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Initial transient process in a simple helical flux compression generator*

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An analytical scheme on the initial transient process in a simple helical flux compression generator, which includes the distributions of both the magnetic field in the hollow of an armature and the conducting current density in the stator, is developed by means of a diffusion equation. A relationship between frequency of the conducting current, root of the characteristic function of Bessel equation and decay time in the armature is given. The skin depth in the helical stator is calculated and is compared with the approximate one which is widely used in the calculation of magnetic diffusion. Our analytical results are helpful to understanding the mechanism of the loss of magnetic flux in both the armature and stator and to suggesting an optimal design for improving performance of the helical flux compression generator.

Keywords: initial transient process, helical flux compression generator, diffusion equation, skin depth
PACC: 5275F, 1340H, 9160P

1. Introduction

Since last three decades, a variety of basic flux compression generators has been proposed and tested.[1−7] Though their sizes and operating regimes are greatly different, it is apparent that the helical magnetic flux compression generator (MFCG) is the most promising concept in relative to current amplification and compactness in dimension. Though the geometry of the helical generator seems to be simple, the understanding of all processes involved is rather difficult. It is apparent from lack of a computer model that utilizes the relevant physical principles to describe the processes without adjusting parameters. Recently, a simple generator was proposed to improve flux and current losses of MFCG that is focused on reliable operation, i.e. the output current amplitude should be highly reproducible.[1] Even though this generator has been tested widely, the basic process in physics and the mechanism of losses of magnetic flux are still expected to be ascertained.[2] Therefore, this paper focuses on the diffusion process of the magnetic field and current densities in the armature and the stator of MFCG, respectively. The transient process of the magnetic field, which is distributed over the cross section in the armature, is investigated in Section 2, where a relationship among the root of the characteristic equation of the magnetic field , frequency of conducting current and decay time of magnetic field in the armature is established too. In Section 3, a relationship between the skin depth around the inner surface of the stator and the frequency of the conducting current density is studied. Section 4 gives a brief conclusion.

2. The transient process in the armature

Consider a simple helical flux compression generator based on Ref.[1], in which the generator is made of aluminium in the armature and of copper in the helical stator. The stator has 32 turns of insulated standard 2-AWG copper wire with a pitch of 3 mm, wound on the surface of a Teflon mandrel with 33 mm outer radius and 31 mm inner radius. The armature is 3 mm in thickness and 19.05 mm in outer radius. Generally, the model of MFCG belongs in two-dimensional problem. But in the initial transient process, because the radius of armature is smaller than its axial size (about one-fifth of the axial size in the present case) and the

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relevant nonlinear process has not been fully understood yet, a one-dimensional linear model is a good approximation and definitely brings the convenience to our calculation. For this helical flux compression generator with an axial magnetic field, $H_z(r,t)$, the cylindrical diffusion equation can be written as

$$\frac{\partial H_z(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial H_z(r,t)}{\partial r}=\frac{1}{k_0}\frac{\partial H_z(r,t)}{\partial t}, \tag{1}$$

where k_0 is the magnetic diffusivity defined as

$$k_0=\frac{1}{\sigma_0\mu_0}, \tag{2}$$

where σ_0 is the vacuum electrical conductivity and $\mu=\mu_0\mu_{\rm R}\approx\mu_0$ is the magnetic permeability in the high magnetic field under consideration.

The distribution of magnetic field in the armature can be considered as the diffusion of the axial magnetic field constrained by a proper boundary and initial conditions in this simplified model

$$\begin{aligned} &H_z(b,t)=H_0{\rm e}^{-wt},\\ &H_z(r,0)=0, \end{aligned} \tag{3}$$

where b is the outer radius of the hollow armature and ω is the circular frequency. The diffusion problem of a hollow cylindrical conductor requires an additional boundary condition on the inner surface. To obtain this condition, it is necessary to apply Faraday's law to the hollow interior of a long conductor where H_z has the same value everywhere inside the cylinder.

First, we integrate one of Maxwell's equations over a surface

$$\int_s(\nabla\times\boldsymbol{E})\cdot{\rm d}\boldsymbol{s}=-\int_s\frac{\partial\boldsymbol{B}}{\partial t}\cdot{\rm d}\boldsymbol{s}. \tag{4}$$

According to Stoke's theorem, we have

$$\oint_c\boldsymbol{E}\cdot{\rm d}\boldsymbol{l}=-\int_s\frac{\partial\boldsymbol{B}}{\partial t}\cdot{\rm d}\boldsymbol{s}=-\mu_0\int_s\frac{\partial\boldsymbol{H}}{\partial t}\cdot{\rm d}\boldsymbol{s}. \tag{5}$$

Define the surface as the intersection of a normal plane with the hollow interior such that

$$-\mu_0A\frac{{\rm d}H_z}{{\rm d}t}=\oint_c\boldsymbol{E}\cdot{\rm d}\boldsymbol{l} \tag{6}$$

where $A=\pi a^2$ is the cross-sectional area of the hollow interior while a is the inner radius of the armature. With $\nabla\times\boldsymbol{H}=\boldsymbol{j}$ and Ohm's law in the interior region of the conductor, we obtain

$$\nabla\times\boldsymbol{H}=\boldsymbol{j}=\sigma_0\boldsymbol{E}. \tag{7}$$

Since the tangential component of $\boldsymbol{E}$ must be continuous on the surface, Eq.(6) takes the forms

$$-\mu_0A\frac{{\rm d}H_z(a,t)}{{\rm d}t}=\left[\frac{1}{\sigma_0}\oint\nabla\times\boldsymbol{h}\cdot{\rm d}\boldsymbol{l}\right], \tag{8}$$

$$-\mu_0\sigma_0\pi a^2\frac{{\rm d}H_z(a,t)}{{\rm d}t}=\left[\oint\nabla\times\boldsymbol{H}\cdot{\rm d}\boldsymbol{l}\right], \tag{9}$$

$$-\frac{1}{k_0}\pi a^2\frac{{\rm d}H_z(a,t)}{{\rm d}t}=2\pi a\left[-\frac{\partial H_z}{\partial r}\right]_{r=a}, \tag{10}$$

$$\frac{{\rm d}H_z(a,t)}{{\rm d}t}=\frac{2k_0}{a}\left[\frac{\partial H_z}{\partial r}\right]_{r=a}. \tag{11}$$

Equations (3) and (11) represent the transient boundary condition for the diffusion of an axial magnetic field into a hollow conductor governed by Eq.(1). The solution of Eq.(1) can be evaluated from mathematical physics. The field $H_z(r,t)$, which is built up from the cavity of the conductor, can be given in the form

$$H_z(r,t)=H_0+\pi H_0\sum_{n=1}^{\infty}{\rm e}^{-k_0\alpha_n^2t}\frac{J_2^2(a\alpha_n)[J_0(\alpha_nr)Y_0(b\alpha_n)-Y_0(\alpha_nr)J_0(b\alpha_n)]}{J_0^2(b\alpha_n)-J_2^2(a\alpha_n)}, \tag{12}$$

where $J_0,J_2,\cdots;Y_0,Y_2,\cdots$ are Bessel functions of the first kind and second kind with their orders $n=0$ and 2. α_n are roots of

$$f(\alpha_n)=J_0(b\alpha_n)Y_2(a\alpha_n)-Y_0(b\alpha_n)J_0(a\alpha_n)=0, \tag{13}$$

where $f(\alpha_n)=0$ is called the characteristic Bessel equation. We plot the first eight roots α_n in Fig.1, where we can see that the value of function between the first root and second root is quite larger than the rest. These roots are listed in Table 1. On the other hand, in Eq.(3) it can be seen that $\omega=2\pi f_n=k_0\alpha_n^2$, where f_n is the frequency. Thus, a relationship among the root, frequency and decay time of the magnetic field in the armature is given by

$$f_n=\frac{1}{2\pi\tau_n}=\frac{k_0\alpha_n^2}{2\pi}. \tag{14}$$

Table 1 shows these characteristic values.

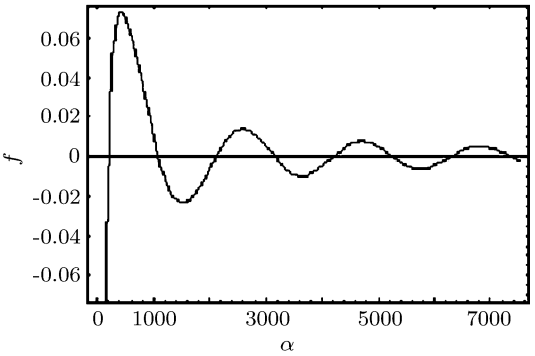


Fig.1. $f(\alpha)$ vs α and first eight roots.

In Eq.(12) it seems that a series number n can vary from 1 to infinity separately. But according to Table 1 and Ref.[1], the decay time is beyond the limit of the physical process if $n = 1$. Therefore, it is reasonable to consider the number n varying from 2 to 8 as the initial transient process. In other words, the time can change from about 0.8 μs to 40 μs because the whole chemical explosive duration is about several tenths of microsecond and the initial physical process (for example, initial current) changes fast until 40 μs which can be seen in Ref.[1].

Table 1. The first eight roots and frequencies and decay times.

n	α_n	f_n/kHz	τ/μs
1	200.42	0.14	1130
2	1085.08	4.14	38.4
3	2113.85	15.72	10.1
4	3154.63	35.01	4.56
5	4198.58	62.02	2.57
6	5243.83	96.74	1.65
7	6289.67	139.16	1.14
8	7335.96	189.33	0.84

Then, Eq.(12), which describes the initial transient process of the armature, can be expressed as

$$H_z(r,t) = H_0 + \pi H_0 \sum_{n=2}^{8} \mathrm{e}^{-k_0\alpha_n^2 t} \frac{J_2^2(a\alpha_n)[J_0(\alpha_n r)Y_0(b\alpha_n) - Y_0(\alpha_n r)J_0(b\alpha_n)]}{J_0^2(b\alpha_n) - J_2^2(a\alpha_n)}. \tag{15}$$

For example, at the time of 40 μs the above formula can be simplified as follows:

$$H_z(r,t) = H_0 + 1.109H_0 \frac{J_2^2(a\alpha_2)[J_0(\alpha_2 r)Y_0(b\alpha_2) - Y_0(\alpha_n r)J_0(b\alpha_2)]}{J_0^2(b\alpha_2) - J_2^2(a\alpha_2)} \tag{16}$$

Figure 2 shows the distribution of magnetic field in the hollow armature of the generator according to Eq.(16) at 40 μs.

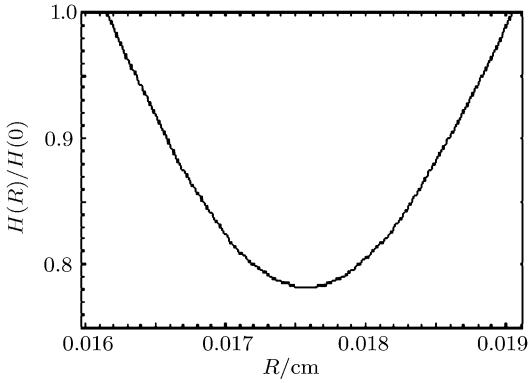


Fig.2. The distribution of magnetic field in the armature at 40 μs.

3. The transient process in the stator

Now we are interested in the distributions of both current density and skin depth in the stator. If the helix coil of stator is wound uniformly and densely, it can be equivalent to a metallic hollow. The diffusion equation of current density can be simplified into a one-dimensional linear equation.[3]

Taking the curl of the magnetic field $\boldsymbol{H}$, the current density $\boldsymbol{j}$ can be derived from the same differential equation, which can be expressed in the form of a cylindrical symmetric coordination as follows:

$$\frac{\partial^2 j(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial j(r,t)}{\partial r} - \frac{j(r,t)}{r^2} = \frac{1}{k_0}\frac{\partial j(r,t)}{\partial t}, \tag{17}$$

where j is an azimuthally conducting current density.

Regarding to a simple helical flux compression generator, no matter what the initial charging process of circuit or a compressive process of explosive driven magnetic flux is, the tendency of current changing with time can be approximately depicted with an exponential form

$$I(r,t) = I(r)\mathrm{e}^{\omega_1 t} \rightarrow j(r,t) = j(r)\mathrm{e}^{\omega_1 t}, \tag{18}$$

where ω_1 is the oscillation frequency of the circuit for the initial charging process, which is proportional to

the relative change rate of current

$$\frac{\mathrm{d}I/\mathrm{d}t}{I} \to \frac{\mathrm{d}j/\mathrm{d}t}{j}. \tag{19}$$

If we insert the exponential form of current density into Eq.(17), then we can see that the diffusion equation of current density is independent of time

$$\frac{\mathrm{d}^2 j(r)}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}j(r)}{\mathrm{d}r} - (\frac{1}{r^2} + \beta^2)j(r) = 0, \tag{20}$$

where $\beta^2 = \dfrac{\omega_1}{k_0}$ is assumed to be a diffusion coefficient of current density in the stator. Together with the boundary condition at the inner surface $r = r_\mathrm{c}$ in the stator, the current density has the following form:

$$j(r) = j_\mathrm{c}, \quad r = r_\mathrm{c}. \tag{21}$$

In regard to the boundary condition of outer surface $r = r_\mathrm{d}$ in the stator, it can be written as follows:

$$2\frac{\mathrm{d}j(r)}{\mathrm{d}r} = \beta^2 r j(r), \qquad r = r_\mathrm{d}. \tag{22}$$

The solution of Eq.(20) for the current density is a linear combination of modified Bessel functions:

$$j(r) = CI_1(\beta r) + DK_1(\beta r), \tag{23}$$

where I_1 and K_1 are modified Bessel functions of the first kind and second kind. C and D are coefficients given below:

$$\begin{aligned} C &= j_\mathrm{c}\frac{K_0(\beta r_\mathrm{d}) + K_2(\beta r_\mathrm{d}) + \beta r_\mathrm{d}K_1(\beta r_\mathrm{d})}{[K_0(\beta r_\mathrm{d}) + K_2(\beta r_\mathrm{d}) + \beta r_\mathrm{d}K_1(\beta r_\mathrm{d})]I_1(\beta r_\mathrm{c}) + [I_0(\beta r_\mathrm{d}) + I_2(\beta r_\mathrm{d}) - \beta r_\mathrm{d}I_1(\beta r_\mathrm{d})]K_1(\beta r_\mathrm{c})}, \\ D &= j_\mathrm{c}\frac{I_0(\alpha r_\mathrm{d}) + I_2(\alpha r_\mathrm{d}) - \alpha r_\mathrm{d}I_1(\alpha r_\mathrm{d})}{[K_0(\beta r_\mathrm{d}) + K_2(\beta r_\mathrm{d}) + \alpha r_\mathrm{d}K_1(\beta r_\mathrm{d})]I_1(\beta r_\mathrm{c}) + [I_0(\beta r_\mathrm{d}) + I_2(\beta r_\mathrm{d}) - \beta r_\mathrm{d}I_1(\beta r_\mathrm{d})]K_1(\beta r_\mathrm{c})}. \end{aligned} \tag{24}$$

The current density around the inner surface j_c can be obtained from the following integration:[4]

$$I_\mathrm{c} = p\int_{r_\mathrm{c}}^{r_\mathrm{d}} j(r)\mathrm{d}r = pC[I_0(\beta r_\mathrm{d}) - I_0(\beta r_\mathrm{c})] + pD[K_0(\beta r_\mathrm{d}) - K_0(\beta r_\mathrm{c})], \tag{25}$$

where p is the pitch of coils. Finally, the current density around inner surface of the stator can be expressed as

$$\begin{aligned} j_\mathrm{c} =& \frac{\beta I_\mathrm{c}}{p} \cdot \{[K_0(\beta r_\mathrm{d}) + K_2(\beta r_\mathrm{d}) + \beta r_\mathrm{d}K_1(\beta r_\mathrm{d})]I_1(\beta r_\mathrm{c}) + \\ & [I_0(\beta r_\mathrm{d}) + I_2(\beta r_\mathrm{d}) - \beta r_\mathrm{d}I_1(\beta R_\mathrm{d})]K_1(\beta r_\mathrm{c})\}/ \\ & \{[K_0(\beta r_\mathrm{d}) + K_2(\beta r_\mathrm{d}) + \beta r_\mathrm{d}K_1(\beta r_\mathrm{d})][I_0(\beta r_\mathrm{d}) - I_0(\beta r_\mathrm{c})] + \\ & [I_0(\beta r_\mathrm{d}) + I_2(\beta r_\mathrm{d}) - \beta r_\mathrm{d}I_1(\beta r_\mathrm{c})][K_0(\beta r_\mathrm{d}) - K_0(\beta r_\mathrm{c})]\}. \end{aligned} \tag{26}$$

With Eq.(26), we can obtain not only the distribution of the current density but also the value of the skin depth in the stator. On the other hand, it is well known that the skin depth can be expressed approximately as

$$\delta = \sqrt{\frac{1}{2\pi f \mu_0 \sigma_0}}. \tag{27}$$

Therefore, we can compare our calculated results with the results from Eq.(27).

Table 2 shows that the skin depth decreases as the frequency increases. The maximum value of skin depth occurs at a frequency of 1 kHz. This kind of residual effect becomes very important in the optimal design of MFCG. For example, the copper wire has a 2 mm diameter in the present case. According to Eq.(27), the skin depth goes through the diameter of the copper wire at 1 kHz, which may result in an extra flux loss. But it does not appear in the experiment of MFCG and can be avoided by our calculated results. Roughly, the difference between the two methods (the present method and Eq. (27)) gradually decreases as the frequency increases. In order to see how the skin depth distributes along the radial distance inside the stator, we present the calculated results as shown in Fig.3.

Table 2. Skin depth in the stator.

Frequency/kHz	Skin depth from Eq.(27)/mm	Skin depth from our scheme/mm
1	2.08	1.01
10	0.66	0.46
100	0.21	0.15
1,000	0.06	0.046
10,000	0.006	0.0047
1,000,000	0.002	0.0015

In Fig.3, from top to bottom, the frequency changes from 1 kHz to 10,000 kHz in proper order. The current density in the stator decreases with the radius increasing while the effect of eddy current becomes more prominent as the frequency of conducting current increases. The higher the frequency, the more the eddy current trends to adhere to the inner surface of the stator, which is due to the current density on the inner surface being attracted by the inducted current density in the armature.

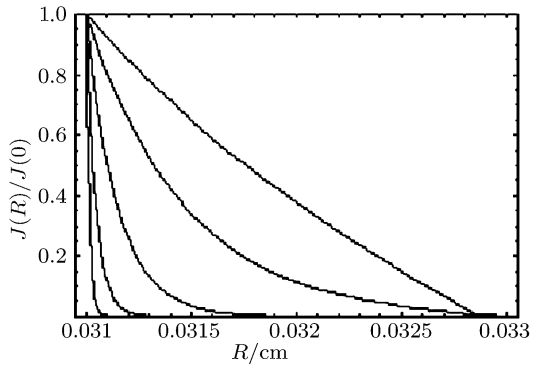


Fig.3. The distribution of current density in the stator.

4. Conclusion

An analytic scheme for a simple MFCG model is proposed to deal with transient processes of the magnetic field in the armature and the current density in the stator with the diffusion equations. Simulation results show that 1) relative steady states in the transient process of the magnetic field or current may still be achieved if the frequency of conducting current reaches a serial separated roots of solution in the characteristic equation of magnetic field; 2) the initial transient process takes place from about 1 μs to 40 μs, which corresponds to the characteristic separating number n from 8 to 2; 3) the higher the frequency is, the faster the magnetic field or current density decays along the radial distance. But if the frequency is very high, the effect of skin depth becomes very obvious; 4) our method for dealing with the effect of skin depth seems more robust than the traditional formula like Eq.(27). Therefore the scheme describing initial transient processes of an MFCG is helpful to the further understanding of the mechanism and optimal design of a helical flux compression generator as well.

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