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A velocity-difference-separation model for car-following theory*

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We introduce a velocity-difference-separation model that modifies the previous models in the literature. The improvement of this new model over the previous ones lies in that it not only theoretically retains many strong points of the previous ones, but also performs more realistically than others in the dynamical evolution of congestion. Furthermore, the proposed model is investigated with analytic and numerical methods, with the finding that it can demonstrate some complex physical features observed in real traffic such as the existence of three phases: free flow, synchronized flow, and wide moving jam; sudden flow drop in flow-density plane; and traffic hysteresis in transition between the free and the synchronized flow.

Keywords: car-following models, inter-vehicle separation, traffic flow
PACC: 6470

1. Introduction

Improvements in modern computing power have made it possible to simulate the flow of large numbers of vehicles on quite extensive road networks by describing the motion of each individual vehicle. This has dramatically increased interest in the mathematical models that describe the dynamics of discrete groups of road vehicles[1−6], namely car-following models. Car-following models describe the dynamical behaviour of a line of closely spaced road vehicles travelling in a single lane without overtaking. This makes them particularly suitable as mathematical models for studying the motion of congested traffic. Therefore, car-following models are not only of great importance with regard to an autonomous cruise control system, but they have also emerged as important evaluation tools for intelligent transportation system (ITS) strategies since the early 1990s.

In this paper, a new car-following model performing more realistically in the dynamical evolution of congestion, which is induced by a small disturbance, is introduced and discussed. Evidence shows that this new model is theoretically an improvement over the full velocity difference model (FVDM), while retaining many strong points in the previous ones. Moreover, it demonstrates and explains some complex physical phenomena observed recently in real traffic flow.

The paper is organized as follows. We review the performance of the previous models on the evolution of congestion induced by a small perturbation, and present the modified car-following model in Section 2. In Section 3, both analytic and numerical methods of this model are used to analyse some observed physical features which exist in real traffic, we shall see that the proposed model works well for our purpose. Finally we conclude the paper in Section 4.

2. Discussion of previous models and modification of FVDM

As basic and important components of microscopic approaches, some car-following theories have been given considerable attention in recent years. In 1995, Bando *et al* proposed an important car-following model called the 'optimal velocity model' (OVM) [7] in which the acceleration of the nth vehicle at time t is determined by the difference between the actual velocity, $v_n(t)$, and an optimal velocity $V(\Delta x_n(t))$, which depends on the headway $\Delta x_n(t)$ to the car in front:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = k[V(\Delta x_n(t)) - v_n(t)], \qquad (1)$$

where k is a positive real number. Helbing and Tilch[8] carried out a calibration of OVM with respect to the empirical data and suggested an optimal velocity function (OVF) as

$$V(\Delta x_n(t)) = V_1 + V_2 \tanh[C_1(\Delta x_n(t) - l_c) - C_2], \quad (2)$$

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where l_c is the length of the vehicles, which can be taken as 5m in simulations. The resulting optimal parameter values are $k = 0.85\text{s}^{-1}$, $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13\text{m}^{-1}$, and $C_2 = 1.57$.

The comparison with field data shows that OVM encountered the problems of too high acceleration and unrealistic deceleration (see Figs.2–4 in Ref.[8]). Therefore, Helbing and Tilch[8] proposed a generalized force model (GFM) to solve the problem. One term is added to the right-hand side of Eq.(1) and the formula of GFM them reads

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = k[V(\Delta x_n(t)) - v_n] + \lambda\Theta(-\Delta v)\Delta v, \quad (3)$$

where $\Delta v = v_{n-1}(t) - v_n(t)$ and Θ is the Heaviside function which is defined as $\Theta(x)x = (x+|x|)/2$. Furthermore, they also performed a calibration to this modified model and the result shows that in GFM, $k = 0.41\text{s}^{-1}$, which is much smaller than that in OVM.

In 2001, Jiang *et al* [9] conducted a detailed analysis of GFM and found out that although the new proposed model has better agreement with the field data than OVM, it cannot predict correctly the delay time of car motion and the kinematic wave speed at jam density. They thought it was due to that GFM does not take the effect of positive Δv on traffic dynamics into account, and they suggested a full velocity difference model (FVDM)[9] which includes the positive Δv factor and obtained a more systematic model as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = k[V(\Delta x_n(t)) - v_n] + \lambda\Delta v. \quad (4)$$

Applying the linear stability analysis theory, they analysed the instability of FVDM and obtained that only when the condition

$$V(\Delta x_n(t)) < \frac{k}{2} + \lambda \quad (5)$$

is met, the traffic is stable and robust when small disturbance is added to the uniform traffic flow.

Most of the earlier works investigated the property of traffic flows in circuit and the boundary condition in simulation was assumed periodic. Now we carry out the simulation of traffic congestion induced from FVDM to check the above analysis, still taking Eq.(2) for the OVF and the parameters are the same as before. For the technical reason of numerical simulation, we take car number $N = 100$, the circuit length $L = 1500$ m. The initial small disturbance is given as follows:

$$x_1(0) = 1\text{m}; x_n(0) = (n-1)L/N, n = 2, \ldots, N, \quad (6)$$

$$\dot{x}_n(0) = V(L/N), n = 1, \ldots, N. \quad (7)$$

The realistic value of λ was investigated by us in real traffic and was found to range between 0.3 and 0.7. First we consider the case of $\lambda = 0.4$. Substituting the values of the parameters into condition (5) we learn that the uniform flow including noise is unstable. Figure 1(a) presents the snapshots of velocity configuration of all vehicles at 200s and 10000s. We clearly see from the figure that vehicle velocities tend to locate on the two typical values as time develops, but at the same time we clearly observe the appearance of the negative velocity at low-density region, which is very unrealistic and unallowed in real traffic. Furthermore, when the traffic flow becomes stationary after sufficiently large time, the motion of all cars forms a loop as shown in Fig.1(b). Changing the value of λ, loops of different size can be obtained in Fig.1(b). Here we can see that the above unrealistic phenomena appear if the condition of $\lambda < 0.5$ is satisfied.

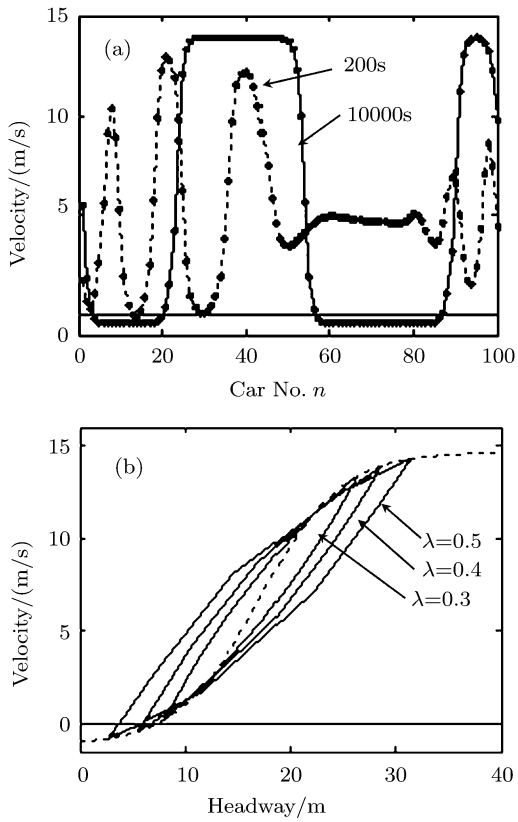


Fig.1. (a) A snapshot of velocities of all cars at $t = 200$s, and 10000s. Circle mark the velocities of the cars. (b) loops for FVDM at different values of λ. The dotted curve represents OVF.

Why does FVDM not behave well in this aspect? We believe it may be because the second term in the right side of (4) makes no allowance of the effect of the inter-car spacing independently of the relative ve-

locity. We think the separation between cars plays an important role in traffic dynamics. The acceleration controlled by a driver is different when he responds to two different stimuli which have the same relative velocity but different headway. Observations in real traffic verify this.

According to the above analysis, we modify the FVDM by taking the separation between cars into account and the dynamics equation becomes

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = k[V(\Delta x_n(t)) - v_n] + \lambda\Theta(\Delta v)\Delta v(1+\tanh(C_1(\Delta x_n(t) - l_c) - C_2))^3 + \lambda\Theta(-\Delta V)\Delta V(1-\tanh(C_1(\Delta x_n(t) - l_c) - C_2))^3. \quad (8)$$

Since this model takes velocity difference dependent on separation into account, we call it a velocity-difference-separation model (VDSM). Note that FVDM may be rewritten as

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = k[V(\Delta x_n(t)) - v_n] + \lambda\Theta(\Delta v)\Delta v + \lambda\Theta(-\Delta v). \quad (9)$$

Compared with Eq.(9), VDSM differs in the expression of interaction term, where VDSM assumes Δv dependent on $\Delta x_n(t)$ contributes to vehicle interaction.

In the following we choose the value of $\lambda = 0.3$ and apply VDSM to carry out the same simulation as aforementioned. We can also obtain a snapshot of velocities of all cars at $t = 10000$s and draw different size of loops by changing the value of λ, the results are shown in Fig.2(a) and 2(b). From Fig.2(a) we can see that four clusters of congestion are formed even from such a tiny disturbance existing in the almost uniform vehicle distribution, but it is more important that the unrealistic negative velocity never appears and the desired values of λ never fall into a negative velocity regime (see Fig.2(b)). From this point of view, VDSM describes the traffic dynamics more exactly, which verifies that the modification in VDSM is reasonable and realistic.

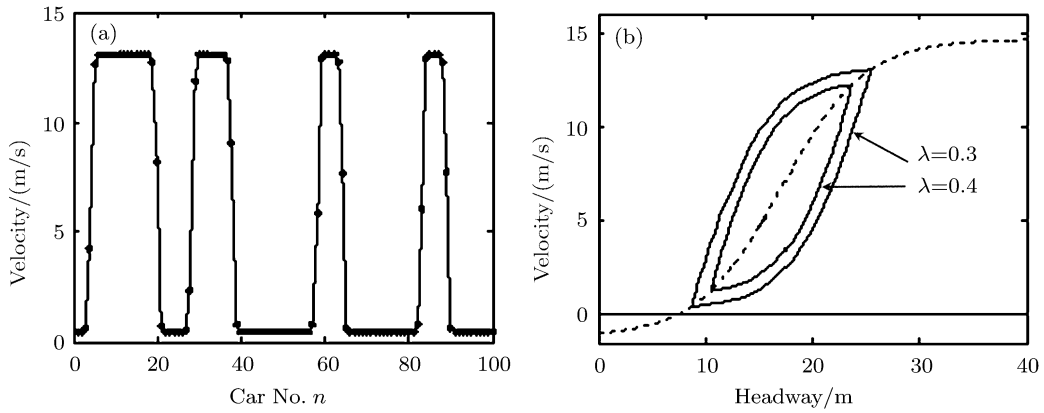


Fig.2. (a) A snapshot of velocities of all cars at 10000s for VDSM, circles represent the velocities of the cars. (b) Loops for the new model at different values of λ. The dotted curve represents OVF.

Next we examine whether the model can predict realistic delay time and the kinematic wave speed just as FVDM. If velocities of two successive cars are given by $v(t)$ and $v(t-T)$ respectively, the delay time of car motion is T, and the kinematic wave speed v_k is equal to the quotient of the headway 7.4m divided by T. As Bando *et al* [10] pointed out, the observed T is of the order of 1s, and Del Castillo and Benizez[11] indicated that v_k ranges between 17 and 23 km/h. Then numerical simulation is carried out to predict these parameters by studying the motion of cars starting from a traffic signal. First a traffic signal is red and all cars are waiting with a headway of 7m, at which the OVF (2) becomes zero. At time $t = 0$, the signal changes to green and cars start. Figure 3 shows the velocities of several cars in a queue for the cases of $\lambda = 0.4$. It can be seen that cars with large number (seventh or more) behave almost in the same manner as its pre-

ceding car and can estimate the delay time T and v_k from them. Table 1 shows the delay time and kinematic wave speed of car motion for various models. From Table 1, we can observe that T of VDSM, which is approximate to that of FVDM, is the most exact of the three models. And v_k falls into the desired range. From this aspect, VDSM retains the strong point that the FVDM modifies the GFM.

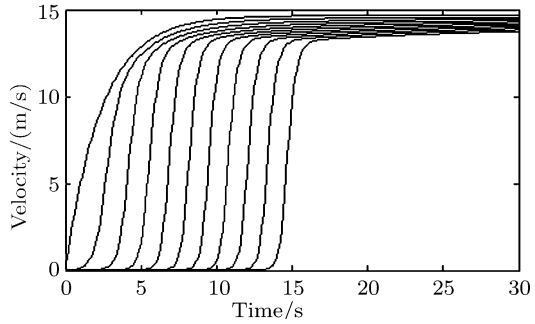


Fig.3. Motion of cars 1–11 for VDSM, each curve shows the velocity of one car.

Table 1. Delay times of car motions from a traffic signal and kinematic wave speed at jam density in different models, the data of GFM, FVDM are reproduced from Ref.[9].

Model	T/s	v_k/(km/h)
GFM ($k = 0.41s^{-1}$)	2.2	12.11
FVDM($k = 0.41s^{-1}$)	1.4	19.03
VDSM($k = 0.41s^{-1}$)	1.3	20.49

3. Observed physical features reproduced by VDSM

Recent observations[12−16] show that traffic flow demonstrates complex physical phenomena, among which are: (1) The existence of three phases: free flow, synchronized flow, and wide moving jam. (2) A prominent feature in traffic flow, the sudden flow drop, which owes to large oscillations in vehicle velocity and headway in the synchronized flow regime induced by a small perturbation. (3) Traffic hysteresis between the free and the synchronized flow, i.e. the transition from free traffic to synchronized flow occurs at a higher density while the inverse transition occurs at a lower density.

First we explore whether the VDSM can demonstrate the existence of three traffic phases by carrying out the previous same circuit simulation of dynamic evolution of congestion. From the simulations we can get the flow-density relation (or the fundamental diagram) of Eq.(8). Let us take car number $N = 100$, increase the circuit length L gradually, and set a same initial disturbance as before. The simulation result is shown in Fig.4 as the solid line, while the dashed line is the fundamental diagram for the homogeneous flow.

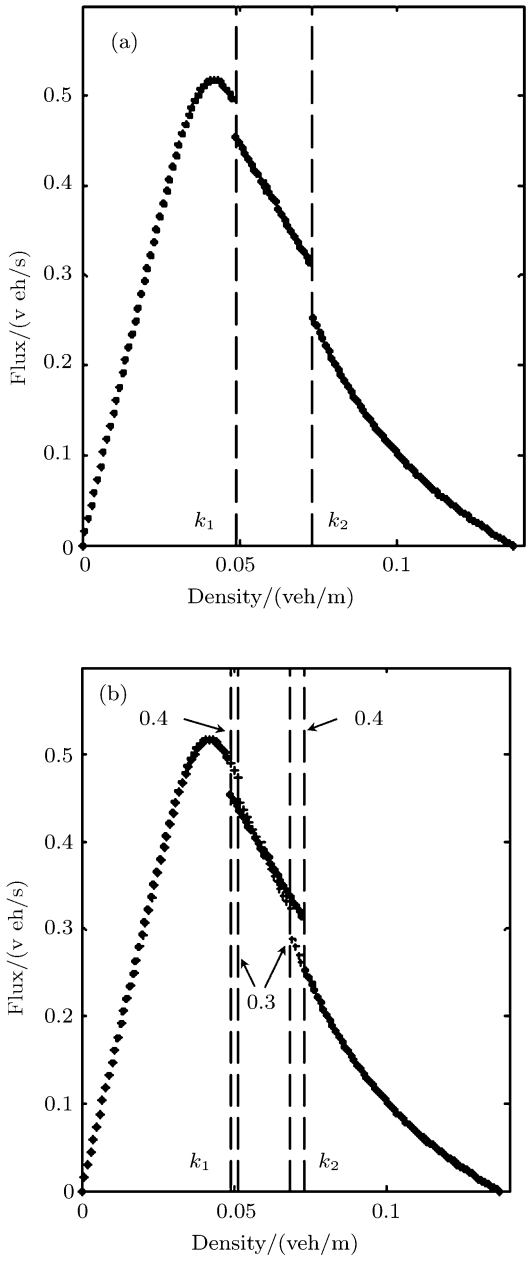


Fig.4. (a)Fundamental diagram plot for $\lambda = 0.3$. Dashed line is the homogeneous solution. (b) Fundamental diagram plot for $\lambda = 0.3$ (dot) and $\lambda = 0.3$ (cross).

It is indicated in Fig.4(a) that when the density k is smaller than a critical value k_1 or greater than another critical value k_2 the flow is the same as the homogeneous solution flow, but when the density falls into the intermediate range ($k_1 < k < k_2$) , the measured flow is considerably lower or higher than the homogeneous flow. Therefore, we can define three regimes in

traffic flow: free flow regime ($k < k_1$), synchronized flow regime ($k_1 < k < k_2$), and coexisting regime of wide moving jam and synchronized flow or free flow ($k > k_2$). In addition, it also should be noted that the synchronized flow range is different for different values of λ , and $(k_2 - k_1)$ increases with decreasing of sensitivity as shown in Fig.4(b). Above findings are supported by the analytical results shown below.

In order to check the above analysis, we analyse the linear stability of an N-car system on a circular lane of length L. It is apparent that the following solution of steady state flow satisfies the dynamical equation (8):

$$x_n^{(0)}(t) = V(b)t + nb,\ b = L/N. \tag{10}$$

To see whether the solution (10) is stable or not, we add a small disturbance $y_n(t) = \mathrm{e}^{\mathrm{i}kn+zt}$

$$x_n(t) = x_n^{(0)}(t) + y_n(t). \tag{11}$$

Neglecting the higher order terms of y_n , the linearized equation is obtained as

$$z^2 + (k - \lambda H(\mathrm{e}^{-\mathrm{i}k} - 1)z - kV'(b)(\mathrm{e}^{-\mathrm{i}k} - 1)) = 0. \tag{12}$$

For simplicity, we let

$$H = \Theta(b > l_\mathrm{c} + \frac{C_2}{C_1})(1 + \tanh(C_1(b - l_\mathrm{c}) - C_2))^3 + \Theta(b \le l_\mathrm{c} + \frac{C_2}{C_1})(1 - \tanh(C_1(b - l_\mathrm{c}) - C_2))^3.$$

The solution of Eq.(12) is:

$$z = z_1(\mathrm{i}k) + z_2(\mathrm{i}k)^2 + \cdots, \tag{13}$$

so the stability condition for VDSM is given as

$$V'(b) < \frac{k}{2} + \lambda H. \tag{14}$$

A schematic phase diagram is shown in Fig.5. In this figure we see three regimes of stability/instability of the homogeneous flow solution for VDSM. If $b > b_2$ (free flow) or $b < b_1$ (coexisting regime of wide moving jam and synchronized flow or free flow) the homogeneous flow solution is stable, and if $b_1 < b < b_2$ (synchronized flow) it is unstable, which sufficiently demonstrates why the traffic flow in free regime and coexisting regime of wide moving jam and synchronized flow or free flow are the same as the homogeneous solution flow, and the flow in synchronized flow regime is greater or lower than that of the homogeneous flow.

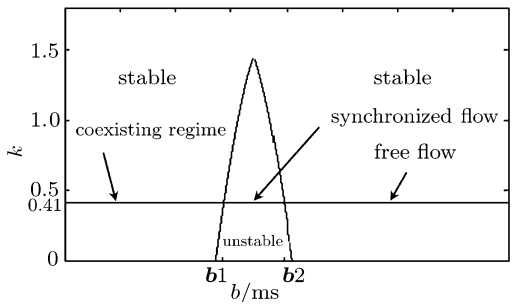


Fig.5. Qualitative schematic showing phases for VDSM.

Regardless of the cause of congestion, congested traffic usually exhibits two prominent features: (1) an initial front that induces sharp flow changes, and (2) traffic hysteresis, i.e. the flow-density plot has the loop structure. In 1983, Koshi *et al* analysed traffic data from the Tokyo Expressway and found that their flow-density plots resemble 'the mirror image of a reversed λ' (Fig.6(a)).[15] The other important feature, i.e. traffic hysteresis, was pointed out by Treiterer and Myers (1974):[16] they found that both flow-density and speed-density plots have loop structures, and Newell (1961) developed a model that contains hysteresis loops (Fig.6(b)).[17]

A traffic simulation is conducted to study these two features of this new theory on a ring road, which simplifies the analysis a great deal because we do not have to specify boundary conditions. We start with an empty circular road and fill it with vehicle gradually. A vehicle enters or exits the ring road one by one. An entering vehicle placed in between two consecutive cars chosen randomly while a leaving vehicle chosen randomly can exit from any location. Our interest is just in the transitions between free and synchronized flow, so we only release 65 vehicles onto the ring road in this manner.

Figure 7 shows the simulation results of VDSM. On the left are the flow-density plots for the entire circular road and on the right are the respective time-speed plots. One should note that the flow-density plots produced show the desired reversed-λ image and two major features: (1) flow-drop, and (2) hysteresis loop (arrowed lines).

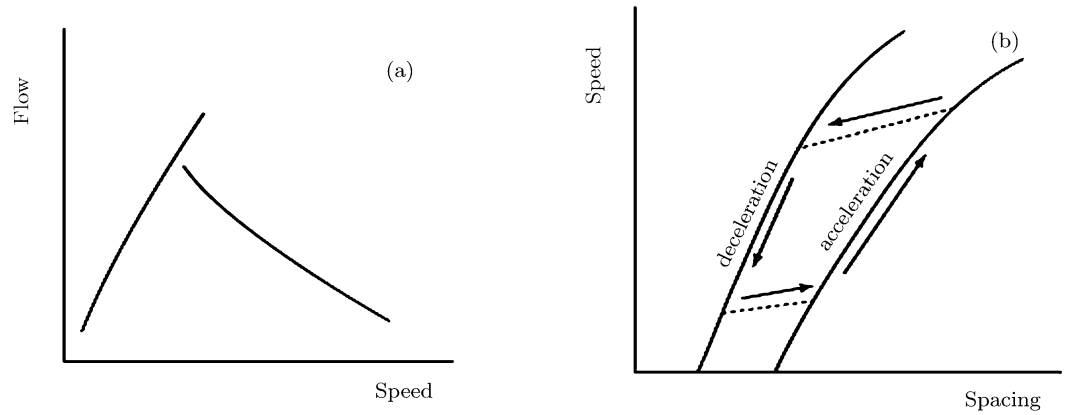


Fig.6. (a) Fundamental diagram proposed by Koshi *et al* (1983)(reproduced) (b)Speed–spacing relationship proposed by Newell (1961)(reproduced)

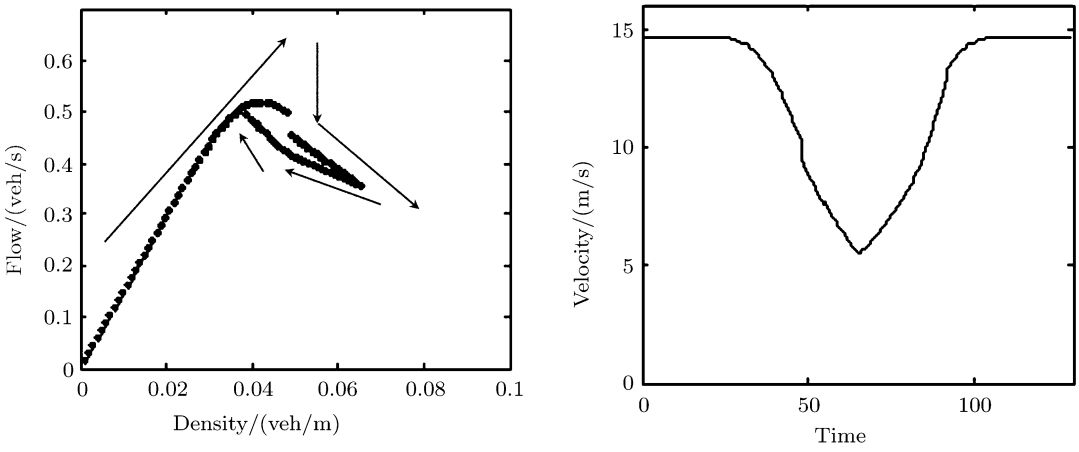


Fig.7. Simulation results of VDSM.

4. Conclusion

This paper introduces a velocity-difference-separation model (VDSM) for a single lane car-following theory based on the OVM, GFM and FVDM. The simulation shows that the model can perform more realistically in predicting the dynamical evolution of congestion induced by a small perturbation, as well as predicting correct delay time of car motion and kinematic wave speed at jam density. Moreover, both theoretical analysis and numerical simulations have demonstrated the potential of the new car-following theory to model some complex traffic patterns in multiphase vehicle traffic flow such as the existence of three traffic phases: free flow, synchronized flow, and wide moving jam; and the sharp flow drop and traffic hysteresis. In this process linear stability analysis for the proposed model has been done and the stability criterion is given.

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