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Simple experimental scheme of teleporting a two-qubit state via linear optical elements*

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We propose a simple experimental scheme in which an unknown two-qubit state is faithfully and deterministically teleported from Alice to Bob. The scheme is constructed with four photons from parametric down conversion, linear optical elements, and conventional photon detectors, all of which are available in current technology. It is shown that the probability of successful teleportation ideally reaches 100% based on single-photon two-qubit-assisted Bell-state measurement, which can distinguish all four Bell-states simultaneously via conventional photon detectors. By generalizing the scheme, the teleportation of an unknown multi-qubit system can also be realized.

Keywords: teleportation, two-qubit state, single-photon two-qubit, linear optical elements
PACC: 0365, 0367, 4250

Quantum teleportation (QT), originally proposed by Bennett *et al*,[1] is a remarkable demonstration of how non-local correlations in quantum mechanics can be used to advantage. The entangled systems, which are divided into two parts, enable one to transmit the quantum information of an unknown quantum state to a remote place while the original state is destroyed. No information about the unknown state is ever revealed in the process of the teleportation. Subsequently, various methods of teleportation have been studied by research groups and the research on QT has undergone rapid development. Some authors theoretically discussed all kinds of teleportation of an unknown state via different quantum channels.[2−21] In particular, a few schemes of teleporting an Einstein–Podolsky–Rosen (EPR) state such as $x\,|01\rangle + y\,|10\rangle$, $x\,|00\rangle + y\,|11\rangle$ were proposed.[5−11] For an arbitrary two-qubit system such as $a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle$, Lee *et al*[12] utilized a four-entangled state to send to the receiver four bits of classical information and Rigonlin[21] explicitly constructed the 16 orthogonal generalized Bell states which can be used to teleport the two-qubit state. Others focused on experimental realization of QT.[22−27] The teleportation of a photon in an unknown polarized state has been experimentally demonstrated by Bouwmeester *et al*[22] and Boschi *et al*[23] by using parametric down-conversion. Kim *et al*[25] reported an experiment of QT in which nonlinear interactions are used for the Bell state measurements.

Recently, more and more attention has been paid to linear optical implementation for the quantum information.[28−34] It is well known that linear optical elements might play an important role in current development of quantum information applications from an experimental point of view.[28] Here, combining these two ideas,[23,34] we present a simple experimental scheme of teleporting an arbitrary two-qubit state via linear optical elements such as polarization beam splitter (PBS), 50/50 beam splitter (BS) and half wave plate (HWP). The prominent feature lies on single-photon two-qubit-assisted Bell-state[23,35] measurement such that the probability of successful teleportation for this scheme ideally reaches 100%. In other words, the post-selection[22,26,27] of successful events by photon detectors is not needed.

We now consider an unknown two-qubit system state[21] which is teleported by the sender Alice to the receiver Bob and has the following form:

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$$
\begin{aligned}
|\Phi\rangle_{12} =&[\alpha|H\rangle_1+\beta|V\rangle_1][\gamma|H\rangle_2+\delta|V\rangle_2]\\
=&\alpha\gamma|HH\rangle_{12}+\alpha\delta|HV\rangle_{12}\\
&+\beta\gamma|VH\rangle_{12}+\beta\delta|VV\rangle_{12}, \qquad (1)
\end{aligned}
$$

where $|H\rangle_i$ ($|V\rangle_i$) represents the state of photon i with horizontal (vertical) polarization. The schematic diagram of this scheme is depicted in Fig.1.

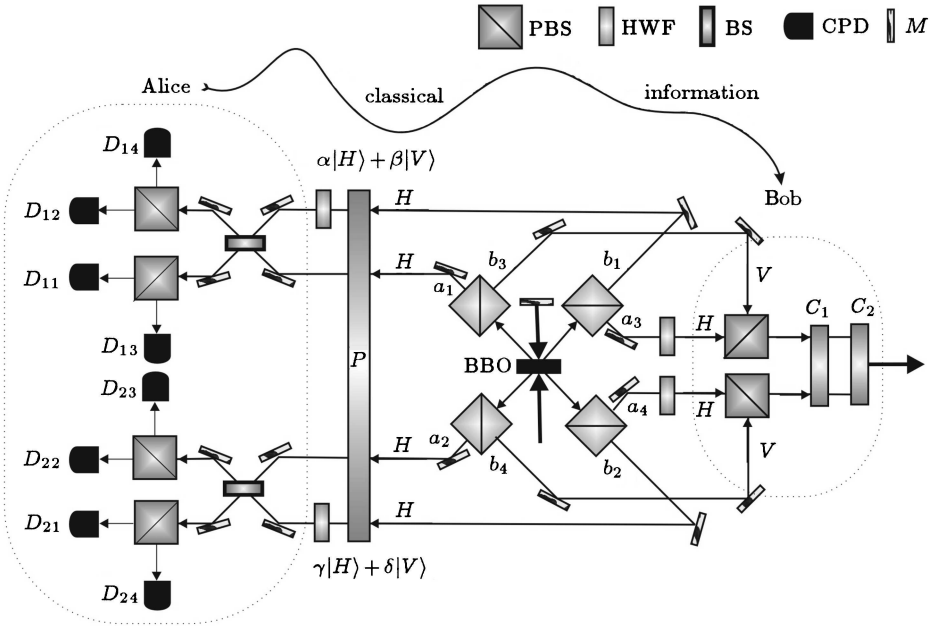


Fig.1. Schematic diagram of teleporting an unknown two-qubit state based on single-photon two-qubit-assisted Bell-state measurement via linear optical elements. The setup consists of a type-II (BBO) spontaneous parametric down-conversion (SPDC) for polarization entangled photons, polarization beam splitters (PBS), half wave plates (HWP) for 90° polarization rotation, 50/50 beam splitters (BS), conventional photon detectors (CPD) for only discriminating the void from one or more photons, mirrors (M), the preparer (P) of the initial quantum states, and polarization transformers (C). See text for details.

Let us describe how to realize this scheme in more detail. Firstly, a pulse of UV light passes a β-barium borate (BBO) crystal twice to produce two entangled photon pairs which may be expressed as[26,27]

$$
\begin{aligned}
|\varphi'\rangle =&\frac{1}{\sqrt{2}}[|H\rangle_1|V\rangle_3+|V\rangle_1|H\rangle_3]\\
&\otimes\frac{1}{\sqrt{2}}[|H\rangle_2|V\rangle_4+|V\rangle_2|H\rangle_4]. \qquad (2)
\end{aligned}
$$

Four photons above are directed to four PBSs which completely transmit horizontal polarization (H) and totally reflect vertical polarization (V), respectively. This transfers the entangled polarization into spatial mode entanglement in the following state:

$$
\begin{aligned}
|\varphi'\rangle\rightarrow|\varphi\rangle =&\frac{1}{2}[|a_1\rangle|a_3\rangle+|b_1\rangle|b_3\rangle]\\
&\times[|a_2\rangle|a_4\rangle+|b_2\rangle|b_4\rangle]\\
&\times|H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4, \qquad (3)
\end{aligned}
$$

where the entanglements of spatial modes 1, 3 and 2, 4 are used as the quantum channel between Alice and Bob. Photons with labels 1, 2 are horizontally polarized which belong to Alice while photons with labels 3, 4 are vertically polarized which belong to Bob. For example, $|a_1\rangle|H\rangle_1$ represents the state of photon 1 with horizontal polarization in path a_1. $|b_2\rangle|H\rangle_2$ denotes the state of photon 2 with horizontal polarization in path b_2.

Secondly, on Alice's side the preparer (P) transforms the H polarization of photons 1 and 2 into an arbitrary product state, i.e. an unknown two-qubit state expressed by Eq.(1). Here, the P affects the polarization in both paths a_1, b_1 and paths a_2, b_2 in the same way via a certain HWP, respectively. After initial state preparation, the state of the whole system composed of photons 1, 2, 3, 4 and quantum channel formed by spatial mode entanglements is given by

$$
\begin{aligned}
|\Psi\rangle =&\frac{1}{2}[|a_1\rangle|a_3\rangle+|b_1\rangle|b_3\rangle]\\
&\times[|a_2\rangle|a_4\rangle+|b_2\rangle|b_4\rangle]\\
&\times[\alpha|H\rangle_1+\beta|V\rangle_1]\\
&\times[\gamma|H\rangle_2+\delta|V\rangle_2]|V\rangle_3|V\rangle_4. \qquad (4)
\end{aligned}
$$

Thirdly, in order to realize teleportation, Alice must perform single-photon two-qubit-assisted Bell-

state measurements[23,35] on the initial state $|\Phi\rangle_{12}$ and on her part of the spatial entanglement, respectively. At the same time, the whole state will project photons 3 and 4 into one of the following states (16 kinds in all):

$$ {}_2\langle\phi^{\pm}|{}_1\langle\phi^{+}|\ \Psi\rangle = \frac{1}{4}[\alpha\gamma\,|a_3\rangle\,|a_4\rangle \pm \alpha\delta\,|a_3\rangle\,|b_4\rangle + \beta\gamma\,|b_3\rangle\,|a_4\rangle \pm \beta\delta\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{5}$$

$$ {}_2\langle\psi^{\pm}|{}_1\langle\phi^{+}|\ \Psi\rangle = \frac{1}{4}[\alpha\delta\,|a_3\rangle\,|a_4\rangle \pm \alpha\gamma\,|a_3\rangle\,|b_4\rangle + \beta\delta\,|b_3\rangle\,|a_4\rangle \pm \beta\gamma\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{6}$$

$$ {}_2\langle\phi^{\pm}|{}_1\langle\phi^{-}|\ \Psi\rangle = \frac{1}{4}[\alpha\gamma\,|a_3\rangle\,|a_4\rangle \pm \alpha\delta\,|a_3\rangle\,|b_4\rangle - \beta\gamma\,|b_3\rangle\,|a_4\rangle \mp \beta\delta\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{7}$$

$$ {}_2\langle\psi^{\pm}|{}_1\langle\phi^{-}|\ \Psi\rangle = \frac{1}{4}[\alpha\delta\,|a_3\rangle\,|a_4\rangle \pm \alpha\gamma\,|a_3\rangle\,|b_4\rangle - \beta\delta\,|b_3\rangle\,|a_4\rangle \mp \beta\gamma\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{8}$$

$$ {}_2\langle\phi^{\pm}|{}_1\langle\psi^{+}|\ \Psi\rangle = \frac{1}{4}[\beta\gamma\,|a_3\rangle\,|a_4\rangle \pm \beta\delta\,|a_3\rangle\,|b_4\rangle + \alpha\gamma\,|b_3\rangle\,|a_4\rangle \pm \alpha\delta\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{9}$$

$$ {}_2\langle\psi^{\pm}|{}_1\langle\psi^{+}|\ \Psi\rangle = \frac{1}{4}[\beta\delta\,|a_3\rangle\,|a_4\rangle \pm \beta\gamma\,|a_3\rangle\,|b_4\rangle + \alpha\delta\,|b_3\rangle\,|a_4\rangle \pm \alpha\gamma\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{10}$$

$$ {}_2\langle\phi^{\pm}|{}_1\langle\psi^{-}|\ \Psi\rangle = \frac{1}{4}[\beta\gamma\,|a_3\rangle\,|a_4\rangle \pm \beta\delta\,|a_3\rangle\,|b_4\rangle - \alpha\gamma\,|b_3\rangle\,|a_4\rangle \mp \alpha\delta\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{11}$$

$$ {}_2\langle\psi^{\pm}|{}_1\langle\psi^{-}|\ \Psi\rangle = \frac{1}{4}[\beta\delta\,|a_3\rangle\,|a_4\rangle \pm \beta\gamma\,|a_3\rangle\,|b_4\rangle - \alpha\delta\,|b_3\rangle\,|a_4\rangle \mp \alpha\gamma\,|b_3\rangle\,|b_4\rangle]\,|V\rangle_3\,|V\rangle_4\,, \tag{12}$$

where

$$|\phi^{\pm}\rangle_i = \frac{1}{\sqrt{2}}[|a_i\rangle|H\rangle_i \pm |b_i\rangle|V\rangle_i,$$

$$|\psi^{\pm}\rangle_i = \frac{1}{\sqrt{2}}[|a_i\rangle|V\rangle_i \pm |b_i\rangle|H\rangle_i \quad (i = 1, 2)$$

are directly analogous to the Bell-states considered in Ref.[1], which are the so-called single-photon two-qubit Bell-states.[35] In experiment, photon i passes through the optical elements such as a HWP for 90° polarization rotation, i.e. $|H\rangle_i \leftrightarrow |V\rangle_i$ in path b_i, an ordinary 50/50 BS for polarizations H and V independent interference, two PBSs and should be detected by one of conventional photon detectors (CPD) D_{i1}, D_{i2}, D_{i3}, or D_{i4}, which is described in Fig.1. This process is equivalent to Bell-state measurements, i.e. $D_{i1} \to |\phi^+\rangle_i$, $D_{i2} \to |\phi^-\rangle_i$, $D_{i3} \to |\psi^+\rangle_i$, $D_{i4} \to |\psi^-\rangle_i$ (see Ref.[23] for details).

Finally, Alice informs Bob which detector registered a photon via a classical communication channel. With this information Bob can reproduce the initial polarizing state as follows. He begins to transform the spatial mode superposition of photons 3 and 4 into the polarizing superposition via a HWP for 90° polarization rotation in path $a_j (j = 3, 4)$ and a PBS in order to combine the paths, i.e. $|a_j\rangle|V\rangle_j \to |H\rangle_j$, $|b_j\rangle|V\rangle_j \to |V\rangle_j$ (see Fig.1). After this, he just switches two optical elements C_1 and C_2 on or off, depending on the information obtained from Alice, to interchange H and V or to provide a relative phase shift of π for V. This transforms the polarization of photons 3 and 4 into the polarized state prepared on photons 1 and 2 in Eq.(1). Table 1 gives Alice's all different results registered by PDs and Bob's corresponding optical operations. Here, X_j and Z_j represent Pauli optical operations, i.e. $X_j|H\rangle_j = |V\rangle_j$, $X_j|V\rangle_j = |H\rangle_j$, $Z_j|H\rangle_j = |H\rangle_j$, $Z_j|V\rangle_j = -|V\rangle_j$, and I is a unity operation.

Table 1. Bob's relevant optical operations corresponding to Alice's all different results registered by PD.

Alice's result	Bob's optical operation	Alice's result	Bob's optical operation
D_{11} D_{21}	$I_3 \otimes I_4$	D_{13} D_{21}	$X_3 \otimes I_4$
D_{11} D_{22}	$I_3 \otimes Z_4$	D_{13} D_{22}	$X_3 \otimes Z_4$
D_{11} D_{23}	$I_3 \otimes X_4$	D_{13} D_{23}	$X_3 \otimes X_4$
D_{11} D_{24}	$I_3 \otimes X_4Z_4$	D_{13} D_{24}	$X_3 \otimes X_4Z_4$
D_{12} D_{21}	$Z_3 \otimes I_4$	D_{14} D_{21}	$X_3Z_3 \otimes I_4$
D_{12} D_{22}	$Z_3 \otimes Z_4$	D_{14} D_{22}	$X_3Z_3 \otimes Z_4$
D_{12} D_{23}	$Z_3 \otimes X_4$	D_{14} D_{23}	$X_3Z_3 \otimes X_4$
D_{12} D_{24}	$Z_3 \otimes X_4Z_4$	D_{14} D_{24}	$X_3Z_3 \otimes X_4Z_4$

From the above analysis, it is certainly shown that the simple experimental scheme of teleporting an unknown two-qubit state between Alice and Bob may be realized within the reach of the current technology. Not only does the investigation cover all possible states of a two-qubit system, unlike Refs.[31, 32] which are restricted to partially entangled states such as the EPR state, but also the probability of successful teleportation is theoretically 100%, which is optimal compared with the existing experimental proposals,[10,31] with the help of single-photon two-qubit Bell-state measurements.

In summary, we have explicitly proposed a simple scheme in which an unknown two-qubit state is faithfully and deterministically teleported from Alice to Bob. The present scheme has beneficial factors as follows: (i) Four Bell-states can fully be distinguished, which may greatly increase the probability of successful teleportation, different from the foregoing experimental proposals.[22,26,27] (ii) It uses only four particles during the whole process, which is more economical, while the general schemes require not less than six particles.[12,16,17,21] (iii) It is an experimentally feasible protocol for implementing teleportation of the polarization-entangled photon via current technology. (iv) It is a scalable protocol which is straightforwardly performed in the case of n-photon superposition on condition that two parts simultaneously establish n pairs of photonic EPRs as their quantum channel. Incidentally, the experimental realization[22,23,26,27] may enhance the possibility to realize our schemes experimentally, which may be useful to the future quantum information processing.

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