Vol 15 No 6, June 2006
1009-1963/2006/15(06)/1384-07

Chinese Physics

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Sensitivity of intrinsic mode functions of Lorenz system to initial values based on EMD method*

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(Received 2 November 2005; revised manuscript received 22 March 2006)

Extreme sensitivity to initial values is an intrinsic character of chaotic systems. The evolution of a chaotic system has a spatiotemporal structure containing quasi-periodic changes of different spatiotemporal scales. This paper uses an empirical mode decomposition (EMD) method to decompose and compare the evolution of the time-dependent evolutions of the x-component of the Lorenz system. The results indicate that the sensitivity of intrinsic mode function (IMF) component is dependent on initial values, which provides some scientific evidence for the possibility of long-range climatic prediction.

Keywords: empirical mode decomposition (EMD), sensitivity, initial values, hierarchical level
PACC: 9260X

1. Introduction

For the nonlinear time series generated by chaotic systems, [1−6] in 1998 Huang presented the Hilbert–Huang transform (HHT), [7] and improved it later.[8] In the HHT, signals (or time series) are first decomposed into a series of stationary waves of different characteristic scales (or hierarchical levels) by using the empirical mode decomposition (EMD) method (i.e. through the stationarization process of the signal, or sifting process), and those stationary components are called intrinsic mode functions (IMF), and behave as the Hilbert transform. The first IMF separated represents the high frequency (low hierarchical level) component of the original series, i.e. random noises. As the decomposition goes on the frequency of IMF component separated gradually reduces, and the last one is the lowest frequency (highest hierarchical level) component, which embodies the overall trend of the evolution of the original series. Those IMF stationary components may be linear or nonlinear, and therefore the EMD decomposition is able to objectively handle the nonlinear problems. Meanwhile, the sum of all IMF components and a mean value can reproduce the original series. Because of the linearization and stationarization of EMD for nonlinear and non-stationary data, the characters of the data are retained in the decomposition process. Therefore, the series obtained from the decomposition is able to reflect the characters of the original series. The merit of the EMD method is to extract the most dominant signal first from the original series, i.e. the first several IMF components separated contain most of the principal information of the original series, and therefore the EMD method is also a new principal component analysis method.

In 1963 meteorologist Lorenz[9,10] first put forward the concept of chaos, which reveals the essential attribute of the world and gave an impetus to the development of nonlinear science. The definition of chaos has been made in a few ways, but it has not been finalized yet. It is clear that the chaotic system possesses three remarkable characteristics: sensitive to

*Supported jointly by the State Key Development Program for Basic Research of China (Grant No 2006CB400503), the National Natural Sciences Foundation of China (Grant Nos 40325015 and 90411008) and the Climate Change Special Foundation of China Meteorological Administration (Grant No CCSF2006-5).

initial values; short-range predictable and long-range unpredictable; and ergodic phase space. Study [11−13] indicates that chaotic systems generally exist in nature and the human society. Due to the extreme sensitivity of a chaotic system to the initial values, an extremely small deviation in the initial values may be magnified in temporal evolution, and lead to a remarkable difference in the future state of the system, even to a completely opposite result. Therefore, the state of a chaotic system is long-range unpredictable, while its short-range state is predictable. This paper investigates the sensitivity of a chaotic system to initial values from a new angle of the spatiotemporal structure of the Lorenz system. The temporal evolution series of the x-component of the Lorenz system at different perturbed initial values is first decomposed using the EMD method, and then the sensitive extents of various decomposed IMF components to the initial perturbations are carefully compared, and which (high or low frequency) component of the system state evolution is first separated is observed. Obviously, the determination of first-separated-component is of significance for the prediction of nonlinear chaos, especially for long-range weather prediction.

2. EMD method and the Lorenz system

If the maximum (or minimum) point of time series $s(t)$ is 2 or greater than the number of upward (downward)-zero-crossings in number, then the time series is non-stationary and needs to be stabilized. The process is as follows: identifying the possible positive (maxima)/negative (minima) peaks of $s(t)$, constructing the upper/lower envelopes of $s(t)$ by the cubic spline method, calculating the mean envelope $m_1(t)$ by averaging the upper and lower envelopes, subtracting the mean envelope from $s(t)$ to produce a new series $h_1(t)$:

$$h_1(t) = s(t) - m_1(t), \tag{1}$$

repeating the above operations k times until the mean envelope approaches zero, thus finally yielding the first IMF component $C_1(t)$:

$$h_{1_{k-1}}(t) - m_{1_k}(t) = h_{1_k}(t);$$

$$C_1(t) = h_{1_k}(t). \tag{2}$$

The first IMF component represents the high frequency parts of the original series. Subtracting the first IMF component $C_1(t)$ from $s(t)$ yields the first residual series $r_1(t)$, which contains no high-frequency components, and is treated as new data. And the next IMF components are calculated in the same process as that described above. Repeating the steps above until the final residual component $r_n(t)$ becomes a monotonic function (the mean value or trend of $s(t)$) and no more IMF can be extracted, yields the following series:

$$r_1(t) - C_2(t) = r_2(t), \cdots, r_{n-1}(t) - C_n(t) = r_n(t). \tag{3}$$

Then, the original series can be represented by the sum of the IMF components and the mean value as follows:

$$s(t) = \sum C_j(t) + r_n(t). \tag{4}$$

The original series $s(t)$ is finally decomposed into a definite number of IMF components by the EMD method, and different IMF components may result from some physical backgrounds. Thus, for a chaotic series, finding out the main characteristic component (hierarchical level), which results from the main physical background, is significant for the prediction of the chaotic series.[11]

The Lorenz model is designed by the following equations:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \sigma(y - x), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= rx - y - xz, \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= xy - bz. \end{aligned} \tag{5}$$

With the physical background taken into consideration, set $\sigma = 10.0$, $b = 8/3$, and $0 < r < \infty$. Letting the left-hand sides of Eqs.(5) be zero yields the stationary solution of the Lorenz system:

$$\begin{aligned} x = 0, y = 0, z = 0, \\ x = \sqrt{b(r-1)}, y = \sqrt{b(r-1)}, z = r - 1, \\ x = -\sqrt{b(r-1)}, y = -\sqrt{b(r-1)}, z = r - 1. \end{aligned} \tag{6}$$

Obviously, only when $r \leq 1$, can the model have a group of real roots, $x = y = z = 0$, meaning a state having no convection, and when $r > 1$, the model has three groups of real roots. It is known from the analysis of stability that when $1 < r < 24.74$, Lorenz system (5) has two symmetric stable equilibrium state solutions. Numerical computation indicates that under the condition of arbitrary initial values the system finally almost approaches one of the two equilibrium

states, when $x = y = z = 0$ it is unstable, and the word 'almost' used here means that there still exists some random evolution for individual state. When $r > 24.74$, there exists nearly no stable state in the whole phase space. This state is the so-called chaotic state.

3. Analysis of the sensitivity to initial values

Select initial values and appropriate parameters, and make the Lorenz system evolve into a chaotic state. Under the condition of different initial values, the initial evolution of $x(t)$ of the Lorenz system (5) is the same; however, with the increase of integration time, $x(t)$ curves separate from each other, showing that the chaotic system is sensitive to its initial value. These one-dimensional time series are then decomposed using the EMD method to find out on which IMF component, i.e. on a small-scale (high frequency) component or a large-scale (low frequency) component, the solution separation of the Lorenz system takes place first. When $\sigma = 10.0, r = 27.74$, and $b = 8/3$, the Lorenz system has three unstable equilibrium points (0, 0, 0), (8.443, 8.443, 26.74) and (-8.443, -8.443, 26.74). Obviously, the Lorenz system is symmetric with respect to the z axis. In the way similar to that in Ref.[4], in this paper we choose (0, 0, 0)→(0.5, 0.5, 0.5)→ ... →(1.0, 1.0, 1.0)→(8.3267, 8.3267, 26) as the integration path; according to different locations of initial values (in the neighbourhood of the two unstable equilibrium points and the area far from the two points), we perform three categories of numerical experiments (initial value points for first category are: (0.5, 0.5, 0.5), (1.0, 1.0, 1.0), and (1.5, 1.5, 1.5); for the second category: (3.5, 3.5, 12.5), (4.0, 4.0, 13.0), (4.5, 4.5, 13.5), and (5.0, 5.0, 14.0); for the third category: (7.0, 7.0, 25.0), (7.5, 7.5, 25.5), and (8.0, 8.0, 26.0); there are 10 initial value points in total), and then investigate the sensitivities of the Lorenz system to the initial values by using various IMF components of the solution.

In the first category of experiments, the Lorenz system is discretized using the Euler method: the time step is 0.001, and the initial value is (0.5, 0.5, and 0.5). In order to examine the sensitivity of the system to the initial value, without any loss of the generality, three perturbations of different amplitudes (0.0001, 0.001. 0.01) are added to the z-component of the initial value, and the three perturbed initial values are (0.5, 0.5, 0.5001), (0.5, 0.5, 0.501), and (0.5, 0.5, 0.51), respectively.

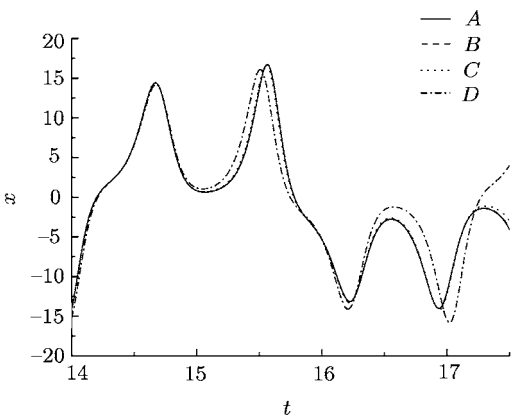


Fig.1. Temporal evolution of the $x(t)$ component of the Lorenz system in the neighbourhood of initial values A (0.5, 0.5, 0.5), B (0.5, 0.5, 0.5001), C (0.5, 0.5, 0.501), and D (0.5, 0.5, 0.51), respectively.

It is seen from Fig.1 that when the integration time $t < 15$, the evolution of the four initial values in the neighbourhood of (0.5, 0.5, and 0.5) is the same. However, when $t = 15$, the four evolving curves start separating, especially when $t > 17.5$, the evolution of the curve with the initial value being (0.5, 0.5, 0.51) is completely different from the others. And meanwhile, other three curves are also separated, but the overall tendency still remains the same. To quantitatively illustrate the problem, the criterion of separation is defined as

$$\delta = \left| \frac{x_i - \overline{x}}{\overline{x}} \right|, \tag{7}$$

where $\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$, N is the total number of initial values in calculation, and $N = 4$ in this paper. when $\delta \geq 0.1$, the separation is considered to happen. According to the expression (7), four curves in Fig.1 start separating at t=14.042, and are completely separated at t=17.232. When $r > 24.74$ the Lorenz system exhibits chaotic behaviours already, but still has a spatiotemporal structure, i.e. the quasi-periodic characteristics of different scales. What we are concerned about is whether the solution of Lorenz system is first separated under the perturbations of initial value, from which the IMF component (hierarchical level) is decomposed by the EMD method.

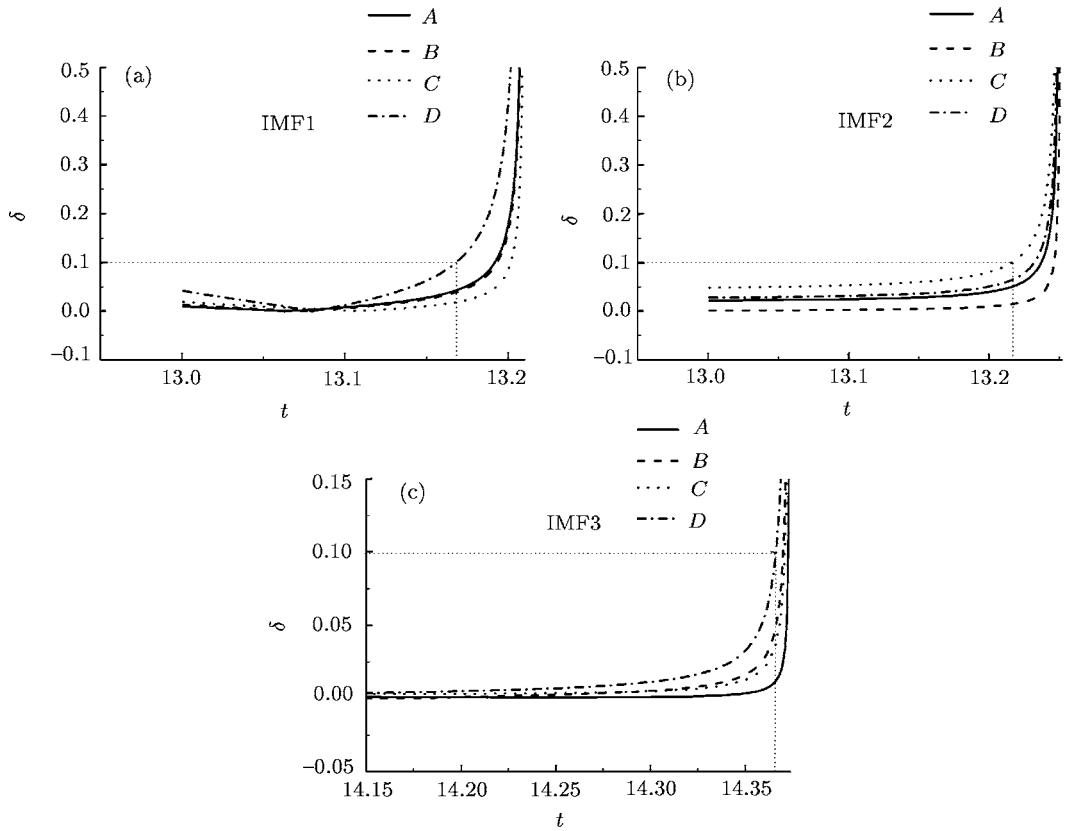


Fig.2. Separations $\delta(t)$ for IMF1(a), IMF2(b) and IMF3(c) of the four evolving curves of the $x(t)$ component of the Lorenz system in the neighbourhood of initial values A (0.5, 0.5, 0.5), B (0.5, 0.5, 0.5001), C (0.5, 0.5, 0.501), and D (0.5, 0.5, 0.51), respectively.

Figure 2 shows the separations $\delta(t)$ of the IMF components for the four $x(t)$ evolving curves from $t = 13$ to $t = 15$. After EMD decomposition, the curves each generate four IMF components, among them IMF1, IMF2, and IMF3, denoting high-to-low frequency components respectively, and IMF4 which represents the linear trend and, therefore, is not analysed here. According to expression (7), the separations of IMF1, IMF2, and IMF3 start at t=13.169, 13.218, and 14.368, respectively, from high (i.e. small scale variation/short periodic variation) to low (i.e. large scale variation/long periodic variation) frequency components, which accords with the conclusion that the state of a chaotic system is long-range unpredictable, but short-range predictable. After a short period of separation from $t = 14.368$ to $t = 14.382$, IMF3 components reunite together until their ending, suggesting that the low frequency part of $x(t)$ is 'almost' not separated. Besides, before the EMD decomposition, $x(t)$ starts separating at $t = 14.046$ (Fig.1); however, after the EMD decomposition, various IMF components of $x(t)$ except IMF3 separate at a time although they are different from each other but earlier than $t = 14.046$, which indicates that the EMD method itself is more sensitive to the initial value.

The initial values for the second category of experiments are (3.5, 3.5, 12.5), (3.5, 3.5, 12.5001), (3.5, 3.5, 12.501), and (3.51, 3.51, 12.51), respectively.

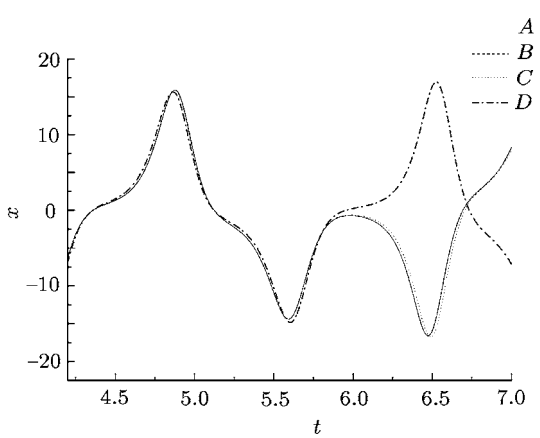


Fig.3. Temporal evolution of the $x(t)$-component of the Lorenz system in the neighbourhood of initial values A (3.5, 3.5, 12.5), B (3.5, 3.5, 12.5001), C (3.5, 3.5, 12.501), and D (3.5, 3.5, 12.51), respectively.

When the initial value is (3.5, 3.5, 12.5), it is observed from Fig.3 that the separation of $x(t)$ starts

after about 5–6 unit time integration, and it happens early compared with the first category. The separation takes place at $t = 4.348$ according to the expression (7), and then one evolving curve completely differs from or is even in phase opposite to the other three ones at $t = 5.842$. Each separation of four evolving curves of $x(t)$ in the second category of experiments is about 10 unit times earlier than that in the first category.

In the way similar to that of the first category of experiments, each of the four $x(t)$ curves is decomposed into the 6 IMF components ordering from high to low frequency. The separations of low frequency component in the second category of experiments take place earliest, especially the separations of IMF3-to-IMF5 almost happen at the beginning of integration, while the separation of high frequency component occurs latest. According to expression (7), the separations of IMF1-to-IMF6 take place at $t = 2.019, 2.002, 2.002, 2.002, 2.002, 2.147$, respectively. This category of experiments suggests that the separation of IMF components starts first from low frequency, while the separation of IMF1 (high frequency) occurs latest, that is to say, the large scale component separates at first, and the long period component is more sensitive to the initial value (figure omitted).

The initial values for the third category of experiments are (7.0, 7.0, 25.0), (7.0, 7.0, 25.0001), (7.0, 7.0, 25.001), and (7.5, 7.5, 25.01), respectively, and close to the fixed point (8.3267, 8.3267, 26.0).

When the initial value lies in the vicinity of point (7.0, 7.0, 25.0), the separations of the four $x(t)$ curves are similar to those of the first category, and they take place later. The separation starts from $t = 9.89$, and the large deviation of the four curves (i.e. the trend of one curve differs completely from those of other three ones) occurs from $t = 12.73$, and such a sudden change happens in all three experiments of the perturbed initial values, though at different times, which indicates that the sensitivities of the Lorenz system to the initial values are different when the system lies in different areas (figure omitted).

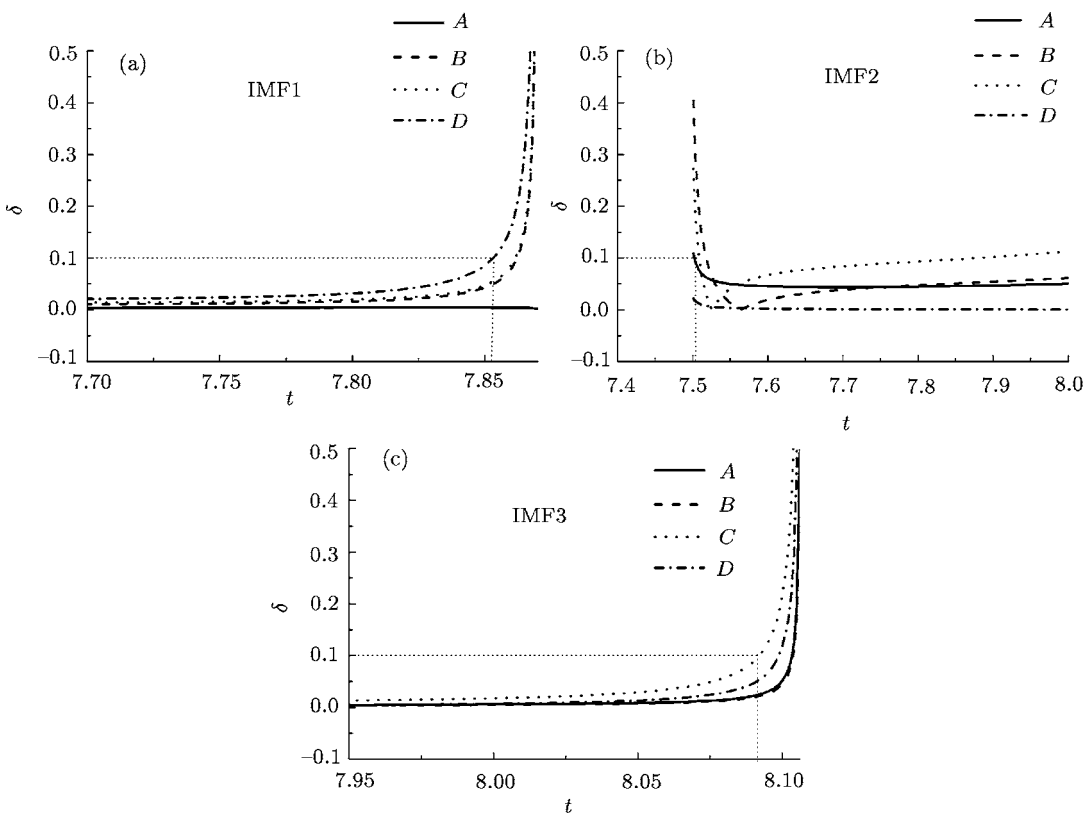


Fig.4. Separations $\delta(t)$ for IMF1(a), IMF2(b) and IMF3(c) of the four evolving curves of the $x(t)$ component of the Lorenz system in the neighbourhood of initial values A (7.0, 7.0, 25.0), B (7.0, 7.0, 25.0001), C (7.0, 7.0, 25.001), and D (7.5, 7.5, 25.01), respectively.

Because the separations of the four evolving cures of the x-component of the Lorenz system in the vicinity of the initial value point (7.0, 7.0, 25.0) all start at $t = 9.89$, the decompositions of the four curves from $t = 7.5$ to $t = 10.0$ are performed. The separations of IMF1, IMF2 and IMF3 components take place at $t = 7.855, 7.502$, and 8.093, respectively, in the way similar to that of the first category, namely from high frequency to low frequency(Fig.4). The results from this category of experiments accord with the theoretical results, i.e. the state of a chaotic system is long-range unpredictable, but it is short-term predictable, which also suggests that when the Lorenz system lies in different areas, its sensitivities to the initial values are different, and thus its predictabilities are also different.

Table 1. Results of numerical experiments for ten groups of initial values.

Group No.	Category	Initial values	ST* of $x(t)$ curves for	ST* for IMFs of $x(t)$ curves except the trend component		
			PIV**	IMF1	IMF2	IMF3
1	First	(0.5, 0.5, 0.5)	14.042	13.169	13.218	14.368
2		(1.0, 1.0, 1.0)	12.228	11.002	11.002	11.002
3		(1.5, 1.5, 1.5)	10.954	9.704	9.502	9.502
4	Second	(3.5, 3.5, 12.5)	4.348	2.019	2.002	2.002
5		(4.0, 4.0, 13.0)	4.281	3.203	3.022	3.180
6		(4.5, 4.5, 13.5)	2.863	1.668	1.692	2.328
7		(5.0, 5.0, 14.0)	0.650	0.131	0.590	0.604
8	Third	(7.0, 7.0, 25.0)	9.890	7.855	7.502	8.093
9		(7.5, 7.5, 25.5)	11.605	10.383	10.252	10.252
10		(8.0, 8.0, 26.0)	15.912	15.291	15.002	15.451

* ST= separation time; ** PIV= perturbed initial value.

4. Results and discussions

It can be seen from Table 1 that the separations of the $x(t)$ component solutions for the first and third categories of one original and three perturbed initial values take place 10 unit time earlier than those for the second category. Because the first and third categories of initial values are close to unstable equilibrium points (0, 0, 0) and (8.4443, 8.4443, 26.74), respectively, while the second category of initial values is far from the two points, it can be seen from Table 1 that due to the initial values lying in different areas, the separation times of $x(t)$ solutions are also greatly different. The separation occurs later and the predictable period of the system is longer, and vice versa: that is to say, for different initial values, the length of predictable period is completely determined by the chaotic nature of the atmosphere. Therefore, the validity of period of prediction depends on not only the error of initial values but also the area which the initial value lies in. The reason for that might be as follows: the errors of initial value in different areas are magnified at different rates. Secondly, it can also be seen from Table 1 that the separations of IMF components of $x(t)$ in the first and third category of experiments take place later than those in the second category, and they also differ from group to group, even in the same category. Besides, the EMD method itself seems to be more sensitive to the initial value, because the separations of almost all IMF components happen earlier than those of $x(t)$ evolving curves themselves. In the experiments with the first category of initial values, the solution separations take place first on high frequency component in the first group of experiments, almost concurrently on various components in the second group, and concurrently on various components in the third group except the separation of highest frequency component being the latest due to the third category of initial values lying in a transition area. In the experiments with the second category of initial values, the solution separation occurs latest on the highest frequency component in the fourth group of experiments; first on the medium frequency component, and then on the low frequency component, and latest on the high frequencies component in the fifth group of experiments; on the components from high to low frequencies in the sixth and seventh group of experiments; first on high and low frequency components, and then on medium frequency component

in the seventh group of experiments. In the experiments with the third category of initial values, the separations of solutions happen first on the medium frequency component, and then on the high frequency component, and finally on the low frequency component. The above separations of solutions evidently show the complexity in nonlinear systems as well as the difficulty in making a long-range prediction.

In this paper only ten groups of experiments for selected different initial values are carried out. The computational results exhibit complicated but interesting phenomena, fully revealing that the actual atmospheric process is a process that happens in a chaotic system containing different spatiotemporal scales and their interactions.[12−14] It is found from the study of the sensitivity of the Lorenz system to the initial value that the sensitivity to the initial value is a general character of nonlinear phenomena. Due to the chaotic system being sensitive to the initial value, a small error of initial value might lead to a big error in the predicted value of future state. Furthermore, as the prediction period increases, the prediction error gradually increases. Meanwhile, the sensitivity of chaotic system to the initial value varies with the initial value area: that is to say, the initial value errors are magnified at different rates in different initial value areas. Therefore, the selection of initial values is also one of factors affecting the predictability. The study of the separations of IMF components of solutions for different initial values reveals that the predictability of the chaotic system is closely related to the area of selected initial values in phase space, and the solution separations exhibit different features in different areas. The results are helpful for the study of long-range forecast and climatic prediction.

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