Vol 15 No 7, July 2006
1009-1963/2006/15(07)/1425-05

Chinese Physics

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The quantum tunnelling radiation of Schwarzschild de Sitter black hole with a global monopole*

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(Received 19 October 2005; revised manuscript received 19 February 2006)

Applying Parikh's quantum tunnelling method, this paper has studied the quantum tunnelling radiation of Schwarzschild de Sitter black hole with a global monopole. The result shows that the tunnelling rates at the event horizon and the cosmological horizon are related to Bekenstein–Hawking entropy and the derived radiation spectrum is not precisely thermal when considering energy conservation and self-gravitation interaction.

Keywords: Schwarzschild de Sitter black hole, energy conservation, Bekenstein–Hawking entropy, tunnelling rate

PACC: 0420, 9760L

1. Introduction

In 1974, Hawking proved that black holes have thermal radiation and the radiation spectrum is precisely thermal after taking the fixed space–time background of the black hole into account,[1] which has effective significance on the cognition and research about the evolution of stars. During the discussion of thermal radiation, there are two points worth highlighting. The first point is the information lost. This means that a pure quantum state will be converted into a mixed one, which disobeys an underlying unitary theory. The second point is in regard to the technical problems. At present, it is known that Hawking radiation has the quantum tunnelling effect, but the causes of the tunnelling potential hill is unclear for us, and the concerned literatures are not discussed in terms of the quantum tunnelling language.[2−13] So, strictly speaking, it is not really quantum tunnelling method. Recently, Pairkh's research shows that when energy conservation and self-gravitation interaction are considered, the horizons will be changed before and after the emission of particles, and we can regard them as the two turning points of the potential barrier. According to this method, Parikh has studied the quantum tunnelling radiation of the static Schwarzschild black hole and Reissner–Nordström black hole.[14−16] The derived result shows that Hawking radiation spectrum is not precisely thermal when energy conservation and self-gravitation interaction are considered.

In this paper, we extend Parikh's semi-classical quantum tunnelling method to discuss the Hawking radiation via tunnelling from Schwarzschild de Sitter black hole with a global monopole. The Amowitt–Deser–Misner (ADM) mass and Bekenstein–Hawking entropy of the black hole are $(1-8\pi\eta^2)$ times than those of Schwarzschild black hole and Reissner–Nordström black hole respectively. Our result shows that the Hawking radiation spectrum deviates from the pure thermal one and the tunnelling rate is related to the change of Bekenstein–Hawking entropy, but it is consistent with an underlying unitary theory. Thus a correct amendment to the Hawking pure thermal spectrum is presented. Finally, reducing the result to Schwarzschild black hole, we can get the same result as Parikh's.

2. The horizons of Schwarzschild de Sitter black hole with a global monopole

According to Ref.[17], we can get the line element of Schwarzschild de Sitter black hole with a global

*Project supported by the National Natural Science Foundation of China (Grant No 10347008).

monopole as

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2\right)\mathrm{d}t_{\mathrm{S}}^2 \\ &+\left(1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2\right)^{-1}\mathrm{d}r^2 \\ &+\left(1-8\pi\eta^2\right)r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right), \end{aligned} \tag{1}$$

where η is an internal global monopole, t_{S} is the coordinate time of Schwarzschild de Sitter black hole with a global monopole, and Λ is the cosmological factor. According to the null super-surface equation

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu}=0, \tag{2}$$

we can get the horizon equation of the black hole as

$$1-\frac{2m}{r}-\frac{1}{3}\Lambda r^2=0. \tag{3}$$

From Eq.(3), we can get

$$\begin{aligned} r_+ &= -2\sqrt{\frac{1}{\Lambda}}\cos\left(\frac{\phi}{3}+\frac{\pi}{3}\right), \\ r_- &= -2\sqrt{\frac{1}{\Lambda}}\cos\left(\frac{\phi}{3}-\frac{\pi}{3}\right), \\ r_{\mathrm{c}} &= 2\sqrt{\frac{1}{\Lambda}}\cos\frac{\phi}{3}, \\ \cos\phi &= -3m\sqrt{\Lambda}, \end{aligned} \tag{4}$$

where r_+ and r_{c} are the event and cosmological horizons respectively, and r_- is a negative root which has no real physical meaning.

3. Painlevé coordinate transformation

Because there exists coordinate singularity in Eq.(1) at the horizons of the black hole, we perform Painlevé coordinate transformation as follows:[6,7]

$$\mathrm{d}t_{\mathrm{S}}=\mathrm{d}t+f'(r)\,\mathrm{d}r. \tag{5}$$

Substituting Eq.(5) into Eq.(1) yields

$$\begin{aligned} \mathrm{d}s^2 =& -\Delta\mathrm{d}t^2-2\Delta f'(r)\,\mathrm{d}t\mathrm{d}r \\ &+\left(\frac{1}{\Delta}-\Delta f'^2(r)\right)\mathrm{d}r^2 \\ &+\left(1-8\pi\eta^2\right)r^2\mathrm{d}\Omega^2, \end{aligned} \tag{6}$$

where $\Delta=1-\dfrac{2m}{r}-\dfrac{1}{3}\Lambda r^2$ and $\mathrm{d}\Omega^2=\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2$. In the flow shape of the four-dimensional space–time, considering flat Euclidean space in radial, we get

$$\frac{1}{\Delta}-\Delta f'^2(r)=1. \tag{7}$$

So Eq.(6) can be rewritten as

$$\mathrm{d}s^2=-\Delta\mathrm{d}t^2\pm2\sqrt{1-\Delta}\mathrm{d}t\mathrm{d}r+\mathrm{d}r^2+(1-8\pi\eta^2)r^2\mathrm{d}\Omega^2, \tag{8}$$

where + sign denotes the space–time line element of the outgoing particles at the event horizon, and – sign denotes the space–time line element of the ingoing particles at the cosmological horizon. Obviously, the line element of Painlevé coordinate transformation has many superior features.[18,19] Firstly, it does not have coordinate singularity. Secondly, the event horizon and the infinite red-shift surface are coincident with each other. Finally, the derived space–time line element is stationary. All of these make it convenient to study Hawking radiation via tunnelling.

4. The quantum tunnelling radiation at the event horizon

According to Eq.(8), the radial null geodesic at the event horizon can be represented as

$$\dot{r}=\frac{\mathrm{d}r}{\mathrm{d}t}=1-\sqrt{1-\Delta}. \tag{9}$$

Considering energy conservation and self-gravitation interaction, when the total ADM mass is held fixed and the black hole mass is allowed to be varied, we conclude that the mass parameter of the black hole m in Eqs.(8) and (9) should be replaced by $m-\omega$ after the particles with energy ω tunnels out. So we can get the new line element at the event horizon as

$$\begin{aligned} \mathrm{d}s^2 =& -\Delta'\mathrm{d}t^2+2\sqrt{1-\Delta'}\mathrm{d}t\mathrm{d}r \\ &+\mathrm{d}r^2+(1-8\pi\eta^2)r^2\mathrm{d}\Omega^2, \end{aligned} \tag{10}$$

where $\Delta'=1-\dfrac{2\,(m-\omega)}{r}-\dfrac{1}{3}\Lambda r^2$. Taking the infinite blue-shift at the event horizon into account, considering that the frequency is arbitrary high and the wave-length is arbitrarily short, so geometric optics limit is reliable here, we can use particle language to describe the outgoing tunnelling rate rather than having to use the second-quantized method to describe the tunnelling effect at the event horizon. In the semi-classical limit, applying the WKB approximation, we can write the relationship between the tunnelling rate and the imaginary part of the particle action as[20,21]

$$\Gamma\sim\mathrm{e}^{-2\mathrm{Im}S}, \tag{11}$$

where S is the particle action across the event horizon of the black hole, whose imaginary part can be represented as

$$\mathrm{Im}S = \mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}} P_r \mathrm{d}r = \mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{0}^{P_r} \mathrm{d}P_r' \mathrm{d}r, \tag{12}$$

where r_i and r_f represent the locations of the event horizon before and after the particles emission, and they are often regarded as the two turning points of the tunnelling potential hill, and the distance between them depends on the energy of the outgoing particles. According to the Hamilton canonical equation, we have

$$\dot{r} = \left.\frac{\mathrm{d}H}{\mathrm{d}P_r}\right|_r,$$

$$\mathrm{d}P_r = \frac{\mathrm{d}H}{\dot{r}}, \tag{13}$$

where $H = M - \left(1-8\pi\eta^2\right)\omega$, ω is the energy of the outgoing particles, M is the space–time total ADM mass. The relationship between M and the black hole mass m satisfies $M = \left(1-8\pi\eta^2\right)m$, so the black hole total energy can be expressed as

$$H = \left(1-8\pi\eta^2\right)(m-\omega). \tag{14}$$

Substituting Eqs.(9), (13) and (14) into Eq.(12), we can obtain the imaginary part of the particles action at the event horizon as

$$\begin{aligned}\mathrm{Im}S =& \mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{0}^{P_r} \mathrm{d}P_r' \mathrm{d}r = \mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{(1-8\pi\eta^2)m}^{\left(1-8\pi\eta^2\right)(m-\omega)} \frac{\mathrm{d}H'}{\dot{r}}\mathrm{d}r \\ =& \left(1-8\pi\eta^2\right)\mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_{(1-8\pi\eta^2)m}^{\left(1-8\pi\eta^2\right)(m-\omega)} \frac{1+\sqrt{\dfrac{2\left(m-\omega'\right)}{r}+\dfrac{1}{3}\Lambda r^2}}{1-\dfrac{2\left(m-\omega'\right)}{r}-\dfrac{1}{3}\Lambda r^2}\mathrm{d}\left(m-\omega'\right)\mathrm{d}r,\end{aligned} \tag{15}$$

where the equation $1-\dfrac{2\left(m-\omega'\right)}{r}-\dfrac{1}{3}\Lambda r^2 = 0$ satisfies the horizon equation after the particles with energy ω' tunnelling out from the event horizon, so there exists a pole in Eq.(15). The integral can be evaluated by deforming the contour around the pole. Computing the integral in Eq.(15), we have

$$\begin{aligned}\mathrm{Im}S =& -\left(1-8\pi\eta^2\right)\mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}} \mathrm{i}\pi r\mathrm{d}r \\ =& -\frac{\pi}{2}\left(1-8\pi\eta^2\right)\left(r_\mathrm{f}^2-r_\mathrm{i}^2\right).\end{aligned} \tag{16}$$

The areas at the event horizon before and after the emission of the particles with energy ω can be expressed as

$$A_\mathrm{i} = \int\sqrt{g}\mathrm{d}\theta\mathrm{d}\varphi = \left(1-8\pi\eta^2\right)4\pi r_\mathrm{i}^2, \tag{17}$$

$$A_\mathrm{f} = \int\sqrt{g}\mathrm{d}\theta\mathrm{d}\varphi = \left(1-8\pi\eta^2\right)4\pi r_\mathrm{f}^2, \tag{18}$$

then the relationship between the tunnelling rate and imaginary part of the particles action satisfies

$$\begin{aligned}\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S} =& \mathrm{e}^{\left(1-8\pi\eta^2\right)\pi\left(r_\mathrm{f}^2-r_\mathrm{i}^2\right)} \\ =& \mathrm{e}^{\frac{1}{4}\left(A_\mathrm{f}-A_\mathrm{i}\right)} = \mathrm{e}^{\Delta S_\mathrm{BH}},\end{aligned} \tag{19}$$

where $\Delta S_\mathrm{BH} = S_\mathrm{BH}\left(m-\omega\right) - S_\mathrm{BH}\left(m\right)$ is the change of Bekenstein–Hawking entropy. So we can see clearly that the tunnelling rate at the event horizon is related to the Bekenstein–Hawking entropy, but it is consistent with an underlying unitary theory, and the derived radiation spectrum is not precisely thermal and correct modification to Hawking precisely thermal spectrum is presented.

5. The quantum tunnelling radiation at the cosmological horizon

Using the same method at the cosmological horizon and considering the absorbed particles with energy ω, we can write the radial null geodesics as

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = -1+\sqrt{1-\Delta''}, \tag{20}$$

and the new line element is

$$\begin{aligned}\mathrm{d}s^2 =& -\Delta''\mathrm{d}t^2 - 2\sqrt{1-\Delta''}\mathrm{d}t\mathrm{d}r + \mathrm{d}r^2 \\ &+ \left(1-8\pi\eta^2\right)r^2\mathrm{d}\Omega^2,\end{aligned} \tag{21}$$

where $\Delta'' = 1 - \dfrac{2\,(m+\omega)}{r} - \dfrac{1}{3}\Lambda r^2$. Applying WKB approximation, we can obtain the relationship between the tunnelling rate and the imaginary part of the particle action as $\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S_\mathrm{c}}$. According to Hamilton canonical equation (13), where ω is the energy of the absorbed particles and $H = -\left(1-8\pi\eta^2\right)(m+\omega)$, we have

$$\begin{aligned}\mathrm{Im}S_\mathrm{c} =& \mathrm{Im}\int_{r_\mathrm{ic}}^{r_\mathrm{fc}}\int_0^{P_r}\mathrm{d}P_r'\mathrm{d}r = \mathrm{Im}\int_{r_\mathrm{ic}}^{r_\mathrm{fc}}\int_{-(1-8\pi\eta^2)m}^{-\left(1-8\pi\eta^2\right)(m+\omega)}\frac{\mathrm{d}H'}{\dot{r}}\mathrm{d}r \\ =& -\left(1-8\pi\eta^2\right)\mathrm{Im}\int_{r_\mathrm{ic}}^{r_\mathrm{fc}}\int_{-(1-8\pi\eta^2)m}^{-\left(1-8\pi\eta^2\right)(m+\omega)}\frac{1+\sqrt{\dfrac{2\,(m+\omega')}{r}+\dfrac{1}{3}\Lambda r^2}}{1-\dfrac{2\,(m+\omega')}{r}-\dfrac{1}{3}\Lambda r^2}\mathrm{d}\,(m+\omega')\,\mathrm{d}r. \end{aligned} \tag{22}$$

In Eq.(22), r_ic and r_fc are the locations of the cosmological horizon before and after the particles of energy ω tunnels into the cosmological horizon respectively, S_c is the particle's action across the cosmological horizon. In Eq.(22), we can easily learn that $1-\dfrac{2\,(m+\omega')}{r}-\dfrac{1}{3}\Lambda r^2 = 0$ is the cosmological horizon equation, so there exists a pole at the cosmological horizon. Deforming the contour around the pole and computing the integral in Eq.(22), we have

$$\begin{aligned}\mathrm{Im}S_\mathrm{c} =& -\left(1-8\pi\eta^2\right)\mathrm{Im}\int_{r_\mathrm{ic}}^{r_\mathrm{fc}}\mathrm{i}\pi r\mathrm{d}r \\ =& -\frac{\pi}{2}\left(1-8\pi\eta^2\right)\left(r_\mathrm{fc}^2-r_\mathrm{ic}^2\right). \end{aligned} \tag{23}$$

So the tunnelling rate at the cosmological horizon can be expressed as

$$\begin{aligned}\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S_\mathrm{c}} =& \mathrm{e}^{\left(1-8\pi\eta^2\right)\pi\left(r_\mathrm{fc}^2-r_\mathrm{ic}^2\right)} \\ =& \mathrm{e}^{\frac{1}{4}\left(A_\mathrm{fc}-A_\mathrm{ic}\right)} = \mathrm{e}^{\Delta S_\mathrm{CH}}, \end{aligned} \tag{24}$$

where A_ic and A_fc are the cosmological horizon areas before and after the particles tunnel into the cosmological horizon, which can be expressed as

$$A_\mathrm{ic} = \int\sqrt{g}\mathrm{d}\theta\mathrm{d}\varphi = \left(1-8\pi\eta^2\right)4\pi r_\mathrm{ic}^2, \tag{25}$$

$$A_\mathrm{fc} = \int\sqrt{g}\mathrm{d}\theta\mathrm{d}\varphi = \left(1-8\pi\eta^2\right)4\pi r_\mathrm{fc}^2, \tag{26}$$

and $\Delta S_\mathrm{CH} = S_\mathrm{CH}\,(m+\omega) - S_\mathrm{CH}\,(m)$ is the change of Bekenstein–Hawking entropy at the cosmological horizon. Equation (24) shows that the tunnelling rate at the cosmological horizon of the black hole is related to the Bekenstein–Hawking entropy, and the Hawking radiation spectrum is not precisely thermal one when energy conservation and self-gravitation interaction are considered.

6. Discussion

From the above discussion, we find that the tunnelling rates at the event horizon and the cosmological horizon of Schwarzschild de Sitter black hole with a global monopole are related to the change of Bekenstein–Hawking entropy, and the factual radiation spectrum deviates from the precisely thermal one when energy conservation and self-gravitation interaction are considered. Especially, when $\Lambda = 0$ and $\eta = 0$, Schwarzschild de Sitter black hole with a global monopole is reduced to Schwarzschild black hole. The locations of the horizons of Schwarzschild black hole before and after the emission of the particles with energy ω are represented as $r_\mathrm{i} = 2m$ and $r_\mathrm{f} = 2(m-\omega)$, respectively. According to Eq.(19), the tunnelling rate at the event horizon can be written as

$$\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S} = \mathrm{e}^{-8\pi\omega\left(m-\frac{\omega}{2}\right)} = \mathrm{e}^{\Delta S_\mathrm{BH}}. \tag{27}$$

Obviously, the result is in accordance with Parikh's.[16]

Acknowledgement

We would like to thank Professor Zhang Jing-Yi and Doctor Ren Jun for their interesting discussions.

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