Vol 15 No 8, August 2006
1009-1963/2006/15(08)/1700-05

Chinese Physics

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Remote atomic information concentration without Bell-state measurement*

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(Received 21 December 2005; revised manuscript received 16 February 2006)

This paper proposes a scheme for information concentration of two remote two-level atoms in cavity QED. This scheme does not involve the Bell-state measurement. During the interaction between atom and cavity, the cavity frequency is large-detuned from the atomic transition frequency, thus the scheme is insensitive to both the cavity decay and the thermal field. This idea can directly be generalized in the case of multi-atom information concentration.

Keywords: concentration, GHZ state, Bell-state measurement
PACC: 0367, 0365, 4250

1. Introduction

Quantum teleportation, proposed by Bennett *et al*[1] and experimentally realized by Bouwmeester *et al*[2] and Boschi *et al*,[3] is one of the most important applications of quantum entanglement. In the usual quantum teleportation scheme, an arbitrary unknown quantum state can be faithfully transmitted to a remote receiver with the establishment of a maximally entangled state between the sender and receiver. It represents the basic building block of future quantum communication network between distant parties.[4] An important application of multiparticle entanglement is in distributed quantum computation,[5] where several distant parties (Alice, Bob, Charlie, etc.) share an initial entangled state and are asked to perform a given computational task using only local operations and classical communications (LOCC). A strategy for the one-to-many-recipient case (to distribute quantum information of a particle from a sender to many distant receivers) has been suggested in Ref.[6]. The process is called telecloning.[7] Now the question we ask is, once a state has been telecloned to spatially separated parties, can it then be recreated by using only LOCC? The answer is yes, and several theoretical schemes have been suggested. Murao and Vedral[8] considered that quantum information of a single qubit was previously distributed to three spatially separated qubits by a symmetric telecloning procedure, and then could be remotely concentrated back to a single qubit by an unlockable bound entangled state[9] with LOCC. More recently, Yu *et al*[10] compared the remote information concentration by a Greenberger–Horne–Zeilinger (GHZ) state with an unlockable bound entangled state. However, the theoretical information concentration process they both considered are the three qubit case, and they only discussed the three qubit state in the cloning state after the asymmetric telecloning procedure. In addition, to realize all the above protocols mentioned in Refs.[8] and [10], the joint Bell-state measurement (BSM) is one of the key steps, but it is still very complex in experiment.[11] As we know, cavity QED provides an effective tool to deal with the problem on atom and cavity.[12−15] In experiment, atoms can be stored for long times and can act as the optimal carrier of quantum information for quantum computation. So the study on atomic states is of practical significance.

Following several recent contributions on teleportation of unknown atomic states with successful probability,[16,17] entanglement generation and concentration,[18−20] and quantum logic gate [21,22] in cavity QED, we propose this cavity QED scheme to realize remote atomic information concentration. Our scheme is a universal one which contains two or more remote two-level atomic information concentration.The distinct feature of the scheme is that it does not require the BSM on two atomic qubits. Mean-

*Project supported by the National Natural Science Foundation of China (Grant No 10374025).
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while, we also notice that the telecloning state [6,7] of three atoms $|\psi_c\rangle = a\sqrt{\frac{2}{3}}\Big[|g\rangle|gg\rangle + \frac{1}{2}|e\rangle(|ge\rangle + |eg\rangle)\Big] + b\sqrt{\frac{2}{3}}\Big[|e\rangle|ee\rangle + \frac{1}{2}|g\rangle(|eg\rangle + |ge\rangle)\Big]$ can be concentrated by a GHZ state using the method which we will discuss in the following. Before the measurements of quantum states, our scheme involves atom–field interaction with a large detuning and a classical field is simultaneously accompanied, no exchange of energy will happen between the atoms and the cavity field. Thus, the scheme is insensitive to both the cavity decay and thermal field. These advantages can warrant the current scheme more feasible in experiments.

2. Information concentration of two remote atoms

Now we discuss the scheme in detail. Firstly, we will consider the two-atom case. The two remote two-level atomic information concentration begins with a situation where two separate parties Alice and Bob hold the two atoms 1 and 2 respectively. Without loss of generality, the two atoms are in one of the following states:

$$|\psi\rangle_{12} = a|g\rangle_1|g\rangle_2 + b|e\rangle_1|e\rangle_2, \quad (1)$$

$$|\psi'\rangle_{12} = a|g\rangle_1|e\rangle_2 + b|e\rangle_1|g\rangle_2, \quad (2)$$

where a and b are complex numbers and satisfy $|a|^2 + |b|^2 = 1$, $|a| > |b|$. The original single atomic qubit state $|\phi\rangle = a|g\rangle + b|e\rangle$ is recreated at the location of a receiver Charlie. In other words, the goal of the scheme in this section is: $|\psi\rangle_{12} \to |\phi\rangle$. The schematic picture of the two remote atomic information concentration is illustrated in Fig.1.

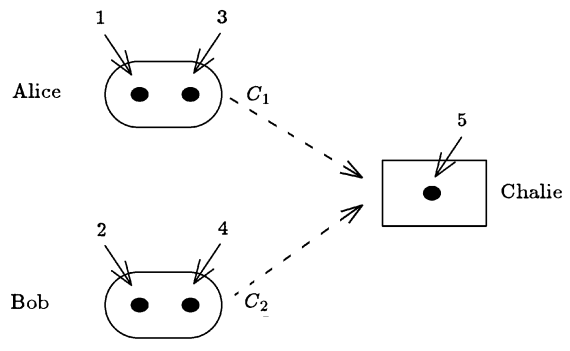


Fig.1. Schematic illustration of two remote atomic information concentration.

We consider that a three-atom GHZ state is shared in advance among the three parties Alice, Bob and Charlie; they hold the atomic qubits 3, 4, 5, respectively:

$$|\psi\rangle_{345} = \frac{1}{\sqrt{2}}(|g\rangle_3|g\rangle_4|g\rangle_5 + |e\rangle_3|e\rangle_4|e\rangle_5). \quad (3)$$

At Alice's station, a single-mode cavity C_1 will be introduced. Then Alice will send the two atoms 1,3 into the cavity. We assume that the two-level atoms are identical and the two atoms simultaneously interacting with the cavity field and driven by a classical field. In the case $\omega_0 = \omega$, $\delta \gg g/2$, $2\Omega \gg \delta, g$, (δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω, Ω is the Rabi frequency of the classical field, g is the atom–cavity coupling strength), there is no quantum information transformation between the atomic system and the cavity. The effective Hamiltonian(assuming $\hbar = 1$) for this interaction can be described as follows:[18]

$$H_e = \lambda[\frac{1}{2}\sum_{j=1,3}(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) + \sum_{j,k=1,3,j\neq k}(S_j^+ S_k^+ + S_j^+ S_k^- + \text{H.c.})], \quad (4)$$

where $\lambda = g^2/2\delta$, $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, and $|e_j\rangle$ and $|g_j\rangle$ are the excited and ground states of the jth atom. Then the evolution operator of the system is given by:[18] $U(t) = \mathrm{e}^{-\mathrm{i}H_0 t}\mathrm{e}^{-\mathrm{i}H_e t}$, where $H_0 = \Omega\sum_{j=1,3}(S_+ + S_-)$. Choosing the interaction time to satisfy $\lambda t = \pi/4$, we modulate the driving field $\Omega t = k\pi$, where k is an integer (which is the same in the whole text). After the interaction, the states of atoms 1, 3 will become

$$\begin{aligned}
|g\rangle_1|g\rangle_3 &\to \frac{1}{\sqrt{2}}(|g\rangle_1 g\rangle_3 - \mathrm{i}|e\rangle_1 e\rangle_3),\\
|g\rangle_1|e\rangle_3 &\to \frac{1}{\sqrt{2}}(|g\rangle_1 e\rangle_3 - \mathrm{i}|e\rangle_1 g\rangle_3),\\
|e\rangle_1|g\rangle_3 &\to \frac{1}{\sqrt{2}}(|e\rangle_1 g\rangle_3 - \mathrm{i}|g\rangle_1 e\rangle_3),\\
|e\rangle_1|e\rangle_3 &\to \frac{1}{\sqrt{2}}(|e\rangle_1 e\rangle_3 - \mathrm{i}|g\rangle_1 g\rangle_3). \quad (5)
\end{aligned}$$

In the same way, single-mode cavities C_2 driven by the classic field are required for Bob. He perform the same operations as Alice on his pairs of atomic qubits 2 and 4. Then the evolution of the total system can be expressed as (assuming the initial state is in the form of Eq.(1))

$$
\begin{aligned}
|\psi\rangle_{12}\otimes|\psi\rangle_{345} =&(a|g\rangle_1|g\rangle_2+b|e\rangle_1|e\rangle_2)\otimes\frac{1}{\sqrt{2}}(|g\rangle_3|g\rangle_4|g\rangle_5+|e\rangle_3|e\rangle_4|e\rangle_5)\\
=&\frac{1}{\sqrt{2}}[a|gg\rangle_{13}|gg\rangle_{24}|g\rangle_5+a|ge\rangle_{13}|ge\rangle_{24}|e\rangle_5\\
&+b|ee\rangle_{13}|ee\rangle_{24}|e\rangle_5+b|eg\rangle_{13}|eg\rangle_{24}|g\rangle_5]\\
\rightarrow&\frac{1}{2\sqrt{2}}[|gg\rangle_{13}|gg\rangle_{24}(a|g\rangle_5-b|e\rangle_5)-\mathrm{i}|gg\rangle_{13}|ee\rangle_{24}(a|g\rangle_5+b|e\rangle_5)\\
&-\mathrm{i}|ee\rangle_{13}|gg\rangle_{24}(a|g\rangle_5+b|e\rangle_5)-|ee\rangle_{13}|ee\rangle_{24}(a|g\rangle_5-b|e\rangle_5)\\
&+|ge\rangle_{13}|ge\rangle_{24}(a|e\rangle_5-b|g\rangle_5)-\mathrm{i}|eg\rangle_{13}|ge\rangle_{24}(a|e\rangle_5+b|g\rangle_5)\\
&-\mathrm{i}|ge\rangle_{13}|eg\rangle_{24}(a|e\rangle_5+b|g\rangle_5)-|eg\rangle_{13}|eg\rangle_{24}(a|e\rangle_5-b|g\rangle_5)]. \qquad (6)
\end{aligned}
$$

After the atom–cavity interaction, Alice and Bob inform their respective final measurements results to Charlie, and Charlie can reconstruct the original state $|\phi\rangle = a|g\rangle + b|e\rangle$ by corresponding unitary transformation. For instance, if the combined results are $|gg\rangle_{13}|gg\rangle_{24}$, with the probability of $\frac{1}{8}$, i.e. Alice detects atoms 1,3 in the state $|gg\rangle$, and Bob detects atoms 2,4 in the state $|gg\rangle$ too, then Charlie only needs to perform the operation of the operator σ_z on his atomic qubit 5 to retrieve the state $|\phi\rangle$. If the measurement results are listed in the above formula such as $|gg\rangle_{13}|ee\rangle_{24}$, $|ee\rangle_{13}|gg\rangle_{24}$, $|ee\rangle_{13}|ee\rangle_{24}...|eg\rangle_{13}|eg\rangle_{24}$, the concentration also can succeed with the same probability (after Charlie performs a corresponding operation of the Pauli operator in the set $\{I, \sigma_x, \sigma_y, \sigma_z\}$). Thus the total success probability is $\frac{1}{8}\times 8 = 1$.

3. Information concentration of three remote atoms

In this section, we will turn to discuss the case of information concentration of three remote atoms. Here, three spatially separated parties (Alice, Bob and Charlie) share one of the following states:

$$
\begin{aligned}
|\psi\rangle_{123} &= a|g\rangle_1|g\rangle_2|g\rangle_3+b|e\rangle_1|e\rangle_2|e\rangle_3,\\
|\psi'\rangle_{123} &= a|g\rangle_1|g\rangle_2|e\rangle_3+b|e\rangle_1|e\rangle_2|g\rangle_3,\\
|\psi''\rangle_{123} &= a|g\rangle_1|e\rangle_2|g\rangle_3+b|e\rangle_1|g\rangle_2|e\rangle_3. \qquad (7)
\end{aligned}
$$

Instead of the previous three-atom GHZ state, a four-atom GHZ state

$$
|\psi\rangle_{4567}=\frac{1}{\sqrt{2}}(|g\rangle_4|g\rangle_5|g\rangle_6|g\rangle_7+\mathrm{i}|e\rangle_4|e\rangle_5|e\rangle_6|e\rangle_7), \quad (8)
$$

is preshared among Alice, Bob, Charlie and David who will reconstruct the singe atomic qubit $|\phi\rangle$. Just like the two-atom case, Alice Bob and Charlie have access to the auxiliary single-mode cavity C_1, C_2, C_3 driven by classic field respectively. Likewise, Alice send atoms 1 and 4 into cavity C_1, Bob send atoms 2 and 5 into cavity C_2, Charlie send atoms 3 and 6 into cavity C_3. According to the same model, and after the atom–field interaction, they communicate their results with David. Then David can choose to perform an appropriate transformation on his atomic qubit 7 to achieve the concentration task.

Next, we will realize the concentration of the three-atomic telecloning state. Three atomic qubits in a cloning state after the asymmetric telecloning procedure[6,7] is

$$
\begin{aligned}
|\psi_{\mathrm{c}}\rangle =&a\frac{1}{\sqrt{N}}(|g\rangle_1|gg\rangle_{23}+p|e\rangle_1|ge\rangle_{23}+q|e\rangle_1|eg\rangle_{23})\\
&+b\frac{1}{\sqrt{N}}(|e\rangle_1|ee\rangle_{23}+p|g\rangle_1|eg\rangle_{23}\\
&+q|g\rangle_1|ge\rangle_{23}), \qquad (9)
\end{aligned}
$$

where $q = 1-p$, $p > q$, N is a normalization factor given by $N = 1+p^2+q^2$. We only need to put $|\psi_{\mathrm{c}}\rangle$ in the place of $|\psi\rangle_{123}$ either in Eqs.(7–9); other steps are the same as before. If the combining results are $|gg\rangle_{14}|gg\rangle_{25}|gg\rangle_{36}$, with the probability of $\frac{1}{16N}$, then David only needs to perform the operator I on his atomic qubit 5 to retrieve the state $|\phi\rangle$. It is obvious that the total concentration success probability of the telecloning state in our scheme is $\frac{1}{4N}+\frac{p^2}{4N}+\frac{q^2}{4N}=\frac{1}{4}$.

4. Remote multi-atom information concentration

We also note that the scheme in the above section can directly be generated to obtain the remote atomic information concentration of multiatoms (more than 3 atoms). In this section, we will consider the generalization of our schemes. Here, we consider that the quantum information of single atomic qubit was previously distributed to n spatially separated qubits. That is, n distant parties (Alice, Bob, Charlie...) hold n atoms (1 2 3...n) respectively. The state $|\psi\rangle_{123...n}$ has the following form:

$$|\psi\rangle_{123...n} = a\frac{1}{\sqrt{N}}\sum_i q_i P_i(|l\rangle_1|l\rangle_2|l\rangle_3...|l\rangle_n) + b\frac{1}{\sqrt{N}}\sum_i q_i \bar{P}_i(|l\rangle_1|l\rangle_2|l\rangle_3...|l\rangle_n), \quad (10)$$

where N is the normalization factor given by $N = \sum_i q_i^2$, P_i denotes an operator which creates a possible state represented by permutation of the elements $\{|l\rangle_1, |l\rangle_2, |l\rangle_3, ...|l\rangle_n\}$ for $|l\rangle_m \in \{|g\rangle, |e\rangle\}$ and $|l\rangle_{m+1} \geq |l\rangle_m (m = 1, 2, ..., n)$. $\bar{P}_i(|l\rangle_1|l\rangle_2|l\rangle_3...|l\rangle_n)$ is the complete symmetry state of $P_i(|l\rangle_1|l\rangle_2|l\rangle_3...|l\rangle_n)$, in other words, if the element $|l\rangle_m$ in $P_i(|l\rangle_1|l\rangle_2|l\rangle_3...|l\rangle_n)$ denotes $|g\rangle(|e\rangle)$, then the corresponding locational element in $\bar{P}_i(|l\rangle_1|l\rangle_2|l\rangle_3...|l\rangle_n)$ is $|e\rangle(|g\rangle)$, and vice versa. It is obvious that the telecloning state also belongs to the form expressed in Eq.(10). In the same way, a $(n+1)$-atom GHZ state is preshared among the above n parties and David (at his station, the task of reconstruction will be accomplished). When n is even, the GHZ state has the following form:

$$|\psi\rangle_{n+1n+2...2n2n+1} = \frac{1}{\sqrt{2}}(|g\rangle_{n+1}|g\rangle_{n+2}...|g\rangle_{2n}|g\rangle_{2n+1} + |e\rangle_{n+1}|e\rangle_{n+2}...|e\rangle_{2n}|e\rangle_{2n+1}), \quad (11)$$

When n is odd, the GHZ state is given by

$$|\psi\rangle_{n+1n+2...2n2n+1} = \frac{1}{\sqrt{2}}(|g\rangle_{n+1}|g\rangle_{n+2}...|g\rangle_{2n}|g\rangle_{2n+1} + \mathrm{i}|e\rangle_{n+1}|e\rangle_{n+2}...|e\rangle_{2n}|e\rangle_{2n+1}). \quad (12)$$

Thus, the initial state of the system is $|\psi\rangle_{123...} \otimes |\psi\rangle_{n+1n+2...2n+1}$, where atoms 1 and n + 1 are sent to Alice, 2 and n + 2 are sent to Bob...David owns the (2n + 1)th atom. n cavities ($C_1C_2...C_n$) driven by the classic field are needed in hands of Alice, Bob, Charlie, etc., too. Figure 2 shows the schematic picture of remote multi-atom information concentration.

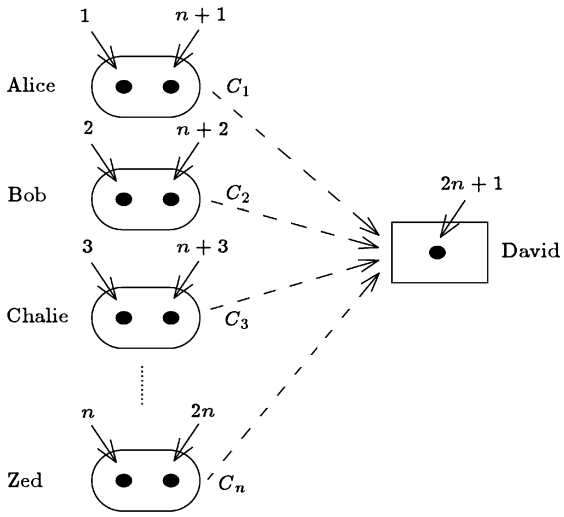


Fig.2. Schematic illustration of remote multi-atom information concentration.

The whole process can be expressed in the similar way as before. When we consider the simplest case described in Eq.(12), i.e. that the state $|\psi\rangle_{123...n}$ is given by

$$|\psi\rangle_{123...n} = a|g\rangle_1|g\rangle_2|g\rangle_3...|g\rangle_n + b|e\rangle_1|e\rangle_2|e\rangle_3...|e\rangle_n, \quad (13)$$

and we suppose n is even, then the process can be represented by the following formula:

$$\begin{aligned}
&|\psi\rangle_{123...n} \otimes |\psi\rangle_{n+1n+2...2n} \\
=&\frac{1}{\sqrt{2}}[a|gg\rangle_{1n+1}|gg\rangle_{2n+2}...|gg\rangle_{n2n}|g\rangle_{2n+1} + a|ge\rangle_{1n+1}|ge\rangle_{2n+2}...|ge\rangle_{n2n}|e\rangle_{2n+1} \\
&+ b|ee\rangle_{1n+1}|ee\rangle_{2n+2}...|ee\rangle_{n2n}|e\rangle_{2n+1} + b|eg\rangle_{1n+1}|eg\rangle_{2n+2}...|eg\rangle_{n2n}|g\rangle_{2n+1}] \\
\rightarrow&\frac{1}{(\sqrt{2})^{n+1}}\{|gg\rangle_{1n+1}|gg\rangle_{2n+2}...|gg\rangle_{n2n}[a|g\rangle_{2n+1} + (-\mathrm{i})^n b|e\rangle_{2n+1}]
\end{aligned}$$

$$
\begin{aligned}
&+ |gg\rangle_{1n+1}|ee\rangle_{2n+2}...|gg\rangle_{n2n}[(-\mathrm{i})a|g\rangle_{2n+1} + (-\mathrm{i})^{n-1}b|e\rangle_{2n+1}] \\
&+ ... \\
&+ |gg\rangle_{1n+1}|gg\rangle_{2n+2}...|ee\rangle_{n2n}[(-\mathrm{i})a|g\rangle_{2n+1} + (-\mathrm{i})^{n-1}b|e\rangle_{2n+1}] \\
&+ ... \\
&+ |ee\rangle_{1n+1}|ee\rangle_{2n+2}...ee\rangle_{n2n}[(-\mathrm{i})^{n}a|g\rangle_{2n+1} + b|e\rangle_{2n+1}] \\
&+ |ge\rangle_{1n+1}|ge\rangle_{2n+2}...|ge\rangle_{n2n}[a|e\rangle_{2n+1} + (-\mathrm{i})^{n}b|g\rangle_{2n+1}] \\
&+ ... \\
&+ |eg\rangle_{1n+1}|eg\rangle_{2n+2}...|eg\rangle_{n2n}[(-\mathrm{i})^{n}a|e\rangle_{2n+1} + b|g\rangle_{2n+1}] \\
&+ ...\}. \qquad (14)
\end{aligned}
$$

Similarly, no joint operations among atomic qubits belonging to different parties are allowed. The n parties measure the states of their atoms in hand, and communicate the results with David, respectively. Finally, David determines an appropriate unitary transformation on his atomic qubit $2n+1$ and gets the original atomic qubit in the state $|\phi\rangle$ with certainty.

5. Discussion and conclusion

In conclusion, we have presented a general scheme for remote information concentration of two or more two-level atoms without Bell-state measurement by using techniques of cavity QED. We only require detecting the states of atoms, and the atomic states can be detected by the ionization detectors with high efficiency in the experiments of Haroche's group.[14,23] It is noted that the states $|eg\rangle_{ij}$ and $|ge\rangle_{ij}$ $(i \neq j)$ for the measurement results here is distinguishable, because we can distinguish the two atoms via sending the two atoms cross the cavity at different positions and finally detecting separately in two detectors D_1 and D_2 after they fly out of the cavity, as in Ref.[24]. Because the interaction between driven atoms and quantized cavity mode we used here is in the large detuning limit, our scheme is insensitive to the cavity decay and thermal field. Thus we believe that the remote atomic information concentration has potential value for real application in the practice of quantum communication networks in the near future.

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