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Multi-party dense coding in non-symmetric quantum channel

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A three- and an $(N+1)$-party dense coding scheme in the case of non-symmetric Hilbert spaces of the particles of a quantum channel are investigated by using a multipartite entangled state. In the case of the $(N+1)$-party dense coding scheme, we show that the amount of classical information transmitted from N senders to one receiver is improved.

Keywords: non-symmetric Hilbert space, dense coding, unitary transformation
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1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics and has been used in quantum information processing, such as quantum computer,[1,2] quantum teleportation,[3−7] and quantum secret sharing.[8−10] Quantum dense coding (QDC) is also one of the exhibitions of entanglement in quantum communication. Since Bennett *et al*[11] first proposed the QDC scheme, it has been deeply developed in Refs.[12]–[19]. In these proposed schemes, the quantum channels are symmetric: that is to say, the dimension of the Hilbert space of the particle of a sender is the same as that of the particle of a receiver. Bose *et al*[20] have presented a dense coding scheme among multi-parties. In this scheme, the quantum channel is non-symmetric. The particles of senders are in a 2^N-dimensional Hilbert space, while the particle in a two-dimensional Hilbert space belongs to the receiver. Yan and Wang[21] have given a scheme for dense coding by using bipartite entangled state and Fan *et al*[22,23] have presented a scheme for dense coding by using the direct product state, in the case of non-symmetric Hilbert spaces of the particles of the quantum channel.

In this paper, we study a three-party dense coding and an $(N+1)$-party dense coding by using a multipartite entangled state, in the case of non-symmetric Hilbert spaces of the particles of the quantum channel. In the case of the $(N+1)$-party dense coding scheme, we show that the amount of classical information transmitted from N senders to one receiver is improved, compared with the previous scheme.

2. Three-party dense coding by using the non-symmetric quantum channel

Suppose that Alice, Charlie, and Bob initially share a maximally entangled state in a $3\times3\times2$-dimensional Hilbert space in the following form:

$$|\mu_1\rangle_{123}=\frac{1}{\sqrt{2}}\left(|000\rangle+|111\rangle\right)_{123}, \tag{1}$$

where particle 1 belongs to Alice in a three-dimensional Hilbert space, particle 2 belongs to Charlie in a three-dimensional Hilbert space, and particle 3 belongs to Bob in a two-dimensional Hilbert space.

Firstly, Alice encodes her message on qutrit 1 by performing one of the six unitary transformations as follows:

$$U_{00}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},$$

$$U_{01}=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix},$$

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$$U_{10}=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix},$$

$$U_{11}=\begin{bmatrix}0&0&1\\1&0&0\\0&-1&0\end{bmatrix},$$

$$U_{20}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix},$$

$$U_{21}=\begin{bmatrix}0&-1&0\\0&0&1\\1&0&0\end{bmatrix}. \quad (2)$$

Secondly, Charlie encodes his message on qutrit 2 by performing one of the three unitary transformations $\{U_{00},U_{10},U_{20}\}$. Then, the collective unitary transformations can be written as

$$\begin{aligned}
U_1^+&=U_{00}\otimes U_{00}, & U_1^-&=U_{01}\otimes U_{00},\\
U_2^+&=U_{10}\otimes U_{00}, & U_2^-&=U_{11}\otimes U_{00},\\
U_3^+&=U_{20}\otimes U_{00}, & U_3^-&=U_{21}\otimes U_{00},\\
U_4^+&=U_{00}\otimes U_{10}, & U_4^-&=U_{01}\otimes U_{10},\\
U_5^+&=U_{10}\otimes U_{10}, & U_5^-&=U_{11}\otimes U_{10},\\
U_6^+&=U_{20}\otimes U_{10}, & U_6^-&=U_{21}\otimes U_{10},\\
U_7^+&=U_{00}\otimes U_{20}, & U_7^-&=U_{01}\otimes U_{20},\\
U_8^+&=U_{10}\otimes U_{20}, & U_8^-&=U_{11}\otimes U_{20},\\
U_9^+&=U_{20}\otimes U_{20}, & U_9^-&=U_{21}\otimes U_{20}. \quad (3)
\end{aligned}$$

The above collective unitary transformations will transform the states in Eq.(1) into the corresponding state respectively,

$$U_1^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|000\rangle\pm|111\rangle\right)_{123}=|\mu_1^{\pm}\rangle, \quad (4)$$

$$U_2^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|100\rangle\pm|211\rangle\right)_{123}=|\mu_2^{\pm}\rangle, \quad (5)$$

$$U_3^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|200\rangle\pm|011\rangle\right)_{123}=|\mu_3^{\pm}\rangle, \quad (6)$$

$$U_4^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|010\rangle\pm|101\rangle\right)_{123}=|\mu_4^{\pm}\rangle, \quad (7)$$

$$U_5^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|110\rangle\pm|201\rangle\right)_{123}=|\mu_5^{\pm}\rangle, \quad (8)$$

$$U_6^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|210\rangle\pm|001\rangle\right)_{123}=|\mu_6^{\pm}\rangle, \quad (9)$$

$$U_7^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|020\rangle\pm|101\rangle\right)_{123}=|\mu_7^{\pm}\rangle, \quad (10)$$

$$U_8^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|120\rangle\pm|201\rangle\right)_{123}=|\mu_8^{\pm}\rangle, \quad (11)$$

$$U_9^{\pm}|\mu_{00}\rangle=\frac{1}{\sqrt{2}}\left(|220\rangle\pm|001\rangle\right)_{123}=|\mu_9^{\pm}\rangle. \quad (12)$$

These 18 states form an orthonormal and complete basis: $\sum_i\sum_j|\mu_i^j\rangle\langle\mu_i^j|=I$ and $\langle\mu_i^j|\mu_{i'}^{j'}\rangle=\delta_{ii'}\delta_{jj'}$, where $|\mu_i^j\rangle$ (or $|\mu_{i'}^{j'}\rangle$) is one of the above 18 states in Eqs.(4)–(12); $i,i'\in[0,9]$; $j,j'\in\{+,-\}$. From now on , we refer to each of these 18 states as the generalized Bell basis.

Thirdly, after performing one of these 18 collective unitary transformations, Alice sends her particle 1 to Bob and Charlie sends his particle 2 to Bob. With the three particles being in his possession, Bob takes only a generalized Bell measurement in the generalized Bell basis $\{|\mu_i^j\rangle,\ i\in[0,9]$ and $j\in\{+,-\}\}$. In this way, Bob not only knows precisely what operations Alice and Charlie have performed, but also obtains $\log_2 18$ bits of classical information.

3. $(N+1)$-party dense coding by using the non-symmetric quantum channel

The method described in the preceding section can readily be generalized to the case where a great number of particles are involved. The specific steps of the process are depicted in Fig.1. Let us follow the state through this circuit. We assume that $(N+1)$-parties initially share a maximally entangled state in a $3^N\times 2$-dimensional Hilbert space in the following form:

$$|\mu_1'\rangle=\frac{1}{\sqrt{2}}\left(\prod_{m=1}^{N+1}|0\rangle_m+\prod_{m=1}^{N+1}|1\rangle_m\right), \quad (13)$$

where N particles belong to N senders in a 3-dimensional Hilbert space, and particle $(N+1)$ belongs to the receiver in a 2-dimensional Hilbert space. We now explain how to implement $(N+1)$-party dense coding by using the non-symmetric quantum channel in Eq.(13). It consists of four steps: (i) Any of the N senders encodes his message on his qutrit by performing one of the six unitary operations $\{U_{00},U_{01},U_{10},U_{11},U_{20},U_{21}\}$. (ii) Each of the $(N-1)$ remainders among the senders encodes his message on his qutrit by performing one of the three unitary operations $\{U_{00},U_{10},U_{20}\}$. (iii) After performing their unitary operations, each of the N senders sends his particle to the receiver. (iv) The receiver performs a collective measurement on the $(N+1)$ particles and identifies the state. Thus, the receiver can learn about the operations that the N senders have performed.

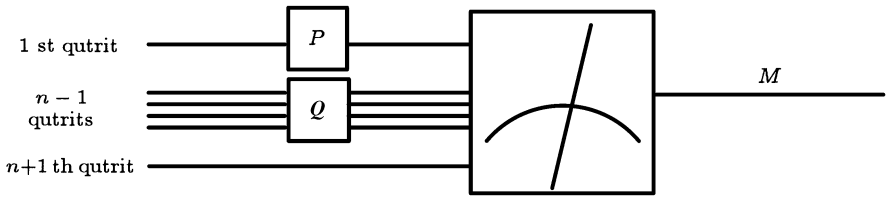


Fig.1. Quantum circuit implementing $(N+1)$-party dense coding. $P \in \{U_{00}, U_{01}, U_{10}, U_{11}, U_{20}, U_{21}\}$, $Q \in \{U_{00}, U_{10}, U_{20}\}$; M denotes measurement; $n = N$.

In this way, an amount of the classical information transmitted from N senders to one receiver by using the non-symmetric $(N+1)$-particle quantum channel in Eq.(13) can be expressed as

$$C = \log_2(3^N \times 2) = 1 + N \log_2 3. \tag{14}$$

Now let us compare the efficiency of the above scheme with that in Ref.[20]. In Ref.[20], the receiver can obtain only $(N+1)$ bits of classical information from N senders. With a little simple algebra, the difference in the amount of classical information transmitted between our scheme and that in Ref.[20] satisfies the following equation:

$$C_1 - C_2 = N \log_2 3/2, \tag{15}$$

where C_1 denotes the amount of classical information in our scheme and C_2 denotes the amount of classical information in Ref.[20]. Obviously, on average, the amount of classical information transmitted by each of the particles in a three-dimensional Hilbert space is $\log_2 3/2$, more than that transmitted by each of the particles in a 2-dimensional Hilbert space.

4. Summary

It must be stressed that our scheme is constructive. (i) We succeed in not only a three-party dense coding but also an $(N+1)$-party dense coding by using a multipartite entangled state, in the case of non-symmetric Hilbert spaces of the particles of the quantum channel. (ii) Meanwhile, the amount of transmitted classical information in our scheme is more than that in Ref.[20]. (iii) Comparing $(N+1)$-party dense coding with bipartite dense coding by the quantum channel in Eq.(13), one receiver can obtain the messages from N-senders, rather than one sender. In a word, our dense coding scheme is a good extension of the dense coding scheme by employing a non-symmetric quantum channel.

In conclusion, we have investigated a three- and an $(N+1)$-party dense coding by using a multipartite entangled state, in the case of non-symmetric Hilbert spaces of the particles of the quantum channel. In the case of $(N+1)$-party dense coding scheme, we have shown that the amount of classical information transmitted from N senders to one receiver is improved, compared with the previous scheme. We hope that this new scheme can be useful for further study of quantum dense coding.

References

[1] Barenco A 1996 *Contemp. Phys.* **37** 375

[2] Vedral V and Plenio M B 1998 *Prog. Quantum Electron* **22** 1

[3] Gao T 2005 *Chin. Phys.* **14** 893

[4] Yuan H C and Qi K G 2005 *Chin. Phys.* **14** 1716

[5] Lin X and Li H C 2005 *Chin. Phys.* **14** 1724

[6] Zheng Y Z, Gu Y J and Guo G C 2003 *Chin. Phys.* **12** 1070

[7] Sun L L, Fan Q B and Zhang S 2005 *Chin. Phys.* **14** 1313

[8] Hillery M, Bužek V and Berthiaume A 1999 *Phys. Rev.* A **59** 1829

[9] Koashi M and Imoto N 1997 *Phys. Rev. Lett.* **79** 2383

[10] Karimipour V, Bahraminasab A and Bagherinezhad S 2002 *Phys. Rev.* A **65** 042320

[11] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881

[12] Mattle K, Weinfurter H, Kwiat P G and Zeilinger A 1996 *Phys. Rev. Lett.* **76** 4656

[13] Lee H J, Ahn D and Hwang S W 2002 *Phys. Rev.* A **66** 024304

[14] Bose S, Plenio M B and Vedral V 2000 *J. Mod. Opt.* **47** 291

[15] Gorbachev V N, Trubilko A I, Zhiliba A I and Yakovleva E S 2000 quant-ph/0011124

[16] Cereceda J L 2001 quant-ph/0105096

[17] Fu C B, Xia Y, Liu B X, Zhang S, Yeon K H and Um C I 2005 *J. Korean Phys. Soc.* **46** 1080

[18] Liu X S, Long G L, Tong D M and Li F 2002 *Phys. Rev.* A **65** 022304

[19] Rigolin G 2004 quant-ph/0407193

[20] Bose S, Vedral V and Knight P L 1998 *Phys. Rev.* A **57** 822

[21] Yan F L and Wang M Y 2004 *Chin. Phys. Lett.* **21** 1195

[22] Fan Q B and Zhang S 2006 *Phys. Lett.* A **348** 160

[23] Fan Q B, Sun L L, Li F Y, Jin Z, Zhang S, Yeon K H and Um C I 2005 *J. Korean Phys. Soc.* **46** 769