part of the particle action satisfies[21,22]

$$\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S}. \tag{15}$$

Considering that the matter–gravity system consists of the black hole and the electro-magnetic field outside the hole and the Lagrangian function of the electro-magnetic field corresponding to the generalized coordinate described by A_μ is $-1/4F_{\mu\nu}F^{\mu\nu}$, we can find that the generalized coordinate A_{t} is an ignorable coordinate. In addition, the coordinate φ is also an ignorable coordinate in the dragging angular system. In order to eliminate the two freedoms, the imaginary part of the action should be written as

$$\mathrm{Im}S = \mathrm{Im}\int_{t_i}^{t_f}\left(L - P_\varphi\dot{\varphi} - P_{A_{\mathrm{t}}}\dot{A}_{\mathrm{t}}\right)\mathrm{d}t = \mathrm{Im}\left[\int_{r_i}^{r_f}\int_0^{P_r}\mathrm{d}P_r'\mathrm{d}r - \int_{\varphi_i}^{\varphi_f}\int_0^{P_\varphi}\mathrm{d}P_\varphi'\mathrm{d}\varphi - \int_{A_{ti}}^{A_{tf}}\int_0^{P_{A_{\mathrm{t}}}}\mathrm{d}P_{A_{\mathrm{t}}}'\mathrm{d}A_{\mathrm{t}}\right], \tag{16}$$

where L is the Lagrangian function of the matter-gravity system, $(P_\varphi, P_{A_{\mathrm{t}}})$ are the canonical momentum conjugate to the generalized coordinates $(\varphi, A_{\mathrm{t}})$, and r_i and r_f represent the locations of the horizon of the black hole before and after the charged particle emission respectively, and they are regarded as the two turning points of the tunnelling potential barrier, the distance between them depends on the energy, electric charge of the outgoing particle. According to the Hamilton canonical equation

$$\begin{aligned}
\dot{r} &= \left.\frac{\mathrm{d}H}{\mathrm{d}P_r}\right|_{(r;\varphi,P_\varphi;A_{\mathrm{t}},P_{A_{\mathrm{t}}})} = \frac{\mathrm{d}\left(M-\omega\right)}{\mathrm{d}P_r} = -\frac{\mathrm{d}\omega}{\mathrm{d}P_r},\\
\dot{\varphi} &= \left.\frac{\mathrm{d}H}{\mathrm{d}P_\varphi}\right|_{(\varphi;r,P_r;A_{\mathrm{t}},P_{A_{\mathrm{t}}})} = a\Omega\frac{\mathrm{d}\left(M-\omega\right)}{\mathrm{d}P_\varphi} = -a\Omega\frac{\mathrm{d}\omega}{\mathrm{d}P_\varphi},\\
\dot{A}_{\mathrm{t}} &= \left.\frac{\mathrm{d}H}{\mathrm{d}P_{A_{\mathrm{t}}}}\right|_{(A_{\mathrm{t}};r,P_r;\varphi,P_\varphi)} = \frac{\Phi\mathrm{d}\left(Q-q\right)}{\mathrm{d}P_{A_{\mathrm{t}}}} = -\frac{\Phi dq}{\mathrm{d}P_{A_{\mathrm{t}}}},
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\Omega &= \frac{a\left[\left(r^2+a^2-2\tilde{\beta}r\right)-\tilde{\Sigma}\right]}{\left(r^2+a^2-2\tilde{\beta}r\right)^2-\tilde{\Sigma}a^2\sin^2\theta},\\
\Phi &= \frac{(Q-q)\,r\left(r^2+a^2-2\tilde{\beta}r\right)}{\left(r^2+a^2-2\tilde{\beta}r\right)^2-\tilde{\Sigma}a^2\sin^2\theta},\\
\tilde{\beta} &= -\frac{(Q-q)^2}{2\left(M-\omega\right)},\\
\tilde{\Sigma} &= r^2-2\left[(M-\omega)-\frac{(Q-q)^2}{2\left(M-\omega\right)}\right]r+a^2,
\end{aligned} \tag{18}$$

and substituting Eq.(17) into Eq.(16) we can obtain

$$\begin{aligned}
\mathrm{Im}S &= -\,\mathrm{Im}\int_{r_i}^{r_f}\int_{(0,0)}^{(\omega,q)}\frac{1}{\dot{r}}\left[\left(1-a\Omega_+'\right)\mathrm{d}\omega' - \Phi'\mathrm{d}q'\right]\mathrm{d}r\\
&= -\,\mathrm{Im}\int_{r_i}^{r_f}\int_{(0,0)}^{(\omega,q)}\frac{2\sqrt{[(r^2-2\tilde{\beta}'r+a^2)^2-\tilde{\Sigma}'a^2\sin^2\theta](\tilde{\Delta}'-\tilde{\Sigma}')}}{\sqrt{\tilde{\Delta}'}(r-r_+')(r-r_-')}\left[(1-a\Omega_+')\mathrm{d}\omega' - \Phi'\mathrm{d}q'\right]\mathrm{d}r,
\end{aligned} \tag{19}$$

where

$$\tilde{\Sigma}' = r^2 - 2\left[(M-\omega') - \frac{(Q-q')^2}{2(M-\omega')}\right] r + a^2 = (r - r'_+)(r - r'_-),$$

$$r'_\pm = (M-\omega') - \frac{(Q-q')^2}{2(M-\omega')} \pm \sqrt{\left[(M-\omega') - \frac{(Q-q')^2}{2(M-\omega')}\right]^2 - a^2},$$

$$\Omega'_+ = \frac{a}{a^2 + r'^2_+ + \dfrac{(Q-q')^2}{(M-\omega')} r'_+}, \quad r_i == M - \frac{Q^2}{2M} \pm \sqrt{\left(M - \frac{Q^2}{2M}\right)^2 - a^2},$$

$$r_f = (M-\omega') - \frac{(Q-q')^2}{2(M-\omega')} + \sqrt{\left[(M-\omega') - \frac{(Q-q')^2}{2(M-\omega')}\right]^2 - a^2}, \tag{20}$$

where $r = r'_+$ describes the event horizon of the black hole after emitting out of the particle with the energy ω' and the charge q' from the event horizon, so there exists a pole at the event horizon. The integral can be evaluated by deforming the contour around the pole. Doing firstly the r integral we get

$$\mathrm{Im}S = \mathrm{Im}\int_{(0,0)}^{(\omega,q)} -2\pi i \left[\frac{r'^2_+ - 2\beta' r'_+}{r'_+ - r'_-}\mathrm{d}\omega' - \frac{(Q-q')\, r'_+}{r'_+ - r'_-}\mathrm{d}q'\right], \tag{21}$$

and finishing the integral, we obtain

$$\mathrm{Im}S = -\pi\left[(M-\omega)^2 + (M-\omega)\sqrt{\left[(M-\omega) - \frac{(Q-q)^2}{2(M-\omega)}\right]^2 - a^2} - \frac{(Q-q)^2}{2}\right.$$
$$\left. - M^2 - M\sqrt{\left(M - \frac{Q^2}{2M}\right)^2 - a^2} + \frac{Q^2}{2}\right] = -\frac{1}{8}(A_f - A_i), \tag{22}$$

where A_i and A_f are the areas of the event horizon before and after tunnelling out of the particle with the energy ω and the charge q respectively, they are

$$A_i = 8\pi\left[M^2 - \frac{Q^2}{2} + M\sqrt{\left(M - \frac{Q^2}{2M}\right)^2 - a^2}\right],$$

$$A_f = 8\pi\left[(M-\omega)^2 - \frac{(Q-q)^2}{2} + (M-\omega)\sqrt{\left[(M-\omega) - \frac{(Q-q)^2}{2(M-\omega)}\right]^2 - a^2}\right]. \tag{23}$$

So the tunnelling rate of the charged particle from the event horizon of the black hole can be written as

$$\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S} = \mathrm{e}^{\frac{1}{4}(A_f - A_i)} = \mathrm{e}^{\Delta S_{\mathrm{BH}}}, \tag{24}$$

where $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(M-\omega, Q-q) - S_{\mathrm{BH}}(M,Q)$ is the change of Bekenstein–Hawking entropy at the event horizon. So the tunnelling rate of the charged particle from the event horizon is related to Bekenstein–Hawking entropy, and the true radiation spectrum of the black hole deviates from the strictly thermal one.

4. Concluding remarks

Note that Eq.(24) agrees with the underlying unitary theory since the tunnelling rate in quantum mechanics is expressible as

$$\Gamma(i \to f) \sim |M_{fi}|^2 \cdot \quad (\text{phase space factor}), \tag{25}$$

where $|M_{fi}|^2$ is the square of the amplitude for the tunnelling process, and the phase-space factor is given by summing over the final states and averaging over the initial states. Since the number of the initial/final

states is the exponent of the initial/final entropy, the tunnelling rate

$$\Gamma \sim \frac{\mathrm{e}^{S_{\mathrm{final}}}}{\mathrm{e}^{S_{\mathrm{initial}}}} = \mathrm{e}^{\Delta S} \tag{26}$$

is consistent with our result obtained by applying Parikh's semi-classical tunnelling model. Hence, Eq.(24) satisfies the underlying unitary theory and provides a right correction to Hawking precisely thermal radiation spectrum.

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