Vol 15 No 7, July 2006
1009-1963/2006/15(07)/1454-06
Chinese Physics
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and IOP Publishing Ltd

An observer based asymptotic trajectory control using a scalar state for chaotic systems

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(Received 19 January 2006; revised manuscript received 8 February 2006)

A state-observer based full-state asymptotic trajectory control (OFSTC) method requiring a scalar state is presented to asymptotically drive all the states of chaotic systems to arbitrary desired trajectories. It is no surprise that OFSTC can obtain good tracking performance as desired due to using a state-observer. Significantly OFSTC requires only a scalar state of chaotic systems. A sinusoidal wave and two chaotic variables were taken as illustrative tracking trajectories to validate that using OFSTC can make all the states of a unified chaotic system track the desired trajectories with high tracking accuracy and in a finite time. It is noted that this is the first time that the state-observer of chaotic systems is designed on the basis of Kharitonov's Theorem.

Keywords: trajectory control, chaotic control, state observer, a unified chaotic system
PACC: 0545

1. Introduction

During the last two decades, much attention has been paid to controlling chaotic systems for engineering applications.[1−9] However, most tracking control approaches involved driving a chaotic system to an equilibrium point (e.g. the origin), and in addition, some strategies (e.g. our previous reports Ref.[7–8]) drove only one system state to a desired trajectory. To date, to our knowledge, driving all chaotic system states to arbitrary desired trajectories has not been fully reported. Some tracking control methods (e.g., neural network, fuzzy and variable structure control) requiring all system states exhibited some drawbacks in practice. It is still significant to propose a tracking approach by using only a scalar state.

In this paper, we present a state observer based full-state asymptotic tracking control (OFSTC) requiring a scalar state for chaotic systems. Since an asymptotic state-observer requiring only a scalar state is proposed to estimate all the states of chaotic systems, it is easy for OFSTC to guarantee sufficiently good control performance as desired. A unified chaotic system (UCS)[10] is illustrated as an example to validate that the proposed method can asymptotically drive all the states of the UCS to arbitrary desired trajectories. We have briefly introduced the developed synchronization approaches and tracking control methods (e.g. Refs.[11–18]) for the UCS in Ref.[19] and also proposed a full-state asymptotic control strategy therein. Compared with our previous result Ref.[19], the proposed approach here adopts a state-observer and hence achieves better control performance; in addition, it is more general and can also be applied to many chaotic systems. Indeed our previous approach considered for the case that only the second state of the UCS is measurable, whereas the proposed approach here can be applied to more cases. As we see below, we use Kharitonov's Theorem[20,21] to solve the synchronization problem (or observability problem). To our knowledge, this is the first time Kharitonov's Theorem has been used for the synchronization problem (or observability problem). By simulation, a sinusoidal wave and two chaotic variables were taken as illustrative tracking trajectories to validate that using OFSTC can make all the states of the UCS track the desired trajectories with high tracking accuracy and in a finite time.

2. Principle of OFSTC

Consider a general chaotic system modelled by

$$\dot{x}_i = f_i(x) + u_i, \quad 1 \le i \le n, \quad s = x_j, \qquad (1)$$

where $x = (x_1, \ldots, x_n)^{\mathrm{T}}$; u_i are control signals; and s is a measurable scalar state.

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For system (1), OFSTC is designed as

$$
\begin{aligned}
&u_i = \rho_i - \sum_{k=1}^{n} \eta_{ik}(x_{\mathrm{d}}) \cdot (\hat{x}_k - x_{k\mathrm{d}}),\\
&\dot{\hat{x}}_i = f_i(\hat{x}_1, \ldots, \hat{x}_{j-1}, s, \hat{x}_{j+1}, \ldots, \hat{x}_n) + u_i - \sigma_i(\hat{x}_j - s),\\
&\rho_i = \dot{x}_{i\mathrm{d}} - f_i(x_{\mathrm{d}}),
\end{aligned} \tag{2}
$$

where $1 \le i \le n$; $x_{\mathrm{d}} = (x_{1d}, \ldots, x_{nd})^{\mathrm{T}}$ is the desired trajectory vector; $\eta_{ik}(x_{\mathrm{d}})$ and σ_i are parameters to be specified.

Using Eq.(2) in Eq.(1) we get

$$
\begin{aligned}
\dot{e}_i =& f_i(x) - f_i(x_{\mathrm{d}}) - \sum_{k=1}^{n} \eta_{ik}(x_{\mathrm{d}}) \cdot e_k\\
&- \sum_{k=1}^{n} \eta_{ik}(x_{\mathrm{d}}) \cdot w_k
\end{aligned} \tag{3a}
$$

and

$$
\begin{aligned}
\dot{w}_i =& f_i(\hat{x}_1, \ldots, \hat{x}_{j-1}, s, \hat{x}_{j+1}, \ldots, \hat{x}_n)\\
&- f_i(x) - \sigma_i w_j
\end{aligned} \tag{3b}
$$

for $1 \le i \le n$, where $e_i = x_i - x_{i\mathrm{d}}$, $w_i = \hat{x}_i - x_i$.

Obviously, for system (1), the full-state trajectories control can be achieved if (i) the state-observer (3b) can estimate all the states asymptotically; and (ii) $\eta_{ik}(x_{\mathrm{d}})$ are appropriately chosen such that system

$$
\dot{e}_i = f_i(x) - f_i(x_{\mathrm{d}}) - \sum_{k=1}^{n} \eta_{ik}(x_{\mathrm{d}}) \cdot e_k
$$

with $1 \le i \le n$ are asymptotically stable at the origin.

For the condition (i), we design the state-observer here in terms of the ordinary differential equation theorem and Kharitonov's Theorem instead of the widely-used Lyapunov function. Since $f_i(.)$ are always Lipshitz functions for most chaotic systems and hence theoretically we can design large enough Hurwitz polynomial coefficients $\eta_{ik}(x_{\mathrm{d}})$ such that condition (ii) is satisfied. As we see below, using OFSTC in the UCS we can asymptotically drive all the states of the UCS to arbitrary desired trajectories.

3. OFSTC for the UCS

The controlled UCS can be described as

$$
\begin{aligned}
&\dot{x}_1 = (25\alpha + 10)(x_2 - x_1) + u_1,\\
&\dot{x}_2 = (28 - 35\alpha)x_1 - x_1 x_3 + (29\alpha - 1)x_2 + u_2,\\
&\dot{x}_3 = x_1 x_2 - (\alpha + 8)x_3/3 + u_3,
\end{aligned} \tag{4}
$$

where u_1, u_2, u_3 are control inputs.

We will use the following Theorems 1 and 2 to design OFSTC for two cases, namely $s = x_1$ and $s = x_2$.

Theorem 1 When $s = x_1$, adopt following control law in system (4):

$$
\begin{aligned}
&u_1 = \rho_1 - \eta_1(\hat{x}_1 - x_{1\mathrm{d}}) - (25\alpha + 10)(\hat{x}_2 - x_{2\mathrm{d}}),\\
&u_2 = \rho_2 - (\eta_2 + 29\alpha - 1)(\hat{x}_2 - x_{2\mathrm{d}}),\\
&u_3 = \rho_3 - \eta_3(\hat{x}_3 - x_{3\mathrm{d}}) - x_{1\mathrm{d}}(\hat{x}_2 - x_{2\mathrm{d}}),
\end{aligned} \tag{5a}
$$

$$
\begin{aligned}
&\dot{\hat{x}}_1 = (25\alpha + 10)(\hat{x}_2 - s) - K(\hat{x}_1 - s) + u_1,\\
&\dot{\hat{x}}_2 = (28 - 35\alpha)s - s\hat{x}_3 + (29\alpha - 1)\hat{x}_2\\
&\qquad - 29K(\hat{x}_1 - s)/25 + u_2,\\
&\dot{\hat{x}}_3 = s\hat{x}_2 - (\alpha + 8)\hat{x}_3/3 + u_3,
\end{aligned} \tag{5b}
$$

where $x_{1\mathrm{d}}$, $x_{2\mathrm{d}}$ and $x_{3\mathrm{d}}$ are the desired smooth trajectories, η_1, η_2, $\eta_3 > 0$ and $K > 28 + 5\max(s^2)/63$, and

$$
\begin{aligned}
&\rho_1 = \dot{x}_{1\mathrm{d}} - (25\alpha + 10)(x_{2\mathrm{d}} - x_{1\mathrm{d}}),\\
&\rho_2 = \dot{x}_{2\mathrm{d}} - (29\alpha - 1)x_{2\mathrm{d}} - (28 - 35\alpha)x_{1\mathrm{d}} + x_{1\mathrm{d}}x_{3\mathrm{d}},\\
&\rho_3 = \dot{x}_{3\mathrm{d}} - x_{1\mathrm{d}}x_{2\mathrm{d}} + (\alpha + 8)x_{3\mathrm{d}}/3.
\end{aligned} \tag{6}
$$

Then all the states of system (4) can asymptotically track the desired trajectories, i.e., $x_1 = x_{1\mathrm{d}}$, $x_2 = x_{2\mathrm{d}}$ and $x_3 = x_{3\mathrm{d}}$ when $t \to \infty$.

Proof: Using (5b) in (4) we get

$$
\begin{aligned}
&\dot{w}_1 = -Kw_1 + (25\alpha + 10)w_2,\\
&\dot{w}_2 = -29Kw_1/25 + (29\alpha - 1)w_2 - sw_3,\\
&\dot{w}_3 = sw_2 - (8 + \alpha)w_3/3
\end{aligned} \tag{7}
$$

with $w_1 = \hat{x}_1 - x_1$, $w_2 = \hat{x}_2 - x_2$, $w_3 = \hat{x}_3 - x_3$, i.e.,

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} w_1\\ w_2\\ w_3 \end{pmatrix} &= \begin{bmatrix} -K & 25\alpha + 10 & 0\\ -29K/25 & 29\alpha - 1 & -s\\ 0 & s & -(8 + \alpha)/3 \end{bmatrix}\\
&\quad \times \begin{pmatrix} w_1\\ w_2\\ w_3 \end{pmatrix} \stackrel{\mathrm{def}}{=} A \begin{pmatrix} w_1\\ w_2\\ w_3 \end{pmatrix}.
\end{aligned} \tag{8}
$$

Let $\det(\lambda I - A) = 0$, then

$$
\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{9}
$$

where

$$
\begin{aligned}
&a_1 = K + (8 + \alpha)/3 - 29\alpha + 1,\\
&a_2 = 63K/5 + (8 + \alpha)(K - 29\alpha + 1)/3 + s^2,\\
&a_3 = 21(8 + \alpha)K/5 + K\,s^2.
\end{aligned}
$$

In terms of Kharitonov's Theorem,[20,21] the asymptotic stability of the system (7) is guaranteed by the following inequalities:

$$a_1,\ \underline{a}_2, \underline{a}_3 > 0, \qquad a_1\underline{a}_2 > \bar{a}_3 \tag{10}$$

with $\underline{a}_2 = \min(a_2)$, $\underline{a}_3 = \min(a_3)$, $\bar{a}_3 = \max(a_3)$.

Note that $a_1 \geq K - 25$ and $\underline{a}_2 > 63K/5$ are satisfied for all $\alpha \in [0,\ 1]$. Then Eq.(10) is achieved if the following inequalities are guaranteed:

$$K > 25, \quad 63(K-25)/5 > 21(8+a)/5 + \max(s^2). \tag{11}$$

Clearly (11) is satisfied if $K > 28 + 5\max(s^2)/63$ is designed. Then

$$\lim_{t\to\infty} w_1(t) = \lim_{t\to\infty} w_2(t) = \lim_{t\to\infty} w_3(t) = 0. \tag{12}$$

On the other hand, substituting Eq.(5a) into Eq.(4) we obtain

$$\begin{aligned}
\dot{e}_1 =& -(25\alpha + 10 + \eta_1)e_1 - \eta_1 w_1 - (25\alpha + 10)w_2,\\
\dot{e}_2 =& (28 - 35\alpha - x_{3\mathrm{d}})e_1 - \eta_2 e_2 - x_{1\mathrm{d}}e_3 - e_1e_3\\
&- (\eta_2 + 29\alpha - 1)w_2,\\
\dot{e}_3 =& x_{2\mathrm{d}}e_1 - [\eta_3 + (\alpha+8)/3]e_3\\
&+ e_1e_2 - \eta_3 w_3 - x_{1\mathrm{d}}w_2,
\end{aligned} \tag{13}$$

where $e_1 = x_1 - x_{1\mathrm{d}}$, $e_2 = x_2 - x_{2\mathrm{d}}$, $e_3 = x_3 - x_{3\mathrm{d}}$.

Therefore, when $t \to \infty$, using Eq.(12) in Eq.(13) we can write the remaining equations of Eq.(13) as

$$\begin{aligned}
\dot{e}_1 =& -(25\alpha + 10 + \eta_1)e_1,\\
\dot{e}_2 =& (28 - 35\alpha - x_{3\mathrm{d}})e_1 - \eta_2 e_2 - x_{1\mathrm{d}}e_3 - e_1e_3,\\
\dot{e}_3 =& x_{2\mathrm{d}}e_1 - [(\alpha+8)/3 + \eta_3]e_3 + e_1e_2.
\end{aligned} \tag{14}$$

For the first equation in Eq.(14), we have

$$\dot{e}_1 = -(25\alpha + 10 + \eta_1)e_1, \tag{15}$$

which indicates

$$\lim_{t\to\infty} e_1(t) = 0. \tag{16}$$

Substituting Eq.(16) into the third equation we get

$$\dot{e}_3 = -[(\alpha+8)/3 + \eta_3]e_3, \tag{17}$$

which leads to

$$\lim_{t\to\infty} e_3(t) = 0. \tag{18}$$

Using Eqs.(16) and (18) in the second equation of Eq.(14) we have

$$\dot{e}_2 = -\eta_2 e_2, \tag{19}$$

which implies that

$$\lim_{t\to\infty} e_2(t) = 0. \tag{20}$$

Thus Theorem 1 is proved.

Theorem 2 When $s = x_2$, adopt the following control law in system (4):

$$\begin{aligned}
u_1 =& \rho_1 - \eta_1(\hat{x}_1 - x_{1\mathrm{d}}) - (25\alpha + 10)(\hat{x}_2 - x_{2\mathrm{d}}),\\
u_2 =& \rho_2 - (\eta_2 + 29\alpha - 1)(\hat{x}_2 - x_{2\mathrm{d}}),\\
u_3 =& \rho_3 - \eta_3(\hat{x}_3 - x_{3\mathrm{d}}) - x_{1\mathrm{d}}(\hat{x}_2 - x_{2\mathrm{d}}),
\end{aligned} \tag{21a}$$

$$\begin{aligned}
\dot{\hat{x}}_1 =& (25\alpha + 10)(s - \hat{x}_1) + u_1,\\
\dot{\hat{x}}_2 =& (28 - 35\alpha)\hat{x}_1 - \hat{x}_1\hat{x}_3 + (29\alpha - 1)s - K(\hat{x}_2 - s) + u_2,\\
\dot{\hat{x}}_3 =& \hat{x}_1 s - (\alpha+8)\hat{x}_3/3 + u_3,
\end{aligned} \tag{21b}$$

where $x_{1\mathrm{d}}$, $x_{2\mathrm{d}}$ and $x_{3\mathrm{d}}$ are the desired smooth trajectories; K, η_1, η_2, $\eta_3 > 0$, and

$$\begin{aligned}
\rho_1 =& \dot{x}_{1\mathrm{d}} - (25\alpha + 10)(x_{2\mathrm{d}} - x_{1\mathrm{d}}),\\
\rho_2 =& \dot{x}_{2\mathrm{d}} - (29\alpha - 1)x_{2\mathrm{d}} - (28 - 35\alpha)x_{1\mathrm{d}} + x_{1\mathrm{d}}x_{3\mathrm{d}},\\
\rho_3 =& \dot{x}_{3\mathrm{d}} - x_{1\mathrm{d}}x_{2\mathrm{d}} + (\alpha+8)x_{3\mathrm{d}}/3.
\end{aligned} \tag{22}$$

Then all the states of system (4) can asymptotically track the desired trajectories, i.e., $x_1 = x_{1\mathrm{d}}$, $x_2 = x_{2\mathrm{d}}$ and $x_3 = x_{3\mathrm{d}}$ when $t \to \infty$.

Proof Using Eq.(21b) into Eq.(4) we get

$$\begin{aligned}
\dot{w}_1 =& -(25\alpha + 10)w_1,\\
\dot{w}_2 =& (28 - 35\,\alpha - \hat{x}_3)w_1 - K\,w_2 - x_1\,w_3,\\
\dot{w}_3 =& x_2 w_1 - (8 + \alpha)w_3/3.
\end{aligned} \tag{23}$$

For the first equation in Eq.(23), we have

$$\lim_{t\to\infty} w_1(t) = 0. \tag{24}$$

Since x_2 is chaotic and hence bounded, using Eq.(24) in the third equation of Eq.(23) we get

$$\dot{w}_3 = -(8 + \alpha)w_3/3, \tag{25}$$

which implies that

$$\lim_{t\to\infty} w_3(t) = 0. \tag{26}$$

Note that x_3 is chaotic and thus bounded and in addition $w_3 = \hat{x}_3 - x_3$. Then Eq.(26) indicates that $\hat{x}_3$ also is bounded. In addition, x_1 is chaotic and hence bounded. Using Eqs.(24) and (26) in the second equation of Eq.(23) we obtain

$$\dot{w}_2 = -Kw_2, \tag{27}$$

which indicates

$$\lim_{t\to\infty} w_2(t) = 0. \tag{28}$$

Then we have

$$\lim_{t\to\infty} w_1(t) = \lim_{t\to\infty} w_2(t) = \lim_{t\to\infty} w_3(t) = 0. \tag{29}$$

Substituting Eq.(21a) into Eq.(4) we get

$$\begin{aligned}
\dot{e}_1 =& -(25\alpha + 10 + \eta_1)e_1 - \eta_1 w_1 - (25\alpha + 10)w_2,\\
\dot{e}_2 =& (28 - 35\alpha - x_{3\mathrm{d}})e_1 - \eta_2 e_2 - x_{1\mathrm{d}} e_3 - e_1 e_3\\
&- (\eta_2 + 29\alpha - 1)w_2,\\
\dot{e}_3 =& x_{2\mathrm{d}} e_1 - [\eta_3 + (\alpha + 8)/3]e_3\\
&+ e_1 e_2 - \eta_3 w_3 - x_{1\mathrm{d}} w_2.
\end{aligned} \tag{30}$$

Clearly Eq.(30) has the same form as Eq.(13), then

$$\lim_{t\to\infty} e_1(t) = \lim_{t\to\infty} e_2(t) = \lim_{t\to\infty} e_3(t) = 0.$$

This implies that Theorem 2 has been proved.

Remark 1 When the desired trajectories are taken to be chaotic states which are chosen from partial or full states of a chaotic system or from partial states of different chaotic systems, the OFSTC can make the UCS synchronize to the chosen chaotic states and also generate a new chaotic system as desired.

Remark 2 It is clear from Eqs.(5) and (21) that both controllers use the derivatives of the desired trajectories. Therefore, the desired trajectories must be differentiable. This is the reason why we assume that the desired trajectories are smooth. In addition, in the case that the desired trajectories are differentiable almost everywhere, the same results can still be guaranteed except at the singularity points.

Remark 3 We have proposed two state-observers for both cases, namely $s = x_1$ and $s = x_2$ respectively. This implies that the two state-observers can directly be applied to the synchronization of two different UCSs by a coupling signal x_1 and x_2 respectively.

4. Simulation

Parameters of the controlled UCS were chosen as

$$\begin{aligned}
&\alpha = 0.8, \qquad x_1(0) = x_3(0) = -1,\\
&x_2(0) = 1, \qquad \hat{x}_1(0) = \hat{x}_2(0) = \hat{x}_3(0) = 0.
\end{aligned} \tag{31}$$

The desired trajectories were taken to be $x_{1\mathrm{d}} = z_1$, $x_{2\mathrm{d}} = \theta_1$ and $x_{3\mathrm{d}} = \sin(t)$ respectively, where

$$\begin{aligned}
\dot{z}_1 &= z_2,\\
\dot{z}_2 &= \bar{a}\, z_1 + \bar{b} z_1^3 + \bar{c} z_2 + f\cos(\bar{w}t),\\
x_{1\mathrm{d}} &= z_1
\end{aligned} \tag{32a}$$

with parameters $\bar{a} = 1$, $\bar{b} = -1$, $\bar{c} = -0.25$, $f = 0.4$, $\bar{w} = 1$, $z_1(0) = 0$, $z_2(0) = 1$; and

$$\begin{aligned}
&\dot{\theta}_1 = -\theta_2 - \theta_3, \qquad \theta_2 = \theta_1 + \bar{\bar{a}}\theta_2,\\
&\theta_3 = \bar{\bar{b}} + (x_1 - \bar{\bar{c}})\theta_3, \quad x_{2\mathrm{d}} = \theta_1
\end{aligned} \tag{32b}$$

with parameters $\bar{\bar{a}} = 0.2$, $\bar{\bar{b}} = 0.2$, $\bar{\bar{c}} = 5.7$, $\theta_1(0) = 0$, $\theta_2(0) = \theta_3(0) = 1$.

Obviously the desired trajectories $x_{1\mathrm{d}}$ and $x_{2\mathrm{d}}$ are the states of the well-known *Duffing* and Rössler systems, respectively. Systems (32a) and (32b) exhibit chaotic behaviour with the chosen parameters. Clearly the desired trajectories $x_{1\mathrm{d}}$, $x_{2\mathrm{d}}$ and $x_{3\mathrm{d}}$ were typical.

Simulation for the case $s = x_1$ is plotted in Figs.1–3, where $\eta_1 = \eta_2 = \eta_3 = K = 10$ is chosen in the control law (5).

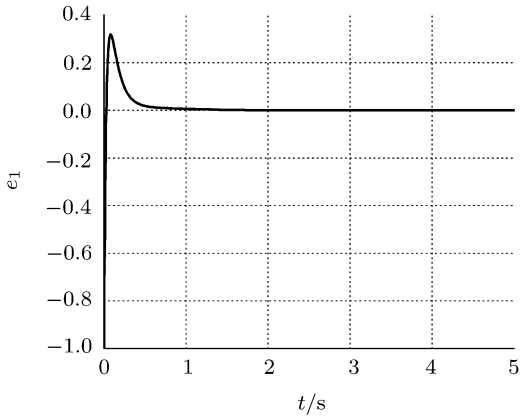


Fig.1. Time response of tracking error $e_1(t)$.

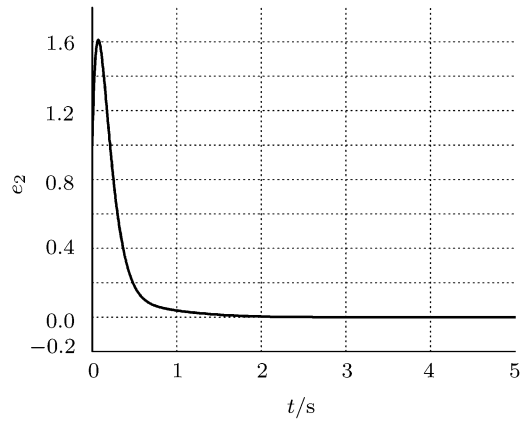


Fig.2. Time response of tracking error $e_2(t)$.

For the case $s = x_2$, we have simulation plotted in Figs.4–6, where $\eta_1 = \eta_2 = \eta_3 = 10$ and $K = 30$ are designed in the control law (21). It is easily seen

from Figs.1–3 that states x_1, x_2 and x_3 can track $x_{1\mathrm{d}}$, $x_{2\mathrm{d}}$ and $x_{3\mathrm{d}}$ respectively, with high accuracy and in a finite time. From Figs.4–6, we also find that states x_1, x_2 and x_3 can track $x_{1\mathrm{d}}$, $x_{2\mathrm{d}}$ and $x_{3\mathrm{d}}$ respectively, with high accuracy and in a finite time. By simulation, we have also validated state-observers (5b) and (21b) can estimate the states x_1, x_2 and x_3 with high accuracy and in a finite time (not plotted here for the sake of compactness).

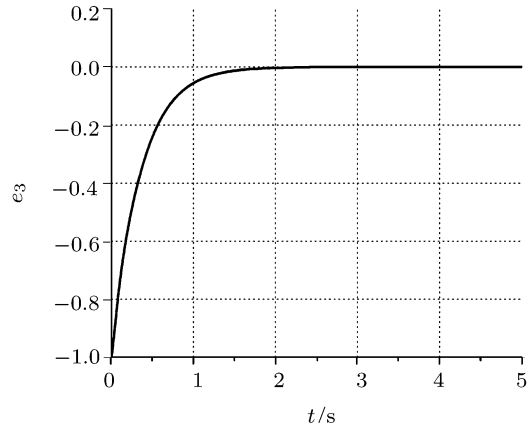


Fig.3. Time response of tracking error $e_3(t)$.

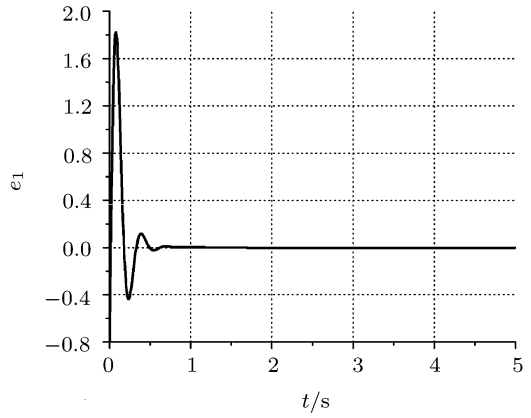


Fig.4. Time response of tracking error $e_1(t)$.

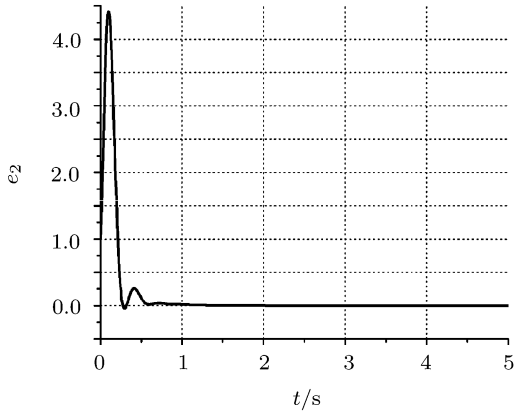


Fig.5. Time response of tracking error $e_2(t)$.

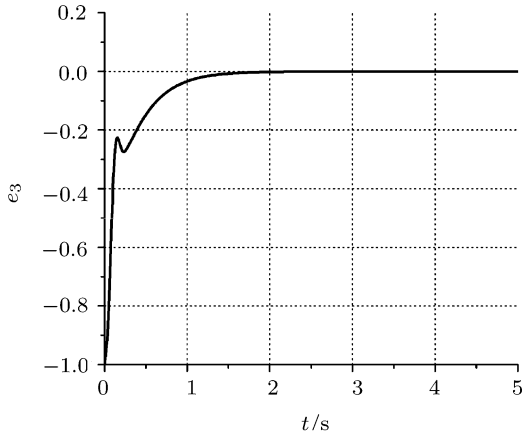


Fig.6. Time response of tracking error $e_3(t)$.

5. Conclusions

In this paper, we present the OFSTC approach to asymptotically drive all the states of chaotic systems to arbitrary desired trajectories. Significant OFSTC requires only a scalar state. A sinusoidal wave and two chaotic variables were taken as illustrative tracking trajectories to validate that using OFSTC in the UCS can make all the states track the desired trajectories asymptotically.

Since chaotic synchronization and anti-chaos problems can be expressed as a tracking control problem, the proposed OFSTC can also be applied to the synchronization and the chaos generation. Indeed, by analysis and simulation, we have validated that the state-observer used for the UCS can directly be applied to the synchronization of two different UCSs by a coupling signal x_1 or x_2.

It is deserves to be noted that this is the first time to design the state-observer of chaotic systems in terms of Kharitonov's Theorem. We have systematically investigated the synchronization according to Kharitonov's Theorem in another forthcoming paper submitted to the *Physical Review E*.

References

[1] Pecora L and Carroll T 1990 *Phys. Rev. Lett.* **64** 821

[2] Chen G and Dong X N 1993 *IEEE Trans. Circ. Syst.* I **40** 591

[3] Gao J F, Luo X J, Ma X K, Pan X Q and Wang J K 1999 *Acta Phys. Sin.* **48** 1618 (in Chinese)

[4] Guan X P, Fan Z P, Peng H P and Wang Y Q 2001 *Acta Phys. Sin.* **50** 2108 (in Chinese)

[5] Liu D, Ren H P and Kong Z Q *Acta Phys. Sin.* 2003 **52** 531 (in Chinese)

[6] Yue D D and Jun Y 2003 *Acta Phys. Sin.* **52** 292 (in Chinese)

[7] Yu D C and Meng Q H 2005 *Acta Phys. Sin.* **54** 1092 (in Chinese)

[8] Yu D C and Wu A G 2005 *Chin. Phys.* **14** 914

[9] Yu D C and Wu A G 2006 *Chin. Phys.* **15** 95

[10] Lü J H, Chen G, Cheng D Z and Celikovsky S 2002 *Int. J. Bifurcation and Chaos* **12** 2917

[11] Tao C H, Lu J A and Lü J H 2002 *Acta Phys. Sin.* **51** 1497 (in Chinese)

[12] Tao C H and Lu J A 2003 *Acta Phys.Sin.* **52** 281 (in Chinese)

[13] Liu J, Chen S H and Lu J A 2003 *Acta Phys. Sin.* **52** 1595 (in Chinese)

[14] Liu J, Chen S H and Xie J 2004 *Chaos, Solitons & Fractals* **19** 533

[15] Chen S H and Lü J H 2002 *Chaos, Solitons and Fractals* **14** 643

[16] Park J 2005 *Chaos, Solitons and Fractals* **25** 69

[17] Lu J A, Wu X, Han X and Lü J 2004 *Phys. Lett.* A **329** 327

[18] Lu J A, Wu X and Lü J H 2002 *Phys. Lett.* A **305** 365

[19] Yu D C and Wu A G 2006 *Chin. Phys.* **15** 306

[20] Kharitonov V L 1979 *Differential Equations* **78** 1483

[21] Anderson B, Jury E and Mansour M 1987 *IEEE Trans. Automat. Control* **32** 909