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Method of determining cosmological parameter ranges with samples of candles with an intrinsic distribution*

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In this paper, the effect of the intrinsic distribution of cosmological candles is investigated. We find that in the case of a narrow distribution the deviation of the observed modulus of sources from the expected central value can be estimated within a ceratin range. We thus introduce lower and upper limits of χ^2, $\chi^2_{\min}$ and $\chi^2_{\max}$ to estimate cosmological parameters by applying the conventional minimizing χ^2 method. We apply this method to a gamma-ray burst (GRB) sample as well as to a combined sample including this GRB sample and an SN Ia sample. Our analysis shows that: a) in the case of assuming an intrinsic distribution of candles of the GRB sample, the effect of the distribution is obvious and should not be neglected; b) taking into account this effect would lead to a poorer constraint of the cosmological parameter ranges. The analysis suggests that in the attempt of constraining the cosmological model with current GRB samples, the results tend to be worse than was previously anticipated if the mentioned intrinsic distribution does exist.

Keywords: cosmological parameters, observational cosmology, distance scale
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1. Introduction

One of the greatest achievements obtained in the past few years in astrophysics is the determination of cosmological parameters with Type Ia supernovae (SN Ia), which suggests an accelerating universe on a large scale.[1−6] The cosmic acceleration was also confirmed, independently of the SN Ia magnitude-redshift relation, by the observations of the cosmic microwave background anisotropies[7] and the large-scale structure in the distribution of galaxies.[8,9] It is well known that all known types of matters with positive pressure generate attractive forces and decelerate the expansion of the universe. Given this, a dark energy component with negative pressure is generally suggested to be the invisible fuel that drives the current acceleration of the universe. There are a huge number of candidates for the dark energy component in the literature, such as a cosmological constant Λ,[10] an evolving scalar field (referred to as quintessence[11,12]), the phantom energy, in which the sum of the pressure and energy density is negative,[13] the so-called 'X-matter',[14−17] the Chaplygin gas,[18−20] the Cardassion model,[21−25] and the brane world model.[26−28]

Samples of SN Ia sources available in the early analysis contain only sources with redshifts $z < 1$. Although observations of the fluctuation in the cosmic microwave background can constrain the cosmological model up to redshifts as high as $z \sim 1000$,[29] a more direct measurement of the universe with objects located at very large distances is strongly desired. Fortunately, recent observations have extended the SN Ia sample to sources with redshifts as large as $z = 1.7$. The previous result was confirmed by these high redshift sources, and the analysis revealed that before its acceleration the universe had underwent a period of deceleration.[6] The success in including high redshift SN Ia sources inspires us to make great efforts to

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search for cosmological rulers with much higher redshifts. Based on the $E_p - E_\gamma$ relation found recently in a class of gamma-ray bursts (GRBs),[30] Dai *et al* (2004)[31] assumed that the GRB sources obeying this relation could be used to measure the universe. Of the 12 GRBs contained in their sample, two have redshifts $z > 2$. Soon after their work, the same issue was investigated by many researchers.[32−36] It was found that current GRB data lacking in low redshift sources could be used to marginalize some parameters in their reasonable ranges (see Ref.[35] for a detailed explanation), or they could be employed to constrain the cosmological model with a new Bayesian method.[34] Although the size of the current GRB sample is small and low redshift sources are missed, the idea that some high redshift extragalactic sources other than SN Ia might be employed to determine the cosmological model is quite interesting and promising.

It would be natural to assume the kind of source which could serve as candles to have a distribution of luminosity, which is reasonable due to fluctuation. As discussed in Ref.[37], the uncertainty of a source must include both the systematic uncertainty and the magnitude dispersion. We argue that if there exists a distribution of luminosity of the candles, the expected luminosity itself (or the corresponding deduced luminosity distance) can be different from source to source, which is due to an intrinsic property rather than to the measurement uncertainty. This raises a topic of finding an appropriate method to estimate cosmological parameter ranges with candles with a certain distribution of luminosity.

When employing candles such as SN Ia or GRBs to measure the universe, the confidence level associated with the fit of the theoretical curve to the luminosity distance data is described by a statistic χ^2 which is determined on the assumption that the measurement uncertainty is the only cause of the deviation of the data from the curve. The best fit will be obtained when one reaches the minimum value of χ^2. However, for candles with a certain distribution of luminosity, the deviation of the observed luminosity from the expected curve must be caused by both the measurement uncertainty and the distribution itself. When taking into account the distribution of luminosity, the statistic χ^2 could not be determined if the distribution itself is unknown. The minimizing χ^2 method will not be applicable if the statistic itself cannot be determined.

In the following, we will study how to deal with this matter and investigate what one can expect from the analysis. A corresponding method will be proposed and further illustrated with two samples.

2. The method

In this section, we propose a method to deal with candles with a certain distribution of luminosity when employing them to constrain the cosmological model. As mentioned above, the statistic χ^2 could not be determined for candles with a certain distribution of luminosity if the distribution itself is unknown. Even if the distribution is known, the statistic is still undetermined since there is no way to know the real luminosity of each source. These difficulties lead to two problems. One is that the well-known minimizing χ^2 method is not be applicable without the determination of the statistic. The other is that the probability associated with the statistic χ^2, if we determine it when taking into account the deviation arising from the distribution, is not available (since the real luminosity of each source is unknown).

It is known that the convolution of two Gaussian curves is still a Gaussian curve with a width that is given by the quadratic sum of the widths of the two original distributions. That is $\sigma^2 = \sigma_1^2 + \sigma_2^2$, where σ_1^2 and σ_2^2 are the variances of the two Gaussian functions concerned and σ^2 is variance of the resulting Gaussian function.

Let us consider the deviation of an observed luminosity distance modulus, $\mu_{\rm ob}$, of a source from the real value of the quantity, $\mu_{\rm th}$, which follows

$$\begin{aligned}&(\mu_{\rm ob} \pm \sigma_{\rm ob}) - \mu_{\rm th}(z; H_0, \Omega_{\rm m}, \Omega_\Lambda)\\ =&(\mu_{\rm ob} \pm \sigma_{\rm ob}) - [\mu_{\rm th,0}(z; H_0, \Omega_{\rm m}, \Omega_\Lambda) + \Delta\mu_{\rm th}],\end{aligned} \tag{1}$$

where $\sigma_{\rm ob}$ is the measurement uncertainty of $\mu_{\rm ob}$, $\mu_{\rm th,0}$ is the central value of $\mu_{\rm th}$, which is the real value of the modulus expected in the case where there is no distribution of luminosity in candles, and $\Delta\mu_{\rm th}$ represents the deviation of $\mu_{\rm th}$ from $\mu_{\rm th,0}$. Suppose that the luminosity distribution of candles is narrow enough so that the absolute value of the deviation of $\mu_{\rm th}$ from $\mu_{\rm th,0}$, $|\Delta\mu_{\rm th}|$, is small. According to the error transform formula, the uncertainty of $\mu_{\rm ob}$ relative to $\mu_{\rm th,0}$ can be determined by

$$\sigma_{\rm ob,0} = \sqrt{\sigma_{\rm ob}^2 + (\Delta\mu_{\rm th})^2}. \tag{2}$$

Relative to the expected central moduli, the χ^2 statis-

tic of a sample of the candles could be determined by

$$\chi^2 = \sum_i \frac{[\mu_{\rm ob,}i - \mu_{\rm th,0,}i(z; H_0, \Omega_{\rm m}, \Omega_\Lambda)]^2}{\sigma^2_{{\rm ob},i} + (\Delta\mu_{{\rm th},i})^2}. \quad (3)$$

Note that, in the case of SN Ia, $\sigma^2_{{\rm ob},i}$ should be replaced by $\sigma^2_{{\rm ob},i} + \sigma^2_v$, where $\sigma_{{\rm ob},i}$ is the uncertainty in the individual distance moduli deduced from the empirical relation between the light-curve shape and luminosity, and σ_v is the uncertainty associated with the dispersion in supernovae redshift (transformed to units of distance moduli) due to peculiar velocities.[6]

It seems that one might be able to evaluate the χ^2 statistic from Eq.(3). But unfortunately this is not true, because there is no way to know $\Delta\mu_{{\rm th},i}$. However, on condition that the distribution of candles is narrow, we can estimate $\Delta\mu_{{\rm th},i}$ by using the width of the distribution. Let $\widetilde{\sigma}_{\rm dis}$ be the width of the distribution of $\mu_{\rm th}/\mu_{\rm th,0}$ that is called the intrinsic distribution of the relative luminosity distance moduli (note that $\mu_{\rm th}/\mu_{\rm th,0}$ should, of course, become unity when there is no deviation of $\mu_{\rm th}$ from $\mu_{\rm th,0}$), and assume $|\Delta\mu_{{\rm th},i}| \simeq \widetilde{\sigma}_{\rm dis}\mu_{{\rm th,0},i}$, then the χ^2 statistic will be estimated by

$$\chi^2 \simeq \sum_i \frac{[\mu_{\rm ob,}i - \mu_{\rm th,0,}i(z; H_0, \Omega_{\rm m}, \Omega_\Lambda)]^2}{\sigma^2_{{\rm ob},i} + \widetilde{\sigma}^2_{\rm dis}\mu^2_{{\rm th,0},i}}. \quad (4)$$

As long as $\widetilde{\sigma}_{\rm dis}$ is known, the χ^2 statistic is then available according to formula (4). For any kind of candle, quantity $\widetilde{\sigma}_{\rm dis}$ could be estimated when the sample employed is large enough and the measurement uncertainty $\sigma_{\rm ob}$ is small enough and when the cosmological model is fixed. Obviously, this could not be realized at present since the cosmological model itself is currently a target to be pursued and for interesting candles the measurement uncertainty is always quite large. But this cannot prevent one from estimating the limits of $\widetilde{\sigma}_{\rm dis}$. As the deviation of $\mu_{\rm ob}$ from $\mu_{\rm th,0}$ is caused by both the distribution of $\mu_{\rm th}$ and the measurement uncertainty of $\mu_{\rm ob}$ itself, $\widetilde{\sigma}_{\rm dis}$ must be smaller than $\widetilde{\sigma}_{\rm dis,max}$, where $\widetilde{\sigma}_{\rm dis,max}$ is the width of the distribution of $\mu_{\rm ob}/\mu_{\rm th,0}$, which is determined by $\widetilde{\sigma}_{\rm dis,max} = \sqrt{\sum_i (\mu_{{\rm ob},i}/\mu_{{\rm th,0},i} - 1)^2/(N-1)}$, with N being the size of the sample. Let us overestimate the effect of the measurement uncertainty in the opposite way. Within the range of $[\mu_{{\rm ob},i} - \sigma_{{\rm ob},i}, \mu_{{\rm ob},i} + \sigma_{{\rm ob},i}]$ we take the value that is the closest to $\mu_{{\rm th,0},i}$ as $\mu^*_{{\rm ob},i}$. Obviously, the distribution of $\mu^*_{\rm ob}/\mu_{\rm th,0}$ would be narrower than the distribution of $\mu_{\rm th}/\mu_{\rm th,0}$ since the deviation caused by the measurement uncertainty is over subtracted. We take the width of the distribution of $\mu^*_{\rm ob}/\mu_{\rm th,0}$ as $\widetilde{\sigma}_{\rm dis,min}$, which is calculated with $\widetilde{\sigma}_{\rm dis,min} = \sqrt{\sum_i (\mu^*_{{\rm ob},i}/\mu_{{\rm th,0},i} - 1)^2/(N-1)}$. Clearly, $\widetilde{\sigma}_{\rm dis}$ must be larger than $\widetilde{\sigma}_{\rm dis,min}$. With these two quantities we have

$$\chi^2_{\rm min} \simeq \sum_i \frac{[\mu_{\rm ob,}i - \mu_{\rm th,0,}i(z; H_0, \Omega_{\rm m}, \Omega_\Lambda)]^2}{\sigma^2_{{\rm ob},i} + \widetilde{\sigma}^2_{\rm dis,max}\mu^2_{{\rm th,0},i}} \quad (5)$$

and

$$\chi^2_{\rm max} \simeq \sum_i \frac{[\mu_{\rm ob,}i - \mu_{\rm th,0,}i(z; H_0, \Omega_{\rm m}, \Omega_\Lambda)]^2}{\sigma^2_{{\rm ob},i} + \widetilde{\sigma}^2_{\rm dis,min}\mu^2_{{\rm th,0},i}}. \quad (6)$$

Since $\widetilde{\sigma}_{\rm dis,min} < \widetilde{\sigma}_{\rm dis} < \widetilde{\sigma}_{\rm dis,max}$, one obtains $\chi^2_{\rm min} < \chi^2 < \chi^2_{\rm max}$. With Eqs.(5) and (6), one can calculate the corresponding probability associated with the χ^2 statistic and confine the conventional confidence contour. In this way, cosmological parameters would be constrained. With this estimating method, the first problem is greatly eased and the second is solved.

3. Application

Let us consider a GRB sample. The sample was presented and studied by Xu *et al*[35] and Xu[38] (the XDL GRB sample), which contains 17 GRBs. As suggested by Ghirlanda *et al*[32] the scatter of the data points of their GRB sample around the correlation of $E_{\rm p} - E_\gamma$ found recently[30] is of a very small order.

To check whether the data of the XDL GRB sample are consistent with those in the case of no scatter beyond the measurement errors in terms of statistics, the simplest method is to calculate the mean of the deviation of the deduced luminosity distance moduli from the expected one of the sample and then compare it with the average of the measurement error. The mean of the deviation is defined as $\sigma_{\rm dev} = \sqrt{\sum_i ((\mu_{{\rm ob},i} - \mu_{{\rm ex},i})/\mu_{{\rm ex},i})^2/(N-1)}$, where $\mu_{\rm ex}$ is the expected value of μ, while the average of the measurement error is calculated with $\sigma_{\rm err} = \sqrt{\sum_i (\sigma_{{\rm ob},i}/\mu_{{\rm ex},i})^2/(N-1)}$. (Note that, as red shifts of these sources are not the same, we consider the relative values.) From the XDL sample we obtain $\sigma_{\rm dev} = 0.0122$ and $\sigma_{\rm err} = 0.0116$, where we adopt $(\Omega_{\rm m}, \Omega_\Lambda, h) = (0.29, 0.71, 0.65)$. It shows that the deviation is slightly larger than the measurement error. (Ignoring the slight difference between the two quantities, the result confirms what was suggested by Ghirlanda *et al*[30,32]). Taking $\mu_{\rm th,0}$ as $\mu_{\rm ex}$ adopted

here, one finds that $\sigma_{\rm dev}$ is identical with $\widetilde{\sigma}_{\rm dis,max}$ determined in the last section. Thus, for the XDL sample, $\widetilde{\sigma}_{\rm dis} < 0.0122$, suggesting that the distribution, if it exists, will be quite narrow. Another approach involves a simulation analysis. We assume that there is no intrinsic distribution of the deduced luminosity distance moduli, and thus the deviation observed is due to the measurement uncertainty. Obviously, under this assumption the distribution of $\mu_{\rm ob}/\mu_{\rm ex}$ should peak at unity. According to the null hypothesis, the observed value of $\mu_{\rm ex}$ for each source is obtained by chance from a parent population of $\mu'_{\rm ob}$ whose distribution is a Gaussian distribution with the measurement uncertainty serving as the width of the Gaussian distribution. For each source one can create a $\mu'_{\rm ob}$ via simulation as long as the expected value $\mu_{\rm ex}$ and the measurement uncertainty are known. In this way, from the 17 $\mu_{\rm ex}$ and the corresponding measurement uncertainties, one can create a set of 17 $\mu'_{\rm ob}$ data by a Monte–Carlo simulation and then obtain a set of 17 $\mu'_{\rm ob}/\mu_{\rm ex}$ data. We perform simulation 100 times, and obtain 100 sets of 17 $\mu'_{\rm ob}/\mu_{\rm ex}$ data. Combining these 100 sets we have a large sample with its size being 1700. The deviation of the relative simulated luminosity distance moduli from the expected one (the unity) is defined as $\sigma'_{\rm dev} = \sqrt{\sum_i(\mu'_{{\rm ob},i}/\mu_{{\rm ex},i}-1)^2/(N-1)}$. Note that $\sigma'_{\rm dev}$ can be written as $\sigma'_{\rm dev} = \sqrt{\sum_i((\mu'_{{\rm ob},i}-\mu_{{\rm ex},i})/\mu_{{\rm ex},i})^2/(N-1)}$, which can thus be directly compared with $\sigma_{\rm dev}$, the deviation of the observed data defined above. From the XDL sample we obtain $\sigma'_{\rm dev} = 0.0113$, which suggests that the deviation associated with observation, denoted by $\sigma_{\rm dev}$, is also slightly larger than that expected from the measurement uncertainties. Two methods come to almost the same results, suggesting that there might be an intrinsic distribution of the relative luminosity distance moduli of the XDL sample, although it would be quite narrow (as the differences between $\sigma'_{\rm dev}$ and $\sigma_{\rm dev}$ and between $\sigma_{\rm err}$ and $\sigma_{\rm dev}$ are small).

To illustrate how to apply the method proposed above to dealing with the data with intrinsic distributions, we assume in the following that there is a distribution of the true value of the deduced relative luminosity distance moduli for the XDL sample, although the distribution, if it exists, might be very narrow (see what is suggested above). For the sake of comparison, we perform the fit with three χ^2 statistics. One is the conventional χ^2 which could be determined by the formula (3) when taking $\Delta\mu_{{\rm th},i} = 0$. The other two are $\chi^2_{\rm min}$ and $\chi^2_{\rm max}$ which are determined by Eqs.(5) and (6) respectively. Each χ^2 statistic is calculated with the XDL GRB sample in many tries. In each try, we adopt a set of parameters, and based on these parameters we deduce both the observed and theoretical luminosity distance moduli. With these moduli and the measurement uncertainties, we are able to evaluate $\widetilde{\sigma}_{\rm dis,min}$ and $\widetilde{\sigma}_{\rm dis,max}$ (see what is proposed in the last section), and then the corresponding χ^2 statistic would be well determined ($H_0 = 65{\rm km\cdot s^{-1}\cdot Mpc^{-1}}$ is adopted throughout this paper). For each χ^2, the best fit will be obtained when the smallest value is reached.

Displayed in Fig.1 and Fig.2 are the Hubble diagram and the confident contour plot of the XDL GRB sample. As concluded previously by other researchers,[32,33,35] currently, employing GRB samples alone cannot tightly constrain the cosmological model. Figures 1 and 2 show that the parameter ranges are indeed poorly constrained even there is no intrinsic distribution of the relative luminosity distance moduli (see solid lines in Fig.2). Taking into account an intrinsic distribution of the moduli leads to much poorer results. This indicates that if there indeed exists an intrinsic distribution of the moduli, the effect arising from the distribution should not be ignored.

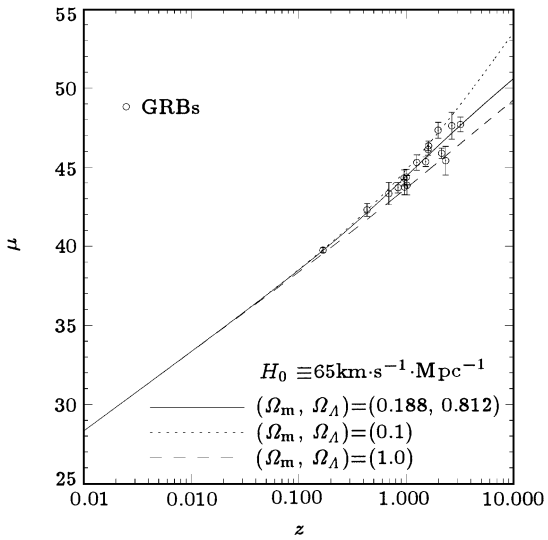


Fig.1. Hubble diagram of the XDL GRB sample, where the two edge curves (the dotted and dashed lines) of the flat universe are also represented.

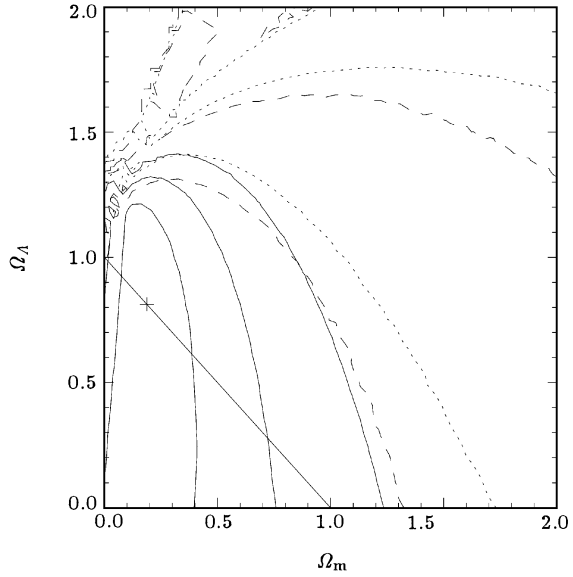


Fig.2. Confident contour plot for the XDL GRB sample, where three dashed lines varying from the innermost to the outmost represent the 1 σ, 2 σ and 3 σ levels of confidence calculated with the statistic $\chi^2_{\rm min}$ respectively, while the three dotted lines represent those associated with the statistic $\chi^2_{\rm max}$ respectively. For the sake of comparison, the confidence levels calculated without considering the distribution of $\widetilde{\mu}_{\rm ob}/\mu_{\rm th}$ are also plotted (the solid lines). The straight line denotes the flat universe and the plus represents the best fit parameters of the flat universe obtained by the conventional minimizing χ^2 method.

Shown in Table 1 are the best fit cosmological parameters for the three kinds of universe, obtained by applying the minimizing χ^2 method to the three χ^2 statistics, where the 1σ errors are estimated from the corresponding 1σ contours in Fig.2. As shown in Fig.2, the 1σ contours are not closed within the ranges of the plots. This leads a poor constraint to the limits of the parameters. Some limits are therefore not able to be determined, which are denoted by '?' in Table 1.

The fact that the parameter ranges are poorly constrained (even when the intrinsic distribution of the relative luminosity distance moduli is ignored) probably is due to the lack of low redshift sources, as it is already known that low redshift sources are important when employing a GRB sample to constrain the cosmological parameters.[34] We thus follow what was done previously[32] to combine an SN Ia sample and the XDL sample to constrain the cosmological model. The SN Ia sample employed is that presented in Ref.[6] (the so-called gold set of SN Ia) which contains 157 sources (where many low redshifts sources are included). In the same way and for the same reason we apply the minimizing χ^2 method to the three χ^2 statistics to find the best fit cosmological parameters. Note that, unlike what is shown in the case of the GRB sample, the deduced luminosity distance moduli of the SN Ia sources do not depend on the adopted cosmological parameters.

Table 1. Best-fit parameters obtained by the least square method.

Sample	Universe	$(\Omega_{\rm m}, \Omega_\Lambda, \chi^2_{0,\nu})$	$(\Omega_{\rm m}, \Omega_\Lambda, \chi^2_{{\rm max},\nu})$	$(\Omega_{\rm m}, \Omega_\Lambda, \chi^2_{{\rm min},\nu})$
SN+GRB	flat	$(0.283^{+0.0314}_{-0.0288}, 0.717, 197.9)$	$(0.288^{+0.0134}_{-0.0201}, 0.712, 193.1)$	$(0.288^{+0.0138}_{-0.000375}, 0.712, 185.4)$
SN+GRB	open	$(0.368^{+0.127}_{-0.114}, 0.857^{+0.371}_{-0.170}, 197.0)$	$(0.281^{+0.0201}_{-0.0201}, 0.717^{+0.0198}_{-0.0353}, 193.2)$	$(0.281^{+0.00669}_{-0.0134}, 0.717^{+0.0100}_{-0.0201}, 185.4)$
SN+GRB	closed	$(0.281^{+0.0334}_{-0.0469}, 0.717^{+0.0296}_{-0.0804}, 198.0)$	$(0.428^{+0.147}_{-0.161}, 0.942^{+0.226}_{-0.246}, 191.3)$	$(0.441^{+0.147}_{-0.0201}, 0.967^{+0.226}_{-0.0351}, 183.1)$
GRB	flat	$(0.188^{+0.200}_{-0.114}, 0.812, 19.69)$	$(0.188^{+1.176}_{-?}, 0.812, 15.14)$	$(0.188^{+1.539}_{-?}, 0.812, 7.43)$
GRB	open	$(0.187^{+0.221}_{-?}, 0.682^{+0.221}_{-?}, 19.67)$	$(0.187^{+?}_{-?}, 0.256^{+?}_{-?}, 14.96)$	$(0.154^{+?}_{-?}, 0.391^{+?}_{-?}, 7.40)$
GRB	closed	$(0.187^{+0.201}_{-0.114}, 0.817^{+0.401}_{-0.206}, 19.67)$	$(0.187^{+1.177}_{-?}, 0.817^{+0.551}_{-?}, 15.14)$	$(0.187^{+1.54}_{-?}, 0.817^{+0.59}_{-?}, 7.44)$

[a] $\chi^2_{0,\nu}$ is the reduced χ^2 calculated from Eq.(4) when assigning $\tilde{\sigma}_{\rm dis}$=0.

It is known that, in estimating the deduced luminosity distance moduli of the SN Ia sources, deviations caused by different magnitudes of the peak luminosity of the sources have been checked. Indeed, we find that the distribution of the relative luminosity distance moduli of the SN Ia sample is very narrow (the figure is omitted). This suggests that, if it still exists (possibly caused by the small deviation from the adopted empirical relation between the light-curve shape and luminosity), the intrinsic distribution must be extremely narrow. Thus we ignore the intrinsic distribution of the relative luminosity distance moduli of the SN Ia sample, and consider only $\widetilde{\sigma}_{\rm dis,min}$ and $\widetilde{\sigma}_{\rm dis,max}$ for the GRB sample when we calculate the corresponding $\chi^2_{\rm min}$ and $\chi^2_{\rm max}$ for the combined sample (including the XDL GRB sample and the gold SN Ia sample).

Shown in Fig.3 and Fig.4 are the Hubble diagram and the confident contour plot of the combined sample. One finds that, including the SN Ia sample significantly improves the constraint of the ranges of cosmological parameters. Once more, the result shows that taking into account the intrinsic distribution of the relative luminosity distance moduli leads to a poorer con-

straint. The effect is still obvious (although it is less obvious than that adopting the GRB sample alone) and therefore should not be neglected. The resulting best fit cosmological parameters as well as their 1σ errors are listed in Table 1.

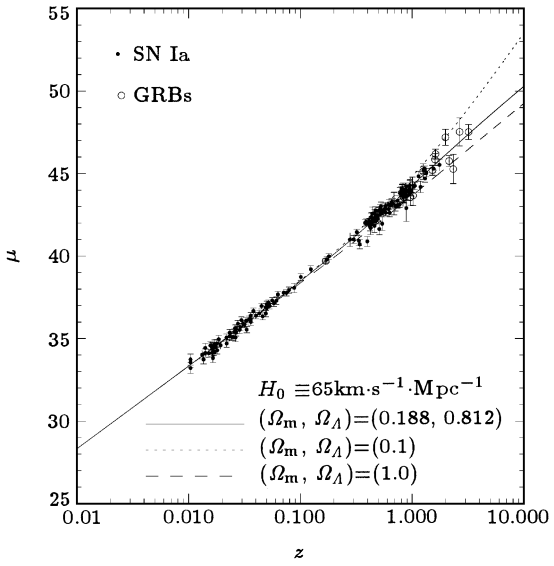


Fig.3. Hubble diagram of the combined sample, where the open circle represents the XDL GRB sample and the solid circle denotes the gold SN Ia sample. Symbols of lines are the same as those denoted in Fig.1.

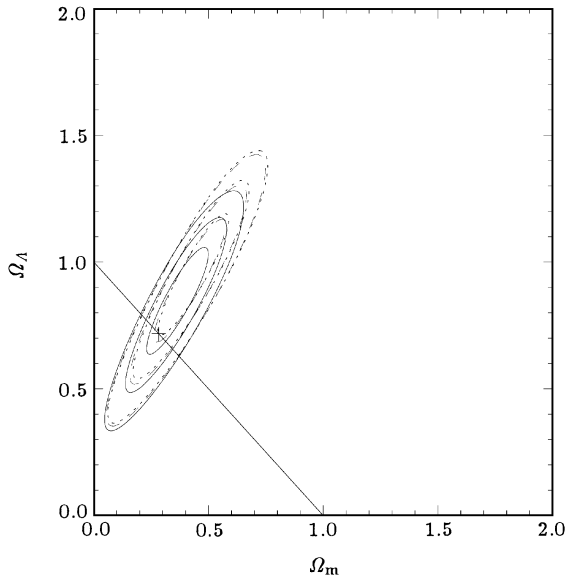


Fig.4. Confident contour plot for the combined sample, where the symbols are the same as those denoted in Fig.2.

4. Discussion and conclusions

The effect of the intrinsic distribution of cosmological candles is investigated in this paper. Due to fluctuation, it is natural that a property (say, the luminosity) of sources serving as a cosmological candle might form a distribution and scatter around a central value. If the distribution does exist, the statistic χ^2 cannot be determined since the distribution itself is unclear and the real value of the property for each source is unknown. However, when the distribution is narrow, the deviation of the observed modulus of each source from the central value could be estimated within a ceratin range. We accordingly determine a lower and upper limits of χ^2, χ^2_{min} and χ^2_{max}, to estimate cosmological parameters via the conventional minimizing χ^2 method. The confidence contours of these two χ^2 statistics can then be plotted in the conventional way, and with these curves the ranges of the parameters could be determined as long as a confidence level is assigned.

With this method, a sample bearing a relatively small width of the intrinsic distribution of the deduced relative luminosity distance moduli would be applicable to constraining the cosmological parameters. To illustrate this method we employ a GRB sample alone and later combine this GRB sample with the gold SN Ia sample, assuming that this GRB sample (the XDL sample) has an intrinsic distribution of the deduced relative luminosity distance moduli while the SN Ia sample has not. The analysis suggests that: a) the effect of the intrinsic distribution of the relative luminosity distance moduli is obvious and therefore should not be neglected if the distribution itself does exist; b) taking into account this effect would lead to a poorer constraint of the ranges of cosmological parameters. This indicates that in the attempt of constraining the cosmological model with GRB samples, the results tend to be worse than what was previously anticipated if the mentioned intrinsic distribution exists, although the distribution is very narrow.

As revealed recently by Wang *et al*,[39] there is a clear evidence for a tight linear correlation between peak luminosities of SN Ia and their $B-V$ colours in about 12 days after the B maximum. They found that this empirical correlation allows one to reduce scatters in estimating their peak luminosities from about 0.5 mag to the levels of 0.18 mag and 0.12 mag in the V and I bands, respectively. We wonder whether taking into account this effect can reduce the measurement uncertainty of the luminosity distance of the SN Ia sources. If so, the ranges of the cosmological parameters might be well constrained as compared with Fig.4 (this will be investigated later).

As encountered in other cases, our method suffers from possible evolution of candles. Quite recently, Fir-

mani *et al*[40] found evidence supporting an evolving luminosity function of long GRBs, where the luminosity scales as $(1+z)^{1.0\pm0.2}$. It is unclear whether the corrected gamma-ray energy, from which the luminosity distance moduli of the adopted GRB sample are deduced, evolves with redshift. If so, the question whether the GRB sample can still be used to constrain the cosmological model should be answered. This deserves a detailed investigation. (It could be done only when the size of the sample is large enough).

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