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Influence of strong magnetic field on β decay in the crusts of neutron stars*

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β decay in the strong magnetic field of the crusts of neutron stars is analysed by an improved method. The reactions ^{67}Ni$(\beta^-)^{67}$Cu and ^{62}Mn$(\beta^-)^{62}$Fe are investigated as examples. The results show that a weak magnetic field has little effect on β decay but a strong magnetic field ($B > 10^{12}$G) increases β decay rates obviously. The conclusion derived may be crucial to the research of late evolution of neutron stars and nucleosynthesis in r-process.

Keywords: β decay, strong magnetic field, neutron star
PACC: 2340, 8760D, 9760J

1. Introduction

Weak interactions, especially β decay and electron capture, play a key role in late evolution of neutron stars and r-process nucleosynthesis of heavy nuclei in the binary neutron star systems.[1,2] Since β decay and electron capture are sensitive to astrophysical environment, it is necessary to investigate the effect of astrophysical environment on them. The existence of strong magnetic field in the crusts of neutron stars have been verified by many experiments and theories. Generally, the magnetic field ranges from 10^9G to 10^{13}G.[3,4] So the occurrence of β decay in the crusts ranges over many orders of magnetic field and plays an essential role in initiating the dynamical instability. Due to the β decay of nuclei, the electron fraction in the crusts will change once the matter in the crusts moves out of β equilibrium. This change leads to a change of the pressure and affects the hydrodynamics, which in turn changes the thermodynamic conditions that determine the β decay rates. Furthermore, the matter of neutron star is heated by the energy from the β decay. Especially, for the binary neutron star systems, because of orbital momentum loss by gravity waves the neutron stars will ultimately merge together, ending as a fast spinning black hole or massive neutron star, depending on the initial total mass and the amount of mass possibly lost from the system during the merger event. β decay plays a key role in the merger event, leading to a quasi-static expansion of the neutron star, which lasts for about 20 seconds.[5]

The β decay rates in strong magnetic field were investigated soon after the pulsar was discovered.[6] Because the importance of Gamow–Teller (GT) resonance transition at high density was not recognized at that time, the effect on the transition of high-lying state was ignored till Bethe *et al* recognized in 1979 that the GT transition is dominant at high densities.[7] Fuller, Flower and Newman (FFN) first adopted the 'brink hypothesis' to deal with the GT resonance transition and obtained the accurate parameters of weak interaction at high densities.[8] They calculated the GT transition matrix element on the basis of single particle orbit of shell model. Afterward, Aufderheide *et al* obtained more accurate β decay rates by using FFN's technique and considering the influence of 'quenching'.[9] Kar *et al* calculated the β decay rates of nuclei with $A > 60$ by using average strength function.[10] The influence of magnetic field on electron capture rate in the crusts of neutron stars was investigated in 1990s.[11,12] The results indicate that weak magnetic field has little effect on electron capture rates, while strong magnetic field has an evident effect, particularly for heavier nucleus whose threshold is negative at high density. Hence, it is interesting to investigate the influence of strong magnetic field on β decay by using the improved method.

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2. β decay rate without magnetic field

β decay is completely suppressed in the core of neutron stars and in the inner crust due to the high electron degeneracy prevailing there. Therefore, only nuclei in the outer crust can decay.[6] The β decay rates for the kth nucleus (Z, A) in thermal equilibrium at temperature T is given by a sum over initial parent states i and final daughter states j:

$$\lambda_k^{\rm bd}(\rho,T,Y_{\rm e}) = \ln 2 \times \sum_i \frac{(2J_i+1)\exp(-E_i/k_{\rm B}T)}{G(Z,A,T)} \times \sum_j \frac{\xi(\rho,T,Y_{\rm e},Q_{ij})}{{\rm ft}_{ij}}, \quad (1)$$

where J_i and E_i are the spin and excitation energy of the parent states, $k_{\rm B}$ is the Boltzmann factor, G is the nuclear partition function, ${\rm ft}_{ij}$ is the comparative half-life connecting states i and j, and Q_{ij} is the nuclear energy difference between states i and j,

$$Q_{ij} = (M_{\rm p} - M_{\rm d})c^2 + E_i - E_j, \quad (2)$$

where $M_{\rm p}$ is the mass of the parent nucleus, $M_{\rm d}$ is the mass of the daughter nucleus and E_j is excitation energy of the jth daughter state. Because of the finite range of this integral, the sum over the daughter states in Eq.(1) is limited to those states for which Q_{ij} are greater than $m_{\rm e}c^2$. ξ is the β decay phase space integral without magnetic field. It can be expressed as

$$\xi(\rho,T,Y_{\rm e},Q_{ij}) = \frac{c^3}{(m_{\rm e}c^2)} \int_0^{\sqrt{Q_{ij}^2 - m_{\rm e}^2c^4}} p^2(Q_{ij}-E_{\rm e})^2 \times \frac{F(Z+1,E_{\rm e})}{1+\exp[(\mu_{\rm e}-E_{\rm e})/k_{\rm B}T]}{\rm d}p, \quad (3)$$

where p, $E_{\rm e}$, $m_{\rm e}$ and $\mu_{\rm e}$ are the electron momentum, energy, mass and chemical potential, respectively. $F(Z+1,E_{\rm e})$ is the Coulomb function which is the ratio of the squares of the electron wavefunction distorted by the coulomb scattering potential to the wavefunction of the free electron,

$$F(Z+1,E_{\rm e}) = 2(1+\gamma)\left(\frac{2p_{\rm e}R}{\hbar}\right)^{-2(1-\gamma)} \times {\rm e}^{\pi\nu}\frac{|\Gamma(\gamma+{\rm i}\nu)|^2}{|\Gamma(2\gamma+1)|^2}, \quad (4)$$

where $\nu = Z\alpha(E_{\rm e}/pc)$, $\gamma = [1-(Z\alpha)^2]^{1/2}$, R is the nuclear radius, Γ is the gamma function, Z is the nuclear charge of the parent state and α is the fine-structure constant.

Following the technique of Auderheide *et al* we break up the sum over parent states into two parts and handle them separately, one for the low energy region near the ground state and the other for the resonance region dominated by GT resonance transition. The total β decay rate can be expressed as[9]

$$\lambda_k^{\rm bd}(\rho,T,Y_{\rm e}) = \lambda_0 + \lambda_{\rm BGT}, \quad (5)$$

where

$$\lambda_0 = \ln 2 \times \frac{(2J_0+1)}{G(Z,A,T)}\exp(-E_{\rm peak}/k_{\rm B}T) \times \frac{\xi(\rho,T,Y_{\rm e},E_{\rm peak}+Q_{00})}{ft_{\rm eff}}, \quad (4)$$

$$\lambda_{\rm BGT} = \ln 2 \times \exp\left(-\frac{E_{\rm BGTR(0)}}{k_{\rm B}T}\right)\frac{G(Z+1,A,T)}{G(Z,A,T)} \times \frac{\xi(\rho,T,Y_{\rm e},E_{\rm BGTR(0)}+Q_{00})}{ft_{0\to{\rm BGTR(0)}}}. \quad (7)$$

Here J_0 is the initial spin of parent state, $E_{\rm BGTR(0)}$ is the energy difference between the orbit that the new neutron occupies in the GT resonance and the ground state, ${\rm ft}_{\rm eff}$ and ${\rm ft}_{0\to{\rm BGTR(0)}}$ are written as[7]

$${\rm ft}_{\rm eff} = 6.06\times10^4, \quad {\rm ft}_{0\to{\rm BGTR(0)}} = \frac{10^{-3.596}}{\left|M_{\rm BGTR(0)}\right|^2}, \quad (8)$$

where $\left|M_{\rm BGTR(0)}\right|^2$ is GT resonance transition matrix element. We approximate the transitions from low-lying parent states to all accessible daughter states by a single transition from a state $E_{\rm peak}$ above the ground state of the parent to the daughter ground state. For the temperature ranging from roughly 0.3MeV to 0.8MeV and electron chemical potentials ranging from 0.5MeV up to at least 8MeV, $E_{\rm peak}$ is approximated by

$$E_{\rm peak} \doteq 5T - Q_{00} + \mu_{\rm e} + \delta(T,\mu_{\rm e}), \quad (9)$$

here $\delta(T,\mu_{\rm e})$ is well fitted by

$$\delta(T,u_{\rm e}) = -0.6604 + 0.9429T - 0.02119\mu_{\rm e} - 0.9432T\mu_{\rm e} - 0.0009524\mu_{\rm e}^2 + 0.06224T\mu_{\rm e}^2. \quad (10)$$

Temperature T in Eqs.(9) and (10) is in units of MeV. If $E_{\rm peak}$ is negative, the transitions from ground state to ground state are computed.

3. β decay rate in strong magnetic field

Due to the influence of strong magnetic field, the state densities of electron gas in phase space are changed and the electrons are prone to residing in the Landau levels. Landau levels are dense in weak magnetic field, but they are sparse under the condition of strong magnetic field and high degeneration. So the electrons can just occupy a few Landau levels in strong magnetic field. The distribution of electrons under zero temperature approximation has been analysed in Ref.[11]. In this paper the β decay rates in the crusts of neutron star at non-zero temperature are calculated. Electron chemical potential will be changed if the electron distribution varies. The variations of the electron chemical potential are obvious in the environments of low temperature ($T < 10^9$K), low density ($\rho/\mu_e < 10^5$mol·cm^{-3}) and very strong magnetic field ($B > 10^{13}$G). For the magnetic field strength of 10^9–10^{13}G we consider here, such variation is slight.[12] Therefore, we may neglect the influence of the strong magnetic field on the electron chemical potential.[6] According to V–A law, the ratio of nucleus transition from initial states to final ones is as follows:[13]

$$\mathrm{d}\lambda_{\mathrm{B}} = \frac{2\pi}{\hbar}\left|H_{ij}\right|^2 \delta(\varepsilon_j - \varepsilon_i)\mathrm{d}\nu_{\mathrm{e}}\mathrm{d}\nu_{\nu}, \tag{11}$$

where the δ function represents energy conservation and H_{ij} is the interaction matrix element. The state density of electron $\mathrm{d}\nu_{\mathrm{e}}$ and that of neutrino $\mathrm{d}\nu_{\nu}$ can be written respectively as

$$\mathrm{d}\nu_{\mathrm{e}} = \frac{\Theta}{4\pi^2\lambda_{\mathrm{e}}^3}\mathrm{d}p_z, \tag{12}$$

$$\mathrm{d}\nu_{\nu} = \frac{V}{8\pi^3\lambda_{\mathrm{e}}^3}q^2\mathrm{d}q\mathrm{d}\Omega_{\nu}, \tag{13}$$

where q is the neutrino momentum (in units of $m_{\mathrm{e}}c$), $\mathrm{d}\Omega_{\nu}$ is the solid angle of neutrino emission, $\Theta = B_{12}/(4.14\times10^{13})$, V is the volume, p_z is the momentum along the direction of magnetic field, $B_{12} = B/10^{12}$, B is the strength of magnetic field.

The most important effect of magnetic field on β decay is the modification of the phase space of electrons. Strictly speaking, all states of nucleons are affected by magnetic field, but such effect is so slight that the modification can be neglected.[6] Therefore, we only modify the phase space integral of electrons while considering the effect of strong magnetic field on β decay. According to the technique of Ref.[11], the phase space integral in magnetic field is written as

$$\xi^{\mathrm{B}} = \frac{\Theta}{2}\sum_{n=0}^{\infty}\theta_n, \tag{15}$$

$$\theta_n = \frac{c^3}{(m_{\mathrm{e}}c^2)}g_{n0}\int_0^{\sqrt{Q_{ij}^2-Q_n^2}}(Q_{ij}-E_{\mathrm{e}})^2 \times \frac{F(Z+1,E_{\mathrm{e}})}{1+\exp[(\mu_{\mathrm{e}}-E_{\mathrm{e}})/k_{\mathrm{B}}T]}\mathrm{d}p, \tag{15}$$

where $Q_n = (m_{\mathrm{e}}^2c^4 - 2n\Theta)^{1/2}$, $g_{n0} = 2-\delta_{n0}$ i.e. g_{n0}=1 for n=0 (or 2 otherwise). ξ in Eq.(1) must be replaced by ξ^{B} when one calculates the β decay rates in strong magnetic field because phase space distribution of electron gas is changed.

4. Results and analysis

The Pauli blocking theory should be considered when the density is high. The outgoing electrons produced in β decay can only occupy the unoccupied states. Because the electrons are prone to residing on the Landau levels, which makes the unoccupied states increase, it is beneficial to improving the electron outgoing rates. Hence we may predict that the β decay rates in magnetic field will become larger than in the case without magnetic field. In order to describe the influence of strong magnetic field on the β decay rates, we define an enhancing factor $C = \lambda_k^{\mathrm{B}}/\lambda_k$, λ_k^{B} and λ_k are β decay rates with/without the magnetic field, respectively. Correspondingly, we also define an enhancing factor $C_0 = \dfrac{\lambda_0^{\mathrm{B}}}{\lambda_0}$, which describes the variation of ground states, and an enhancing factor $C_{\mathrm{GT}} = \dfrac{\lambda_{\mathrm{BGT}}^{\mathrm{B}}}{\lambda_{\mathrm{BGT}}}$, which describes the variation of resonance states.

The variations of three enhancing factors with magnetic field in the reaction of ^{67}Ni(β^-)^{67}Cu are shown in Fig.1. It is found that the magnetic field has little effect on the β decay rates when the temperature is 5×10^9K, the density is 10^8mol·cm^{-3} and the magnetic field strength is smaller than 10^{12}G, since C_0, C_{GT}, and C is close to 1. But when the magnetic field strength is larger than 10^{12}G the strong magnetic field makes the β decay rates increase obviously (about several orders of magnitude). It is also found that the effect of strong magnetic field on C_0 is slightly larger than on C_{GT} and C, which is because the electrons produced by ground transition have lower energy than that of GT resonance transition. The lower the electron energy is, the easier the phase space distribution

of electron is changed. The maximum electron energy of ground state is 5.41MeV and that of GT resonance is 7.01MeV, make the ground states apt to be affected by the strong magnetic field.

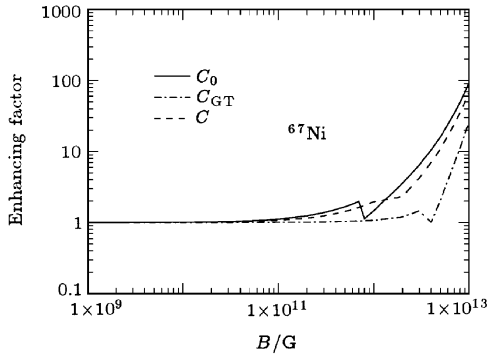


Fig.1. Three enhancing factors as a function of magnetic field. (ρ/μ_e=10^{8}mol·cm^{-3}, T=5×10^{9}K) .

It is seen in Fig.2 that β decay rates increase obviously when magnetic field strength is larger than 10^{13} G, which is much larger than that in Fig.1. The reason is that the energy from the ground state to ground state Q_{00} in the reaction ^{67}Ni(β^-)^{67}Cu is 4.07MeV while Q_{00} of the reaction ^{62}Mn(β^-)^{62}Fe is 10.81MeV, high-energy electrons are so difficult to be changed that β decay rates in the latter reaction are affected only slightly when magnetic field strength is lower than 10^{13}G.

Furthermore, for reaction ^{62}Mn(β^-)^{62}Fe at ρ/μ_e=2 × 10^{9}mol·cm^{-3}, as shown in Fig.2, C is always close to C_{GT}. This is because, at high densities, the GT resonance transition is dominant (about 76% as obtained in our calculation) in the total transition. But for reaction ^{67}Ni(β^-)^{67}Cu at ρ/μ_e=10^{8}mol·cm^{-3}, C is close to C_0 because the ground states transition is dominant (about 63%). Therefore, we think the GT resonance transition should be carefully evaluated in the research of β decay, especially in the pressure of strong magnetic field and high densities.

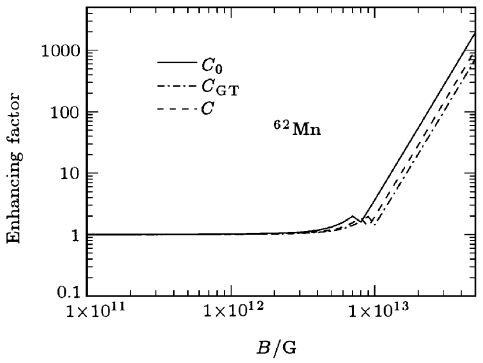


Fig.2. Three enhancing factors as a function of magnetic field. (ρ/μ_e=2 × 10^{9}mol·cm^{-3}, T=5×10^{9}K).

By calculations, we have demonstrated that strong magnetic field has obvious effect on β decay rates. Therefore, it is necessary to consider the influence of strong magnetic field (such as $B > 10^{12}$G) in the research of β decay rates in the crusts of neutron stars. During the last decade, one of the intensive researches is the astrophysical r-process, which creates about half of all nuclei with $A > 70$,[14] proceeding through very neutron-rich and unstable isotopes produced by stellar explosions or other violent events. While the astrophysical site of this process remains a mystery, the main candidate environments are neutrino-heated ejecta from core-collapse supernovae and decompressed ejecta from neutron star mergers.[2] The ultimate abundance of any stable nuclide depends strongly on the β decay half lives of its neutron-rich progenitors. Moreover, primary importance in analysis of r-process nucleosynthesis is the β decay rates, and an alteration in the rates can change the path and pace of the r-process. Hence, the conclusion we obtained has significant influence on further research of nuclear astrophysics, especially for r-process nucleosynthesis in the binary neutron star systems.

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