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Evaluating the dynamical coupling between spatiotemporally chaotic signals via an information theory approach*

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An information-theoretic measure is introduced for evaluating the dynamical coupling of spatiotemporally chaotic signals produced by extended systems. The measure of the one-way coupled map lattices and the one-dimensional, homogeneous, diffusively coupled map lattices is computed with the symbolic analysis method. The numerical results show that the information measure is applicable to determining the dynamical coupling between two directly coupled or indirectly coupled chaotic signals.

Keywords: spatiotemporal chaos, information entropy, dynamical coupling, coupled map lattices
PACC: 0545, 0250

1. Introduction

Recently much attention has been paid to investigating the phenomenon of spatiotemporal chaos occurring in many extended chaotic dynamical systems. The phenomenon exists extensively in real spatiotemporal systems, such as in fluids, optics, chemical reactions, and biology systems, etc. Chaotic-coupled-map lattices being excellent in modelling real spatiotemporal systems have received considerable attention, and have been widely used in the investigation of controlling, synchronization and application of spatiotemporal chaos.[1−8]

It is important to determine the mutual relation between two chaotic signals in the study of spatiotemporal chaos. However, the methods of correlation function and Fourier transform that are used in the analysis of linear relation often fail to determine the nonlinear relation between two chaotic signals. In our previous works[9] we have applied the symbolic analysis method to revealing the mutual relation between spatiotemporally chaotic signals. The method was first introduced by Lehrman and Rechester[10] to quantitatively describe the coupling between two different signals in the Lorenz equation. Then it was used by Liu and Chen [11] to describe the coupling between two signals of two generalized synchronization Lorenz and Rössler systems, and by Azad *et al*[12] to describe the coupling between the climatic time-series data of New Delhi and Sundernagar, respectively. The result of application of symbolic analysis shows that the conditional entropy of two coupled signals has a sharp minimum, from which one can determine whether or not two different signals are coupled, but the conditional entropy is inconvenient for measuring the degree of the dynamical coupling between two signals.

In this paper we introduce an information measure for evaluating the degree of the dynamical coupling between two signals. The measure can be calculated with the symbolic analysis method. Based on the chaotic coupled map lattices we apply the measure in the case of spatiotemporally chaotic signals. The computed results show that the measure is effective in determining the degree of the dynamical coupling.

2. Theory and method

Consider two discrete random variables X and Y with sets of values Ξ and Λ, respectively, probability distribution functions (PDF's) $p(x)$ and $p(y)$, and

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joint PDF $p(x, y)$. The entropy $H(X)$ of a single variable, say X, is defined as[13,14]

$$H(X) = -\sum_{x\in\Xi} p(x)\ln p(x), \tag{1}$$

the joint entropy $H(X, Y)$ of X and Y as

$$H(X, Y) = -\sum_{x\in\Xi}\sum_{y\in\Lambda} p(x, y)\ln p(x, y), \tag{2}$$

and the conditional entropy $H(Y|X)$ of Y given X as

$$H(Y|X) = -\sum_{x\in\Xi}\sum_{y\in\Lambda} p(x, y)\ln p(y|x). \tag{3}$$

We introduce the following information measure[15] to evaluate the degree of the dynamical coupling between X and Y:

$$\mu = \frac{H(Y) - H(Y|X)}{H(Y)}. \tag{4}$$

It is evident that the measure μ represents a normalized quantity from 0 for uncoupled X and Y to 1 for fully coupled X and Y.

Consider two discrete time series $\{x_n\}$ and $\{y_n\}$ regarded as realizations of two discrete random valuables X and Y, which represent the observables of either one multi-variable system or two possibly coupled systems. We use $\{x_n\}$ and $\{y_n\}$ for the calculation of information entropy $H(X)$, $H(Y)$ and $H(Y|X)$. In order to recognize time patterns in the time series $\{x_n\}$ and $\{y_n\}$, we need convert them into symbolic series with the coarse-graining method.[10−12,16−20] Then the information concerning the orbits can be suitably encoded with the symbolic series. We briefly review the coarse-graining method as follows.

Given a set of $m + 1$ critical points, $\{\chi_0, \chi_1, \cdots, \chi_m\}$, and denoting a symbol by an integer, the time series $\{x_n\}$ can be converted into a symbolic series $\{s_n\}$ by the rule defined as

$$\text{If } \chi_k \le x_j < \chi_{k+1}, \text{ then } s_j \equiv k, \tag{5}$$

where $k \in \{0, 1, 2, \cdots, m-1\}$. Partition the resulting symbolic series $\{s_n\}$ into subsequences of a given length L, and denote and identify each subsequence uniquely with an integer, which can be determined with the following formula:

$$l_x(i) = \sum_{p=1}^{L} m^{L-p} s_{p+i-1}. \tag{6}$$

Here i indicates that the subsequence starts at position i along the sequence $\{s_1, s_2, s_3, \cdots\}$. Substituting these subsequences with the corresponding integers, we convert the time series $\{x_n\}$ into the symbolic series $\{l_x(n)\}$. The information content of the time series $\{x_n\}$ contained in the symbolic series $\{l_x(n)\}$ can be quantified by the information entropy,

$$H(X) = -\frac{1}{L}\sum_{l_x} P_{l_x}\ln P_{l_x}, \tag{7}$$

where P_{l_x} is the probability of finding a particular subsequence l_x that is the number of times this subsequence can be found in the long symbolic series $\{l_x(n)\}$, divided by the number of all subsequences.

The coding ability of the symbolic language defined by the rule (5) depends on the number of the critical points and their values. For convenience, we equally partition the phase space of a chaotic time series into 10 domains of which each is signified by a symbol, and take the length of each subsequence to be 2 in the later calculation.

Similarly, we can convert the time series $\{y_n\}$ into a symbolic series $\{l_y(n)\}$ by using the coarse-graining method.

For the time series $\{x_n\}$ and $\{y_n\}$, the joint entropy is defined as

$$H(X, Y) = -\frac{1}{L}\sum_{l_x}\sum_{l_y} P(l_x, l_y)\ln P(l_x, l_y), \tag{8}$$

where $P(l_x, l_y)$ is the joint probability of the sequences l_x and l_y that is the number of joint sequence (l_x, l_y), divided by the total number of the joint sequences of the symbolic series $\{l_x(n)\}$ and $\{l_y(n)\}$. According to information theory,[14] the conditional entropy of $\{x_n\}$ and $\{y_n\}$ can be calculated from the following formula:

$$H(Y|X) = H(X, Y) - H(X). \tag{9}$$

The studies in Refs.[9–12] show that, if the time series $\{x_n\}$ is coupled with $\{y_n\}$, then the conditional entropy $H(Y|X)$ will have a sharp minimum with a shift of time parameter n_0. So we use the minimum of $H(Y|X)$ as the value of $H(Y|X)$ in the calculation of μ that is defined by formula (4).

3. Dynamical coupling of spatiotemporal chaotic signals

Chaotic-coupled-map lattices are excellent in modelling real spatiotemporal systems and, have been widely used in the investigation of controlling, synchronization and application of spatiotemporal chaos. Here we investigate the dynamical coupling between

spatiotemporally chaotic signals produced by the one-way coupled map lattices[2]

$$x_{n+1}(i) = (1-\varepsilon)f[x_n(i)] + \varepsilon f[x_n(i+1)], \qquad (10)$$

and the one-dimensional, homogeneous, diffusively coupled map lattices[3]

$$x_{n+1}(i) = (1-\varepsilon)f[x_n(i)] + \frac{\varepsilon}{2}[f(x_n(i+1)) + f(x_n(i-1))], \qquad (11)$$

where n, i, and ε are the discrete time step, the lattice site index, and the coupling coefficient, respectively. In Eqs.(10) and (11), periodic boundary conditions, $x_n(i) = x_n(i+N)$, with N being the system size, are assumed, and the local mapping function $f(x)$ is chosen to be the logistic map:

$$f(x) = 4x(1-x). \qquad (12)$$

We evenly take 51 different values of ε in the interval $[0, 1]$. For each value of ε, we randomly choose the initial condition $x_0(i)$ in the interval $(0, 1)$ for $i = 1, 2, \cdots, N$. The system size is chosen to be $N = 50$ for the one-way coupled map lattices (10) and, $N = 100$ for the diffusively coupled map lattices (11), respectively. Numerically iterate Eqs.(10) and (11). When the transient dies out, the system runs into the spatiotemporally chaotic states. We take one lattice as an example to consider the dynamical coupling between the time series produced by this lattice and other lattices.

Figure 1 shows the calculation results for the map lattices (10), where Curve 1 presents the dynamical coupling measure μ vs ε for $X = \{x_n(25)\}$ and $Y = \{x_n(26)\}$, and Curve 2 for $X = \{x_n(25)\}$ and $Y = \{x_n(27)\}$, Curve 3 for$X = \{x_n(25)\}$ and $Y = \{x_n(28)\}$, Curve 4 for $X = \{x_n(25)\}$ and $Y = \{x_n(29)\}$, and Curve 5 for $X = \{x_n(25)\}$ and $Y = \{x_n(30)\}$, respectively. From these curves, we can see that the dynamical coupling μ increases approximately with the increase of coupling coefficient ε. There appear two peaks in all these curves, which indicated that the dynamical couplings at the peaks are stronger than those in the neighbouring areas. The reason is that the time series produced by the system (10) with the coefficient ε at the peaks exhibit some periodic behaviours.[18] We also can see that the value of μ matches approximately with the value of ε in Curve 1, where ε can be taken as a reference of the dynamical coupling between $X = \{x_n(25)\}$ and $Y = \{x_n(26)\}$. Figure 1 shows that the value of μ decreases with the increase of the distance between two lattices for the same value of ε. This means that the dynamical coupling strength decreases with the increase of the distance between two lattices.

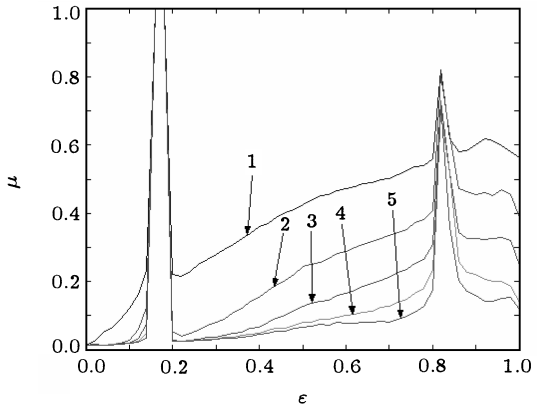


Fig.1. The dynamical coupling measure μ vs the coupling coefficient ε for the one-way coupled map lattices (10). Curve 1 is for $X = \{x_n(25)\}$ and $Y = \{x_n(26)\}$, and Curve 2 for $X = \{x_n(25)\}$ and $Y = \{x_n(27)\}$, Curve 3 for$X = \{x_n(25)\}$ and $Y = \{x_n(28)\}$, Curve 4 for $X = \{x_n(25)\}$ and $Y = \{x_n(29)\}$, and Curve 5 for $X = \{x_n(25)\}$ and $Y = \{x_n(30)\}$, respectively.

Figure 2 shows the calculation results for the map lattices (11), where Curve 1 presents the dynamical coupling measure μ vs ε for $X = \{x_n(50)\}$ and $Y = \{x_n(51)\}$, and Curve 2 for $X = \{x_n(50)\}$ and $Y = \{x_n(52)\}$, Curve 3 for $X = \{x_n(50)\}$ and $Y = \{x_n(53)\}$, Curve 4 for $X = \{x_n(50)\}$ and $Y = \{x_n(54)\}$, and Curve 5 for $X = \{x_n(50)\}$ and $Y = \{x_n(55)\}$, respectively. Similar to Fig.1, Fig.2 shows that the dynamical coupling μ increases approximately with the increase of coupling coefficient ε and, the two peaks in all these curves indicate that the dynamical couplings at the peaks are stronger than those in the neighbouring areas. The reason is that the time series produced by the system (11) with the coefficient ε at the peaks exhibit some periodic behaviours.[18] In addition, we can see that the value of μ matches approximately with the value of $\varepsilon/2$ in Curve 1, where $\varepsilon/2$ can be taken as a reference of the dynamical coupling between $X = \{x_n(50)\}$ and $Y = \{x_n(51)\}$. Figure 2 shows that the value of μ decreases with the increase of the distance between two lattices for the same value of ε. This means the dynamical coupling strength decreases with the increase of the distance between two lattices.

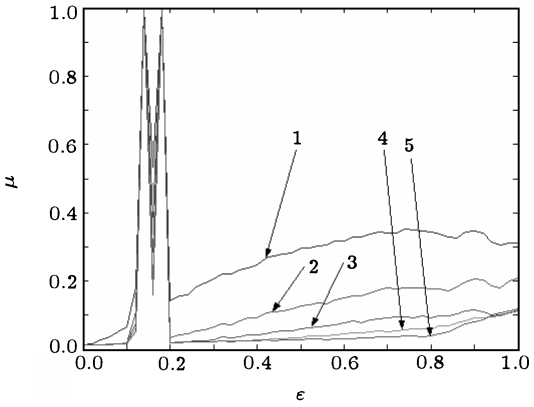


Fig.2. The dynamical coupling measure μ vs the coupling coefficient ε for the one-dimensional, homogeneous, diffusively coupled map lattices (11). Curve 1 is for $X = \{x_n(50)\}$ and $Y = \{x_n(51)\}$, and Curve 2 for $X = \{x_n(50)\}$ and $Y = \{x_n(52)\}$, Curve 3 for $X = \{x_n(50)\}$ and $Y = \{x_n(53)\}$, Curve 4 for $X = \{x_n(50)\}$ and $Y = \{x_n(54)\}$, and Curve 5 for $X = \{x_n(50)\}$ and $Y = \{x_n(55)\}$, respectively.

From the results presented by Figs.1 and 2, we can conclude that if we take the coupling coefficient ε as a reference of the coupling strength for the map lattices (10), or if we take the coupling coefficient $\varepsilon/2$ as a reference of the coupling strength for the map lattices (11), then the introduced information measure μ is an effective index for measuring the dynamical coupling between two time series.

4. Conclusion

In this paper we have introduced an information-theoretic measure to evaluate the dynamical coupling between spatiotemporally chaotic signals produced by extended systems. We have computed the measure of the one-way coupled map lattices and the one-dimensional, homogeneous, diffusively coupled map lattices with the symbolic analysis method. The numerical results show that the dynamical coupling presented by the information measure μ increases approximately with the increase of coupling coefficient ε, and decreases generally with the increase of the distance between two lattices. This characteristic of the dynamical coupling is consistent with the form of the map lattices. So the information measure is applicable to determining the dynamical coupling between two directly coupled or indirectly coupled time series. As the usual correlation function is incompetent in qualifying the dynamical coupling between two time series produced by nonlinear dynamical systems, so it is expected that the information measure introduced in this paper could be used for this purpose.

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