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Restudy of the stability problem of the Schwarzschild black hole*

Tian Gui-Hua(田贵花)[a)b)], Wang Shi-Kun(王世坤)[b)], and Zhao Zheng(赵 峥)[c)]

[a)] *School of Science, Beijing University of Posts and Telecommunications, Beijing* 100876, *China*
[b)] *Key Laboratory of Mathematics Mechanization, Academy of Mathematics and System Science, Chinese Academy of Sciences, Beijing* 100080, *China*
[c)] *Department of Physics, Beijing Normal University, Beijing* 100875, *China*

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The stability of the Schwarzschild black hole is restudied in the Painlevé coordinates. Using the Painlevé time coordinate to define the initial time, we reconsider the odd perturbation and find that the Schwarzschild black hole in the Painlevé coordinates is unstable. The Painlevé metric in this paper corresponds to the white-hole-connected region of the Schwarzschild black hole ($r > 2m$) and the odd perturbation may be regarded as the angular perturbation. Therefore, the white-hole-connected region of the Schwarzschild black hole is unstable with respect to the rotating perturbation.

Keywords: Schwarzschild black hole, angular perturbation, white-hole-connected region of the Schwarzschild black hole ($r > 2m$)
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1. The stability problem

Regge and Wheeler first treated the stability problem of the Schwarzschild black hole in 1957. They divided the perturbations into two classes: odd and even ones.[1] Later, it was found that the odd one really represents the angular perturbation to the metric, while the even one is the radial perturbation to the metric.[2]

The Schwarzschild coordinates have been used to study the stability problem in the work of Regge and Wheeler. The background metric has an apparent singularity at the horizon $r = 2m$ in the Schwarzschild coordinates. So, it is very difficult to discriminate the real divergence in the perturbation at $r = 2m$ from a spurious one caused by the improper choice of the coordinates. The Kruskal reference frame is singularity-free at the horizon $r = 2m$. Vishveshwara[3] proposed a solution to the problem. He transformed the perturbation fields $h^{\rm s}_{\mu\nu}(t_{\rm s}, r, \theta, \phi)$ in the Schwarzschild coordinates into $h^{\rm k}_{\mu\nu}(v, u, \theta, \phi)$ of the Kruskal reference frame, where $(t_{\rm s}, r, \theta, \phi)$ and (v, u, θ, ϕ) are the Schwarzschild and Kruskal coordinates respectively. The perturbation fields $h^{\rm k}_{\mu\nu}(v, u, \theta, \phi)$ should have no spurious divergence, though $h^{\rm s}_{\mu\nu}$ have. Nevertheless, there arises the problem what time coordinate should be chosen to define the initial time. Though $h^{\rm k}_{\mu\nu}(v, u, \theta, \phi)$ apparently depends on the Kruskal coordinates (v, u, θ, ϕ), it is the Schwarzschild time coordinate $t_{\rm s}$ that Vishveshwara used to define the initial time. In this way, the Schwarzschild black hole is proven stable by Vishveshwara.[3]

In fact, the metric in the Schwarzschild coordinates has another obvious drawback: the time coordinate $t_{\rm s}$ (usually denoted as t in references) is not proper for analysis of the physical process at the horizon $r = 2m$.[4] For example, a process near the horizon $r = 2m$ is described in the Schwarzschild time coordinate as from $t_{\rm s} \to 0,\ r \to 2m$ to $t_{\rm s} \to +\infty,\ r \to 2m$. The time coordinate $t_{\rm s}$ changes infinitely in the process. But in Kruskal time coordinate v, which is more realistic physically, it is just from $v \to 0,\ u \to 0$ with $\frac{v}{u} \to 0$ to $v \to 0,\ u \to 0$ with $\frac{u}{v} \to +1$. That is, at $r = 2m$, the infinite change in $t_{\rm s}$ only corresponds to the infinitesimal change in v. The example clearly shows that the Schwarzschild time coordinate $t_{\rm s}$ loses its meaning near or at $r = 2m$.

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†Corresponding author: E-mail: hua2007@126.com cc tgh-2000@263.net

Therefore, it is not unreasonable to suspect the correctness of the conclusion of Vishveshwara's which is obtained by the Schwarzschild time coordinate. One should choose a 'good' time coordinate to study the stability problem and to find what results would be obtained consequently.[5−13]

2. Argument for the Painlevé metric

There are several well-known coordinates for the Schwarzschild black-hole, which are regular at $r = 2m$: for example, the Kruskal coordinate system. Here, we mainly select Painlevé coordinates for the study of the stability problem. The Painlevé coordinates were discovered independently by Painlevé[14] in 1921 and Kraus and Wilczck[15,16] in 1994. The Painlevé metric is

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2m}{r}\right)\mathrm{d}t_{\mathrm{p}}^2 \\ & -2\sqrt{\frac{2m}{r}}\mathrm{d}r\mathrm{d}t_{\mathrm{p}}+\mathrm{d}r^2+r^2\mathrm{d}\Omega^2 \\ =& -\mathrm{d}t_{\mathrm{p}}^2+\left(\mathrm{d}r-\sqrt{\frac{2m}{r}}\mathrm{d}t_{\mathrm{p}}\right)^2+r^2\mathrm{d}\Omega^2. \end{aligned} \tag{1}$$

It is obtained from Schwarzschild metric

$$\mathrm{d}s^2=-\left(1-\frac{2m}{r}\right)\mathrm{d}t_{\mathrm{s}}^2+\left(1-\frac{2m}{r}\right)^{-1}\mathrm{d}r^2+r^2\mathrm{d}\Omega^2 \tag{2}$$

by the transformation

$$t_{\mathrm{p}}=t_{\mathrm{s}}-\left[2\sqrt{2mr}+2m\ln\frac{\sqrt{r}-\sqrt{2m}}{\sqrt{r}+\sqrt{2m}}\right]. \tag{3}$$

The Painlevé metric (1) is well-behaved at the horizon. It is obvious that the radial null geodesics are

$$\frac{\mathrm{d}r}{\mathrm{d}t_{\mathrm{p}}}=\pm1+\sqrt{\frac{2m}{r}}. \tag{4}$$

The upper and lower classes correspond to the outgoing and ingoing null geodesics respectively. In the usual Penrose diagram, part I corresponds to our region ($r > 2m$), parts II and II' are the black-hole ($r < 2m$) and white-hole ($r < 2m$) regions respectively, and part I' is another region ($r > 2m$) not communicating with our region. The metric (1) corresponds to the region I connected with II', and is defined as the white-hole-connected region.

Not only is the Painlevé metric stationary and regular at the horizon,[15−17] but also its time coordinate t_{p} is time-like for $r > 2m$. These good qualities make it more suitable for the study of stability of the Schwarzschild black hole. Of course, to carry out perturbation analysis entirely in the Painlevé metric would be very burdensome. Nevertheless, the perturbation fields $h_{\mu\nu}$ are tensors and the perturbation equations are also tensor equations (see Eq.(5) in the following); they are invariant under the coordinate transformation. One could use Vishveshwara's result in the Schwarzschild coordinates, and transform them to the Painlevé coordinates and use the time coordinate t_{p} to study the stability problem. In this way, we obtain the conclusion that the white-hole-connected region of the Schwarzschild black hole is unstable .

3. New discussion of the Regge–Wheeler equation

Here, we briefly explain the perturbation of the Schwarzschild black hole.[1,18,19] Suppose the background metric is $g_{\mu\nu}$, while the perturbation in it is $h_{\mu\nu}$, then the contract Ricci tensors $R_{\mu\nu}$ and $R_{\mu\nu}+\delta R_{\mu\nu}$ correspond to the metrics $g_{\mu\nu}$ and $g_{\mu\nu}+h_{\mu\nu}$ respectively. The linear perturbation equations are

$$\delta R_{\mu\nu}=-\delta\Gamma^{\beta}_{\mu\nu;\beta}+\delta\Gamma^{\beta}_{\mu\beta;\nu}=0, \tag{5}$$

where the semicolons in the subscripts indicate the covariant differentiation, and

$$\delta\Gamma^{\beta}_{\mu\nu}=\frac{1}{2}g^{\beta\alpha}\left[h_{\mu\alpha;\nu}+h_{\nu\alpha;\mu}-h_{\mu\nu;\alpha}\right] \tag{6}$$

is a tensor. After consideration of the gauge freedom, the odd perturbations are

$$h_{\mu\nu}=\begin{vmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ h_0(r) & h_1(r) & 0 & 0 \end{vmatrix} \mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}\left[\sin\theta\frac{\partial}{\partial\theta}\right]P_l\left(\cos\theta\right),$$

and the even ones (parametrized by H_0, H_1, H_2, K) are

$$h_{\mu\nu}=\begin{vmatrix} H_0(1-\frac{2m}{r}) & H_1 & 0 & 0 \\ H_1 & H_2(1-\frac{2m}{r})^{-1} & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2K\sin^2\theta \end{vmatrix} \mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}P_l\left(\cos\theta\right).$$

Here we mainly discuss the odd perturbation. In the odd perturbation, the linear equations (5) are combined into a single equation: that is, the Regge–Wheeler equation

$$\frac{\mathrm{d}^2Q}{\mathrm{d}r^{*2}}+\left[k^2-V\right]Q=0, \tag{7}$$

where the effective potential V and the tortoise coordinate are

$$V=\left(1-\frac{2m}{r}\right)\left[\frac{l\left(l+1\right)}{r^2}-\frac{6m}{r^3}\right], \tag{8}$$

$$r^*=r+2m\ln\left(\frac{r}{2m}-1\right), \tag{9}$$

respectively. The perturbation fields $h_0(r)$ and $h_1(r)$ are connected with Q by

$$\begin{aligned} h_0(r)=&\frac{\mathrm{i}}{k}\frac{\mathrm{d}}{\mathrm{d}r^*}\left(rQ\right) \\ =&\frac{\mathrm{i}}{k}\left[\left(1-\frac{2m}{r}\right)Q+r\frac{\mathrm{d}Q}{\mathrm{d}r^*}\right] \end{aligned} \tag{10}$$

and

$$h_1(r)=r\left(1-\frac{2m}{r}\right)^{-1}Q. \tag{11}$$

Because the perturbation fields $h_{\mu\nu}$ are tensors and the perturbation equations (5) are also tensor equations, they are invariant under the coordinate transformation. Though the Regge–Wheeler equation was obtained in the Schwarzschild coordinates, it might be regarded as the simplest one from the above viewpoint. That is, whatever stationary coordinate system may be chosen, after simplifying the perturbation equations for calculation, the Regge–Wheeler equation can be obtained. For example, we could calculate the perturbation in the Painlevé coordinates of the black hole (1), the perturbation equations (5) are applied in this coordinates as

$$\begin{aligned} &-g_{\mathrm{p}}^{\beta\alpha}\left[h^{\mathrm{p}}_{\mu\alpha;\nu;\beta}+h^{\mathrm{p}}_{\nu\alpha;\mu;\beta}-h^{\mathrm{p}}_{\mu\nu;\alpha;\beta}\right] \\ &+g_{\mathrm{p}}^{\beta\alpha}\left[h^{\mathrm{p}}_{\mu\alpha;\beta;\nu}+h^{\mathrm{p}}_{\beta\alpha;\mu;\nu}-h^{\mathrm{p}}_{\mu\beta;\alpha;\nu}\right]=0. \end{aligned} \tag{12}$$

If we transform Eq.(12) in two ways: (1) transforming the perturbation fields $h^{\mathrm{p}}_{\mu\nu}$ in the Painlevé coordinates into $h^{\mathrm{s}}_{\mu\nu}$ ($h^{\mathrm{s}}_{\mu\nu}$ is the perturbation fields in the Schwarzschild coordinates), (2) transforming the independent variables (the Painlevé coordinates (t_{p}, r)) into Schwarzschild coordinates (t_{s}, r), then we would obtain the Regge–Wheeler equation definitely after some simplifying calculation.

4. Calculation

Therefore, we could make use of Eq.(7) as done in Ref.[3], and obtain the perturbation field quantities in the Schwarzschild metric coordinates for simplicity. Then we obtain the corresponding quantities in the metric (1) of Painlevé coordinates by transformation. By selecting the time coordinate t_{p} to define the initial condition, we prove that Rek=0, Im$k>0$ is possible for the Schwarzschild black hole in the metric (1).

Using Eq.(3), one obtains the perturbation fields $[h^{\mathrm{p}}_{ij}]$ in the metric (1) of Painlevé coordinates as

$$h^{\mathrm{p}}_{03}=h^{\mathrm{s}}_{03}, \tag{13}$$

$$h^{\mathrm{p}}_{13}=h^{\mathrm{s}}_{13}+\sqrt{\frac{2m}{r}}\left(1-\frac{2m}{r}\right)^{-1}h^{\mathrm{s}}_{03}, \tag{14}$$

where

$$h^{\mathrm{s}}_{03}=h_0\mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}=\frac{\mathrm{i}}{k}\left[(1-\frac{2m}{r})Q+r\frac{\mathrm{d}Q}{\mathrm{d}r^*}\right]\mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}, \tag{15}$$

$$h^{\mathrm{s}}_{13}=h_1\mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}=r\left(1-\frac{2m}{r}\right)^{-1}Q\mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}, \tag{16}$$

(see Eqs.(10) and (11)), therefore, we have

$$h^{\mathrm{p}}_{03}=\frac{\mathrm{i}}{k}\left[\left(1-\frac{2m}{r}\right)Q+r\frac{\mathrm{d}Q}{\mathrm{d}r^*}\right]\mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}, \tag{17}$$

$$\begin{aligned} h^{\mathrm{p}}_{13}=&\left[\frac{\mathrm{i}}{k}\sqrt{\frac{2m}{r}}Q+r\left(1-\frac{2m}{r}\right)^{-1}\right. \\ &\left.\times\left(\frac{\mathrm{i}}{k}\sqrt{\frac{2m}{r}}\frac{\mathrm{d}Q}{\mathrm{d}r^*}+Q\right)\right]\mathrm{e}^{-\mathrm{i}kt_{\mathrm{s}}}. \end{aligned} \tag{18}$$

Now, we solve the differential equation (7)

$$\frac{\mathrm{d}^2Q}{\mathrm{d}r^{*2}}+\left[k^2-V\right]Q=0.$$

As proved in Ref.[3], when $k=\mathrm{i}k_2$, $k_2>0$, the asymptotic solutions of Eq.(7) as $r^*\to\infty$ are $\tilde{A}\mathrm{e}^{\pm k_2r^*}$, and the well-behaved one is $\tilde{A}\mathrm{e}^{-k_2r^*}$, that is,

$$Q_\infty=\tilde{A}\mathrm{e}^{-k_2r^*}\quad\text{when}\quad r^*\to\infty. \tag{19}$$

From Eq.(7), the solution Q_∞ cannot become $A\mathrm{e}^{+k_2r^*}$ as $r^*\to-\infty$. Therefore, the asymptotic solution of Eq.(7) for $r^*\to-\infty$ is[3]

$$Q_{2m}=A\mathrm{e}^{-k_2r^*}\quad\text{when}\quad r^*\to-\infty. \tag{20}$$

It is obvious that Q_{2m} is singular at the horizon $r=2m$(or, $r^*\to-\infty$). Nevertheless, it may be unphysical and caused by the ill-behavedness of the Schwarzschild metric (2) at the horizon $r=2m$. Substituting Eq.(20) into Eqs.(17) and (18), and using the transformation equation (3), then, at $r=2m$, that is, $r^*\to-\infty$, one could obtain the asymptotic forms of the perturbation fields as

$$\begin{aligned}h^{\mathrm{p}}_{03}(t_{\mathrm{p}},r)=&\frac{1}{k_2}\left[(1-\frac{2m}{r})-k_2r\right]A\mathrm{e}^{k_2(t_{\mathrm{s}}-r^*)}\\=&\frac{1}{k_2}\left[(1-\frac{2m}{r})-k_2r\right]A\mathrm{e}^{k_2t_{\mathrm{p}}}\mathrm{e}^{k_2\left[-r+2\sqrt{2mr}-4m\ln\left(1+\sqrt{\frac{r}{2m}}\right)\right]}\end{aligned} \tag{21}$$

and

$$\begin{aligned}h^{\mathrm{p}}_{13}(t_{\mathrm{p}},r)=&\frac{1}{k_2}\sqrt{\frac{2m}{r}}A\mathrm{e}^{k_2(t_{\mathrm{s}}-r^*)}+r\left(1-\frac{2m}{r}\right)^{-1}\left[\frac{1}{k_2}\sqrt{\frac{2m}{r}}(-k_2)+1\right]A\mathrm{e}^{k_2(t_{\mathrm{s}}-r^*)}\\=&\left[\frac{1}{k_2}\sqrt{\frac{2m}{r}}+r(1+\sqrt{\frac{2m}{r}})^{-1}\right]A\mathrm{e}^{k_2t_{\mathrm{p}}}\mathrm{e}^{k_2\left[-r+2\sqrt{2mr}-4m\ln\left(1+\sqrt{\frac{r}{2m}}\right)\right]}.\end{aligned} \tag{22}$$

Actually, the asymptotic forms of the perturbation fields h^{p}_{13} and h^{p}_{03} are the same as those of Eqs.(21) and (22) for $r\to\infty$ due to the fact that Q_∞ and Q_{2m} have similar forms.

The boundaries for the perturbation problem consist of the spatial infinity and the horizon $r=2m$ of the black hole. The boundary condition at the spatial infinity requires that the perturbation fields should not diverge initially as $r\to\infty$. For the boundary condition at the horizon $r=2m$ ($r^*\to-\infty$), Vishveshwara implied that the perturbation fields in a 'good' coordinate system (where the metric is well-behaved at the horizon) do not diverge too at the initial time.[3]

From Eqs.(21) and (22), it is easy to see that h^{p}_{13} and h^{p}_{03} are regular at $r=2m$ and $r\to\infty$ at $t_{\mathrm{p}}=0$. Therefore, the perturbation fields $h^{\mathrm{p}}_{13}(0,r)$ and $h^{\mathrm{p}}_{03}(0,r)$ initially are finite in $(2m,\infty)$, or $(-\infty,+\infty)$ by use of r^*. They ($h^{\mathrm{p}}_{13}(t_{\mathrm{p}},r)$ and $h^{\mathrm{p}}_{03}(t_{\mathrm{p}},r)$) grow into infinity as $t_{\mathrm{p}}\to\infty$.

If the time coordinate t_{p} were suitable for analysis of the physical process at $r=2m$, we would have the conclusion that the Schwarzschild black hole is unstable. Because the time coordinate t_{p} in metric (1) has a drawback as the Schwarzschild time coordinate t_{s} does, the conclusion is not sure. As a matter of fact, the time coordinate t_{p} has a drawback at $r=2m$ due to the fact

$$g_{00}=-\left(1-\frac{2m}{r}\right)=0 \tag{23}$$

at $r=2m$ in the metric (1). Nevertheless, we make further transformation

$$\mathrm{d}R=\sqrt{\frac{r}{2m}}\mathrm{d}r-\mathrm{d}t_{\mathrm{p}}, \tag{24}$$

then

$$\mathrm{d}s^2=-\mathrm{d}t_{\mathrm{p}}^2+\frac{2m}{r}\mathrm{d}R^2+r^2\mathrm{d}\Omega^2. \tag{25}$$

In this metric (25), the time coordinate t_{p} is well-defined. Now, we further transform the perturbation fields into the coordinate system (25), and use the time coordinate t_{p} in the metric (25) to define the initial time. In this case, if $\mathrm{Im}k=k_2>0$ still belongs to the spectrum S of the Schwarzschild black-hole, we could gain the unstable conclusion definitely.[20]

By Eq.(24), the perturbation fields are trans-

formed according to

$$\hat{h}^{\rm p}_{03} = h^{\rm p}_{03} + \sqrt{\frac{2m}{r}} h^{\rm p}_{13}, \tag{26}$$

$$\hat{h}^{\rm p}_{13} = \sqrt{\frac{2m}{r}} h^{\rm p}_{13}, \tag{27}$$

where we denote the perturbation fields in coordinates (25) as $\hat{h}^{\rm p}_{\mu\nu}$. From Eq.(24), we have the variable r as a function of the variables $t_{\rm p}$ and R

$$r(t_{\rm p}, R) = \left[\frac{9m}{2}\right]^{\frac{1}{3}} (t_{\rm p} + R)^{\frac{2}{3}}. \tag{28}$$

So, $\hat{h}^{\rm p}_{03}$ and $\hat{h}^{\rm p}_{13}$ could be written as

$$\begin{aligned}\hat{h}^{\rm p}_{03}(t_{\rm p}, R) =& h^{\rm p}_{03}(t_{\rm p}, r(t_{\rm p}, R)) \\ &+ \sqrt{\frac{2m}{r(t_{\rm p}, R)}} h^{\rm p}_{13}(t_{\rm p}, r(t_{\rm p}, R)),\end{aligned} \tag{29}$$

and

$$\hat{h}^{\rm p}_{13}(t_{\rm p}, R) = \sqrt{\frac{2m}{r(t_{\rm p}, R)}} h^{\rm p}_{13}(t_{\rm p}, r(t_{\rm p}, R)) \tag{30}$$

First, at $t_{\rm p} = 0$, $\hat{h}^{\rm p}_{03}(0, R)$ and $\hat{h}^{\rm p}_{13}(0, R)$ are all well-defined simultaneously at $r = 2m$ and infinity simultaneously; this again ensures that the perturbation fields are finite initially and they $(\hat{h}^{\rm p}_{03}(t_{\rm p}, R),\ \hat{h}^{\rm p}_{13}(t_{\rm p}, R))$ all grow into divergence as $t_{\rm p} \to \infty$ as shown by Eqs.(28)–(30) and (21)–(22). Because the time coordinate $t_{\rm p}$ in the metric (25) is 'good', we could certainly conclude that the Schwarzschild space–time is unstable with respect to this kind of perturbations.

5. Conclusion

The Painlevé metric (1) corresponds to the white-hole-connected region. The white-hole-connected region is connected with the white hole, from which every particle must leave in a finite time. So, the white-hole-connected region is unstable in nature. That is, the white-hole-connected region of the Schwarzschild space-time is unstable with respect to the angular perturbations.

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