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Equivalent circuit with complex physical constants and equivalent-parameters-expressed dissipation factors of piezoelectric materials*

Chen Yu(陈 雨)[a], Wen Yu-Mei(文玉梅)[a)b)†], and Li Ping(李 平)[a)b)]

[a)] *College of Optoelectronic Engineering, Chongqing University, Chongqing 400044, China*

[b)] *Key Laboratory of Optoelectronic Technology and Systems of the Education Ministry of China, Chongqing University, Chongqing 400044, China*

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The equivalent circuit with complex physical constants for a piezoelectric ceramic in thickness mode is established. In the equivalent circuit, electric components (equivalent circuit parameters) are connected to real and imaginary parts of complex physical coefficients of piezoelectric materials. Based on definitions of dissipation factors, three of them (dielectric, elastic and piezoelectric dissipation factors) are represented by equivalent circuit parameters. Since the equivalent circuit parameters are detectable, the dissipation factors can be easily obtained. In the experiments, the temperature and the stress responses of the three dissipation factors are measured.

Keywords: piezoelectric, equivalent circuit, dissipation factor, complex physical coefficient, stress, temperature

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1. Introduction

Loss or hysteresis in piezoelectric materials exhibits both merits and demerits. For positioning actuator applications, hysteresis in the field-induced strain provides a serious problem and, for resonance actuation, such as ultrasonic motors, loss generates significant heat in the piezoelectric materials. Furthermore, in consideration of the resonant strain amplified in proportion to a mechanical quality factor, low (intensive) mechanical loss materials are preferred for ultrasonic motors. In contrast, for force sensors and acoustic transducers, high mechanical loss is helpful to widen a frequency range for receiving signals. Haerdtl[1] wrote a review article on electrical and mechanical losses in ferroelectric ceramics. Losses are considered to consist of four portions: domain wall motion; the fundamental lattice portion, which also occurs in domain-free monocrystalline; the microstructure portion, which occurs typically in polycrystalline samples; and the conductivity portion in highly ohmic samples. However, in the typical piezoelectric ceramic case, the loss due to domain wall motion exceeds the other three contributions significantly. Recently, Uchino and Hirose[2] reported the phenomenological theory to deal with losses in piezoelectric materials, and discussed various techniques for measuring electric, mechanical, and piezoelectric losses represented by complex coefficients. But their conclusions were derived from only a limited number of PZT-based soft and hard piezoelectrics, which is not enough to generalize these conclusions.

In this paper, complex coefficients are introduced in Mason's equivalent model for piezoelectric materials. Based on this model, equivalent circuit parameters are connected with the complex coefficients. Considering that the dissipation factors are defined by complex coefficients, we derive the expression of dissipation factors in terms of equivalent circuit parameters. Such expressions make it feasible to compute dissipation factors by the measurement of equivalent circuit parameters. In our derivation, no specific piezoelectric material is referred to, so the conclusion is generally applicable.

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†Corresponding author. E-mail: ymwen@cqu.edu.cn

2. Equivalent circuit with complex physical constants

A phenomenological description of piezoelectric effects is to use the piezoelectric equations derived from the thermodynamic theory. A typical piezoelectric equation system can be given as[3]

$$\begin{aligned} X_l &= c_{lk}^D x_k - h_{ml} D_m, \\ E_n &= -h_{nk} x_k + \beta_{nm}^S D_m, \end{aligned} \tag{1}$$

where X_l and x_k are the components of the stress and the strain tensors respectively, k, l=1–6. E_n and D_m are the components of electric and dielectric vectors respectively, n, m=1–3; c_{lk}^D is the component of elastic stiffness constant tensor at constant dielectric D; h_{mj} and h_{ni} are the components of piezoelectric stiffness constant tensors, β_{nm}^S is the component of impermeability tensor at constant entropy S. When further precision is not demanded, the coefficients of the piezoelectric equations can be regarded as real numbers, and the piezoelectric materials are taken as lossless systems. But, in some applications, ignoring losses will bring about obvious errors. Land *et al*[4] and Martin[5] generalized the concept of complex dielectric constant and complex elastic compliance to the dielectric loss and the mechanical loss. They introduced complex piezoelectric stiffness constants into the piezoelectric equations. The complex coefficients are

$$\begin{aligned} \beta_{nm}^S &= \beta'^S_{nm} + \mathrm{i}\beta''^S_{nm}, \\ c_{lk}^D &= c'^D_{lk} + \mathrm{i}c''^D_{lk}, \\ h_{ml} &= h_{nk} = h'_{ml} + \mathrm{i}h''_{ml}, \end{aligned} \tag{2}$$

where $\mathrm{i}^2 = -1$, β'^S_{nm} and β''^S_{nm} are the real and imaginary parts of β_{nm}^S, c'^D_{lk} and c''^D_{lk} the real and imaginary parts of c_{lk}^D, h'_{ml} and h''_{ml} the real and imaginary parts of h_{ml} or h_{nk}. Based on the d-type piezoelectric equation, and the definition of Poynting vector, Holland[6,7] presented the physical explanation of the imaginary parts of complex coefficients. He proved that the imaginary part of complex impermeability is connected to the dielectric loss; the imaginary part of complex elastic stiffness constant connected to the mechanical loss; and the imaginary part of complex piezoelectric stiffness constant connected to the piezoelectric loss.

The complex coefficients do not change the form of the piezoelectric equation. Only the coefficients in the solution to the electric circuit state equation and the solution to the mechanical vibration equation take complex numbers. In this paper, we focus on a piezoelectric ceramic plate in thickness mode. According to the research of Yamamoto,[8] the following assumption can be given.

(A) The area of either surface of piezoelectric ceramic element is large enough compared with the dimensions of the thickness and the surface perpendicular to the axis of symmetry of the resulting acoustical wave fronts. This condition implies that the diameter of a ceramic element is many times greater than the wavelength of the sound in the medium, and it is reasonably well satisfied in practice. For example, the ultrasonic wavelength of a 1MHz sound in water is approximately 1.5mm, while the diameter of a ceramic element used to produce or receive the sound lies in the range from 25 to 40mm.

(B) The surfaces of the ceramic element must be electrical equipotential. This condition is also satisfied because the electrical wavelength for the ultrasonic frequency of interest is much greater than the size of the conductor on the surfaces of the transducer, and as a result a uniform electric field is applied to the transducer.

(C) The ceramic element is operated at sufficiently low amplitude to avoid cavitations, localized plastic deformation, and irregular modes of vibration.

On these assumptions, the electric and mechanical boundary conditions can be expressed as

$$\begin{aligned} &x_3 \neq 0,\ x_1 = x_2 = x_4 = x_5 = x_6 = 0, \\ &X_3 \neq 0,\ X_1 = X_2 = X_4 = X_5 = X_6 = 0. \end{aligned} \tag{3}$$

These boundary conditions indicate that the piezoelectric ceramic plate is under uniaxial compressive and tensile stress and strain. Consequently, the piezoelectric equation system of Eq.(1) can be specified as a component form in the 3rd-direction (z-direction) or axial direction of the piezoelectric ceramic plate.

$$\begin{aligned} X_3 &= c_{33}^D x_3 - h_{33} D_3, \\ E_3 &= -h_{33} x_3 + \beta_{33}^S D_3. \end{aligned} \tag{4}$$

According to Mason's statement,[9,10] the equation of the electric circuit state can be written as

$$I = \mathrm{i}\omega A D_3 = \mathrm{i}\omega C_0 V - n(\dot{\xi}_2 + \dot{\xi}_1), \tag{5}$$

where I and V are the current flowing through and the AC voltage across the device respectively, ω is the operational frequency. A is the area of electrode surfaces

of the piezoelectric ceramic plate. $\dot{\xi}_1$ and $\dot{\xi}_2$ are the particle vibrating speeds on the sides of the piezoelectric ceramic plate. C_0 and n the static capacitance and the electromechanical factor of the piezoelectric ceramics respectively, which are connected to the impermeability and the piezoelectric stiffness constant of piezoelectric ceramics as follows:

$$\begin{aligned} C_0 &= \frac{A}{t\beta_{33}^S}, \\ n &= \frac{Ah_{33}}{t\beta_{33}^S}, \end{aligned} \tag{6}$$

where t is the thickness of the piezoelectric ceramic plate. As mentioned above, if the losses are considered in piezoelectric materials the physical constants should be taken as complex numbers. Substituting the complex impermeability and piezoelectric stiffness constant into Eq.(6), we have

$$\begin{aligned} C_0 =& \frac{A}{t\left(\beta_{33}^{\prime S} + \mathrm{i}\beta_{33}^{\prime\prime S}\right)} \\ =& \frac{A}{t\beta_{33}^{\prime S}M^2} - \mathrm{i}\frac{A\lambda}{t\beta_{33}^{\prime S}M^2} \\ =& \frac{A}{t\beta_{33}^{\prime S}M^2}\left(1 - \mathrm{i}\lambda\right), \\ n =& \frac{A\left(h_{33}' + h_{33}''\right)}{t\left(\beta_{33}^{\prime S} + \mathrm{i}\beta_{33}^{\prime\prime S}\right)} \\ =& \frac{A\left(h_{33}' - h_{33}''\lambda\right)}{t\beta_{33}^{\prime S}M^2} - \mathrm{i}\frac{A\left(h_{33}'\lambda + h_{33}''\right)}{t\beta_{33}^{\prime S}M^2} \\ =& \frac{A}{t\beta_{33}^{\prime S}M^2}\left[\left(h_{33}' - h_{33}''\lambda\right) - \mathrm{i}\left(h_{33}'\lambda + h_{33}''\right)\right], \end{aligned} \tag{7}$$

where

$$\begin{aligned} M &= \left(1 + \lambda^2\right)^{1/2}, \\ \lambda &= \frac{\beta_{33}^{\prime\prime S}}{\beta_{33}^{\prime S}}. \end{aligned} \tag{8}$$

Notice that the impedance of the complex static capacitance can be taken as

$$\frac{1}{\mathrm{i}\omega C_0} = \frac{1}{\mathrm{i}\omega\dfrac{A}{t\beta_{33}^{\prime S}}} + \frac{t\beta_{33}^{\prime\prime S}}{\omega A}, \tag{9a}$$

or

$$\begin{aligned} \mathrm{i}\omega C_0 =& \mathrm{i}\omega\frac{A\beta_{33}^{\prime S}}{t\left(\beta_{33}^{\prime S2} + \beta_{33}^{\prime\prime S2}\right)} \\ &+ \frac{\omega A\beta_{33}^{\prime\prime S}}{t\left(\beta_{33}^{\prime S2} + \beta_{33}^{\prime\prime S2}\right)}. \end{aligned} \tag{9b}$$

Equation (9) implies that the complex capacitance C_0 can be divided into two components: a capacitor and a resistor, which are connected in series (9a) or in parallel (9b). They are

$$\begin{aligned} C'_0 &= \frac{A}{t\beta_{33}^{\prime S}}, \\ R_0 &= \frac{t\beta_{33}^{\prime\prime S}}{\omega A}; \end{aligned} \quad \text{(in series)} \tag{10a}$$

$$\begin{aligned} C'_e &= \frac{A\beta_{33}^{\prime S}}{t\left(\beta_{33}^{\prime S2} + \beta_{33}^{\prime\prime S2}\right)}, \\ R_e &= \frac{t\left(\beta_{33}^{\prime S2} + \beta_{33}^{\prime\prime S2}\right)}{\omega A\beta_{33}^{\prime\prime S}}. \end{aligned} \quad \text{(in parallel)} \tag{10b}$$

The resistance resulting from the complex constants explicitly demonstrates the physical meaning of the energy loss. If the energy losses are not considered and the complex constants are not introduced, there would be no such resistance in the equivalent circuit. In Eq.(10a), C'_0 is called static capacitance or cut-off capacitance of the piezoelectric ceramic; R_0, according to our later discussion, is nearly equal to the impedance of the system. In Eq.(10b),C'_{e} and R_{e} are the capacitance and resistance in parallel respectively. Solving the motion equation of piezoelectric ceramics,

$$\frac{\partial^2\xi}{\partial\tau^2} = v^2\frac{\partial^2\xi}{\partial z^2}, \tag{11}$$

where ξ is the particle displacement of piezoelectric ceramics, τ the time, and v the velocity of the longitudinal wave produced by piezoelectric ceramics,

$$v = \sqrt{\frac{c_{33}^D}{\rho}}, \tag{12}$$

with ρ being the density of piezoelectric ceramics, we have the simple harmonic solution for particle displacement

$$\xi = \frac{\xi_1\sin k\left(t - z\right) - \xi_2\sin kz}{\sin kz}. \tag{13}$$

The physical constants in Eqs.(12) and (13) are also substituted by their complex counterparts, so the longitudinal wave velocity v and the wave number k become complex,

$$\begin{aligned} v &= v' + \mathrm{i}v'', \\ k &= \frac{\omega}{v} = k' - \mathrm{i}k'', \end{aligned} \tag{14}$$

where

$$\begin{aligned} v' &= \left(\frac{c_{33}^{\prime D}\left(1 + N\right)}{2\rho}\right)^{1/2}, \\ v'' &= \left[\left(\frac{c_{33}^{\prime\prime D}\eta}{2\rho\left(1 + N\right)}\right)\right]^{1/2}, \end{aligned}$$

$$
\begin{aligned}
k' =& \frac{\omega}{N}\left[\frac{\rho(1+N)}{2c_{33}'^{D}}\right]^{1/2},\\
k'' =& \frac{\omega\eta^2}{N}\left[\frac{\rho}{2c_{33}''^{D}\eta(1+N)}\right]^{1/2},\\
N =& \left(1+\eta^2\right)^{1/2},\\
\eta =& \frac{c_{33}''^{D}}{c_{33}'^{D}}.
\end{aligned}
\tag{15}
$$

Equation (14) reveals that the real part of the wave number k is related to the longitudinal wave velocity, and the imaginary part is related to the attenuation of the wave amplitude. Equation (15) shows that when loss is taken into account, k' is a little less than its real number counterpart k, which is represented by the real elastic stiffness constant c_{33}^{D} ($c_{33}'^{D} \approx c_{33}^{D}$). A decrement in wave number indicates an increment in longitudinal wave velocity. Moreover, according to Eq.(15), k' is affected by ω, indicating that the wave produced by piezoelectric ceramics is dispersive.

The distribution of forces in a piezoelectric ceramic is determined by Eq.(4). Thus we have the force balance equations,

$$
\begin{aligned}
-F_1 =& c_{33}^{D}\left(\frac{\partial\xi}{\partial z}\right)_{z=0} A - h_{33}D_3 A,\\
-F_2 =& c_{33}^{D}\left(\frac{\partial\xi}{\partial z}\right)_{z=t} A - h_{33}D_3 A.
\end{aligned}
\tag{16}
$$

Substituting Eq.(13) in Eq.(16), we have

$$
\begin{aligned}
F_1 =& \left(\frac{\rho v A}{\mathrm{i}\sin kt} - \frac{n^2}{\mathrm{i}\omega C_0}\right)\left(\dot{\xi}_2 + \dot{\xi}_1\right)\\
&+ \mathrm{i}\rho v A\dot{\xi}_1 \tan\frac{1}{2}kt + nV,\\
F_2 =& \left(\frac{\rho v A}{\mathrm{i}\sin kt} - \frac{n^2}{\mathrm{i}\omega C_0}\right)\left(\dot{\xi}_2 + \dot{\xi}_1\right)\\
&+ \mathrm{i}\rho v A\dot{\xi}_2 \tan\frac{1}{2}kt + nV,
\end{aligned}
\tag{17}
$$

where F_1 and F_2 are forces on the surface of a piezoelectric ceramic disc at $z = 0$ and $z = t$ respectively. Based on Eqs.(5) and (17), Mason's electric equivalent circuit can be drawn, as shown in Fig.1. In this electric circuit, the mechanical forces are equivalent to the voltage, and the vibrating velocities of the piezoelectric element are equivalent to the current. And Eq.(17) can be taken as the circuit equations, following KCL and KVL (Kirchhoff's current and voltage laws).[11] Generally, all the components in such an electric equivalent circuit are taken to be real numbers without taking into consideration the losses. But when they are substituted by complex constants, the components become complex numbers, which can be divided into two parts. The imaginary part represents the dissipative component and energy storage component. We then establish an electric equivalent circuit with these divided components.

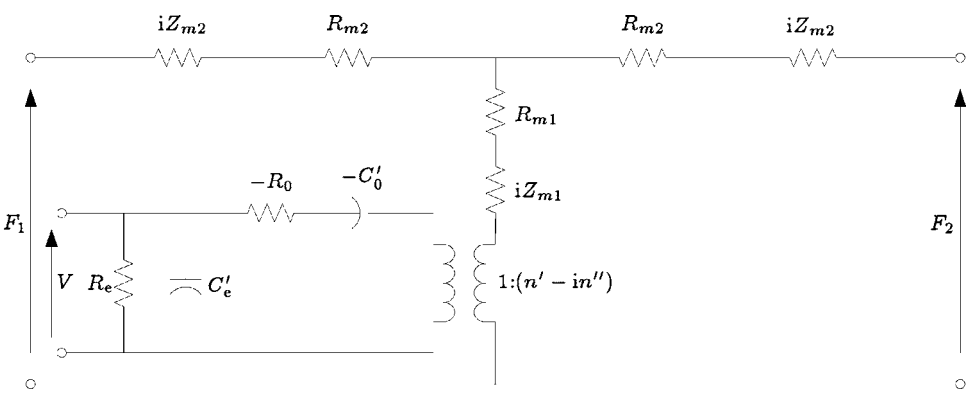


Fig.1. Equivalent circuit with separated components represented by real and imaginary parts of complex physical constants.

Substituting complex physical constants into Eq.(17), we have

$$
\begin{aligned}
\left(\frac{\rho v A}{\mathrm{i}\sin kt} - \frac{n^2}{\mathrm{i}\omega C_0}\right) &= R_{m1} + \mathrm{i}Z_{m1},\\
\mathrm{i}\rho v A\tan\frac{1}{2}kt &= R_{m2} + \mathrm{i}Z_{m2},
\end{aligned}
\tag{18}
$$

where

$$R_{m1}=\frac{\left(2n'n''C'_0+n''^2C''_0-n'^2C''_0\right)}{\omega\left(C'_0+C''_0\right)}\frac{\left(\sin^2 k't\cosh k''t+\cos^2 k't\sinh^2 k''t\right)}{\left(\sin^2 k't\cosh^2 k''t+\cos^2 k't\sinh^2 k''t\right)},$$

$$Z_{m1}=\frac{n'^2C'_0-n''^2C'_0+2n'n''C''_0}{\omega\left(C'_0+C''_0\right)},$$

$$R_{m2}=\frac{\rho A\left(v'\sinh k''t-v''\sin k't\right)}{2\left(\cos^2\frac{1}{2}k't\cosh^2\frac{1}{2}k''t+\sin^2\frac{1}{2}k't\sinh^2\frac{1}{2}k''t\right)},$$

$$Z_{m2}=\frac{\rho A\left(v'\sin k't+v''\sinh k''t\right)}{2\left(\cos^2\frac{1}{2}k't\cosh^2\frac{1}{2}k''t+\sin^2\frac{1}{2}k't\sinh^2\frac{1}{2}k''t\right)}. \tag{19}$$

Generally, the measurements of equivalent circuit parameters are taken when piezoelectric elements are considered to be free of mechanical load. Then, equivalent circuits can be simplified because the forces on the piezoelectric elements in such a case are regarded as null, and the equivalent electric parts in the equivalent circuits are taken to be short circuited. From Fig.1, we can simplify the equivalent circuit to Fig.2.

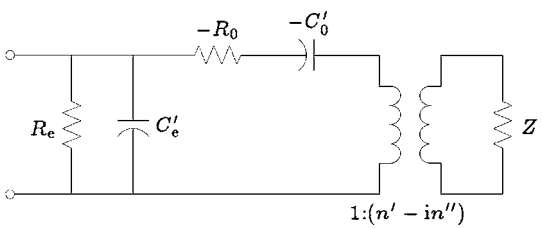


Fig.2. Simplified equivalent circuit in free vibration mode.

In Fig.2, we have

$$\begin{aligned}Z=&R_{m1}+\mathrm{i}Z_{m1}+\frac{R_{m2}+\mathrm{i}Z_{m2}}{2}\\=&\left(\frac{\rho vA}{\mathrm{i}\sin kt}-\frac{n^2}{\mathrm{i}\omega C_0}\right)+\frac{\mathrm{i}\rho vA\tan\frac{1}{2}kt}{2}\\=&\frac{1}{\mathrm{i}\omega C_0}\left(\frac{\omega\rho vA^2}{t\beta_{33}^S\sin kt}-n^2\right.\\&\left.-\frac{\omega\beta_{33}^Sk\rho vA^2\tan\frac{1}{2}kt}{2kt}\right).\end{aligned} \tag{20}$$

We can obtain the equivalent input impedance Z_e of the circuit in Fig.2

$$Z_\mathrm{e}=\frac{1}{\frac{1}{R_e}+\mathrm{i}\omega C'_e+\frac{1}{\frac{Z}{n^2}-\frac{1}{\mathrm{i}\omega C'_0}-R_0}}. \tag{21}$$

According to Eqs.(9) and (10), we have

$$\frac{1}{R_e}+\mathrm{i}\omega C'_e=\frac{1}{R_0+\frac{1}{\mathrm{i}\omega C'_0}}. \tag{22}$$

Thus it leads to

$$\begin{aligned}Z_\mathrm{e}=&\frac{1}{\frac{1}{R_0+\frac{1}{\mathrm{i}\omega C'_0}}+\frac{1}{\frac{Z}{n^2}-\left(R_0+\frac{1}{\mathrm{i}\omega C'_0}\right)}}\\=&R_0+\frac{1}{\mathrm{i}\omega C'_0}-\left(\frac{n^2}{Z}\right)\left(R_0+\frac{1}{\mathrm{i}\omega C'_0}\right)^2.\end{aligned} \tag{23}$$

Substituting Eq.(20) in Eq.(21) and considering the equality

$$\frac{\tan\alpha}{\alpha}=-\sum_v\frac{2}{\alpha^2-\frac{v^2\pi^2}{4}}\quad v=1,\ 3,\ 5,\cdots, \tag{24}$$

we have

$$\begin{aligned}Z_\mathrm{e}=&R_0+\frac{1}{\mathrm{i}\omega C'_0}\\&+\sum_v\frac{1}{\mathrm{i}\omega C'_1+\frac{1}{\mathrm{i}\omega L'_1/v^2}+\frac{1}{R'_1/v^2}+\frac{1}{R''_1/v^2}},\end{aligned} \tag{25}$$

where

$$C'_1=\frac{C_0t^2\omega^2}{8v^2k_{33}^2}=\frac{At\rho\omega^2}{8}\frac{h'^2_{33}-h''^2_{33}}{\left(h'^2_{33}+h''^2_{33}\right)^2},$$

$$\begin{aligned}L'_1=&\frac{8k_{33}^2}{\pi^2C_0\omega^2}\\=&\frac{8}{A\pi^2\omega^2}\left[\frac{\left(h'^2_{33}+h''^2_{33}\right)^2}{c'^D_{33}\left(h'^2_{33}-h''^2_{33}\right)+2c''^D_{33}h'_{33}h''_{33}}\right],\end{aligned}$$

$$R'_1 = \frac{4}{At\rho\omega^3}\frac{\left(h'^2_{33}+h''^2_{33}\right)^2}{h'_{33}h''_{33}},$$
$$R''_1 = \frac{8t}{A\pi^2\omega}\left[\frac{\left(h'^2_{33}+h''^2_{33}\right)^2}{c'^D_{33}\left(h'^2_{33}-h''^2_{33}\right)-2c''^D_{33}h'_{33}h''_{33}}\right]. \quad (26)$$

According to Eq.(25), the equivalent circuit of a piezoelectric ceramic in free vibration mode (Fig.2) can be transformed to another form of equivalent circuit (Fig.3) with separated equivalent-circuit parameters. These parameters are connected to real and imaginary parts of complex physical constants as shown in Eq.(26).

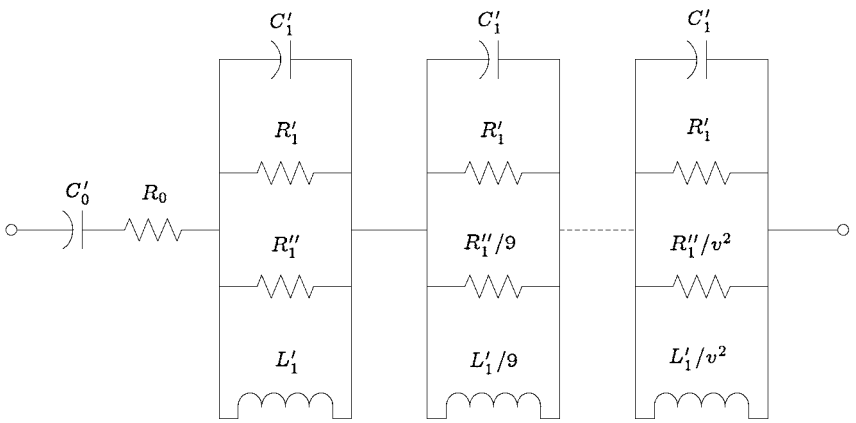


Fig.3. Equivalent circuit with equivalent parameters in free vibration mode.

Commonly, the equivalent circuit parameters in Fig.3 can be detected easily by a network analyser, for example an Anritsu network analyser (MS4630B).

3. Measuring dissipation factors through equivalent circuit parameters

As mentioned above, dissipation factors are used to represent energy losses in piezoelectric materials. However, the measurement of the dissipation factors is always indirect. In Uchino and Hirose's methods,[2] dissipation factors are given by energy ratio between input and output fields. But the field energy is hard to be measured or accurately measured in most applications. In this paper, we propose a method of computing dissipation factors of piezoelectric materials through equivalent circuit parameters. Since the equivalent circuit parameters are easily detectable, dissipation factors can be obtained easily.

In the statement of Uchino and Hirose,[2] three kinds of dissipation factors of piezoelectric materials can be defined as ratios of imaginary and real parts of corresponding complex physical constants. In the thickness mode of piezoelectric ceramics as mentioned above, only one component (in the 3rd-direction) of complex constant tensors is concerned. So we can specify the dissipation factors as follows:

$$\tan\delta = \frac{\beta''^S_{33}}{\beta'^S_{33}},$$
$$\tan\phi = \frac{c''^D_{33}}{c'^D_{33}},$$
$$\tan\theta = \frac{h''_{33}}{h'_{33}}. \quad (27)$$

In researches about the dielectric loss in dielectric materials, a Debye equation is always used to describe the relationship between the relative dielectric constants and the frequency pheonomenologically.[12] In the Debye equation, the ratio of imaginary and real parts of a complex dielectric constant is represented by $\tan\delta$ (dielectric dissipation factor), where δ is the phase delay between an applied electric field and a produced dielectric field. In Eq.(27), other two dissipation factors can be defined similarly. $\tan\phi$ is connected to the mechanical loss and is called the elastic dissipation factor, where ϕ is the phase delay between the stress and the strain; $\tan\theta$ is connected to the piezoelectric loss and is called the piezoelectric dissipation factor, where θ is the phase delay between an applied stress field and the produced dielectric field (direct piezoelectric effect), or the phase delay between an applied electric field and the produced strain field (converse piezoelectric effect). Both delay phases are exactly the same if we introduce the same com-

plex piezoelectric constant h_{33} into the piezoelectric equations.

Considering that equivalent circuit parameters are easy to be detected, we propose a method for measuring dissipation factors through equivalent circuit parameters. From Eq.(10), $\tan\delta$ can be represented by the product of C'_0 and R_0, i.e.

$$\tan\delta = \omega C'_0 R_0. \tag{28}$$

Notice that, in Fig.3, when the frequency is high enough, C'_1 in each parallel branch is short circuited, and R_0 in the series arm can be taken as the impedance of the equivalent circuit at such a frequency. Z_a is the impedance at higher resonance frequency which is also called anti-resonance frequency f_a of a resonator. So, it is reasonable to regard Z_a as a measure of R_0.

In Fig.3, we define a resistance R_1, which represents the parallel resistances of R'_1 and R''_1, i.e.

$$R_1 = \frac{R'_1 + R''_1}{R'_1 R''_1}. \tag{29}$$

R_1 is called dynamic resistance, which also can be detected easily. Considering that the imaginary parts of the complex constants are far smaller than the real parts, the quadratic terms of the imaginary parts in Eq.(26) can be ignored. So we have

$$\begin{aligned} C'_1 R_1 =& \frac{C'_1 (R'_1 + R''_1)}{R'_1 R''_1} \\ =& \frac{2\pi^2 \omega h''_{33} c'^D_{33} + \rho t^2 \omega h'_{33}}{\pi^2 h'_{33} c'^D_{33}}, \\ \frac{L'_1}{R_1} =& \frac{L'_1 R'_1 R''_1}{R'_1 + R''_1} = \frac{\omega \rho t^2 h''_{33}}{\pi^2 h'_{33} c'^D_{33}} + \frac{1}{\omega}. \end{aligned} \tag{30}$$

Solving Eq.(30), $\tan\theta$ is given by

$$\tan\theta = \frac{\omega L'_1 C'_1 - R_1 C'_1}{\omega (R_1 + C'_1 - 2\omega L'_1 C'_1 + 2R_1 C'_1)}. \tag{31}$$

The dynamic inductance L'_1 and capacitance C'_1 are two other parameters which can be measured by many devices. According to Eqs.(26) and (31), the elastic dissipation factor $\tan\phi$ can be given by

$$\tan\phi = \frac{\omega L'_1 - \omega L'_1 \tan^2\theta + 2R_1 \tan\theta}{R_1 - R_1 \tan^2\theta - \omega L'_1 \tan\theta}. \tag{32}$$

Thus, the three dissipation factors are represented by equivalent circuit parameters according to Eqs.(28), (31) and (32). Since all the equivalent circuit parameters mentioned above are detectable, measurements of dissipation factors of piezoelectric materials in thickness mode can be achieved easily.

4. Experimental results

In the following experiments, PZT-4 ceramic elements experience a temperature change from –30°C to 120°C and a stress change from 0MPa to 15Mpa. According to the researches of Berlincourt *et al*,[13–16] the change of temperature or stress will lead to the variation of energy losses in piezoelectric materials.

Figure 4 shows temperature dependence of dissipation factors. They are computed according to the expression in terms of the equivalent circuit parameters. Our samples used for experiments are in stress-free thickness vibration mode. The dielectric dissipation factor $\tan\delta$ decreases initially in proportion to the temperature, and exhibits a tendency to saturation above a certain temperature (about 20°C). Uchino's experiments gave a result from 20°C to 100°C, but he ignored the lower temperature range. The temperature dependences of piezoelectric and elastic dissipation factors are not mentioned in Uchino's research either. In Fig.4, the piezoelectric dissipation factor $\tan\theta$ increases gradually with increasing temperature, indicating that higher temperature results in greater piezoelectric loss. The elastic dissipation factor shows different features in lower and higher temperature regions. When the temperature is lower than about 80°C, $\tan\phi$ increases with the temperature, and falls down with the increase of temperature at temperatures higher than about 90°C.

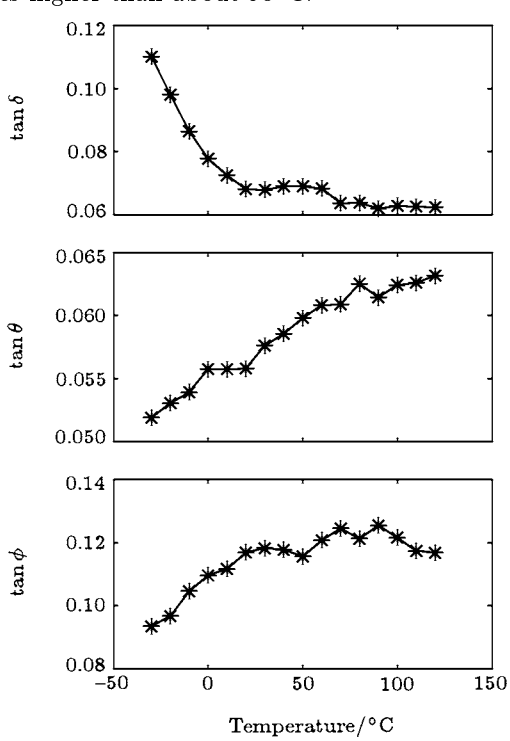


Fig.4. Dissipation factors vs. temperature.

The stress dependences of dissipation factors are shown in Fig.5. Dielectric dissipation factor $\tan\delta$ increases with the stress lower than about 9MPa. When stress keeps increasing, $\tan\delta$ reaches its saturation and then begins to decrease with the stress. In Uchino's experiments, the piezoelectric dissipation factor $\tan\theta$ was measured under both electric field and compressive stress. A correlation factor was used to represent $\tan\theta$ under such a condition. In Fig.5, we give an individual stress dependence of $\tan\theta$. Without the effect of electric field, $\tan\theta$ decreases with the increase of stress. This fact indicates that higher voltage will enlarge the piezoelectric loss. In our experiments, a similar increase of elastic dissipation factor $\tan\phi$ with stress is also found as in Uchino's experiment.

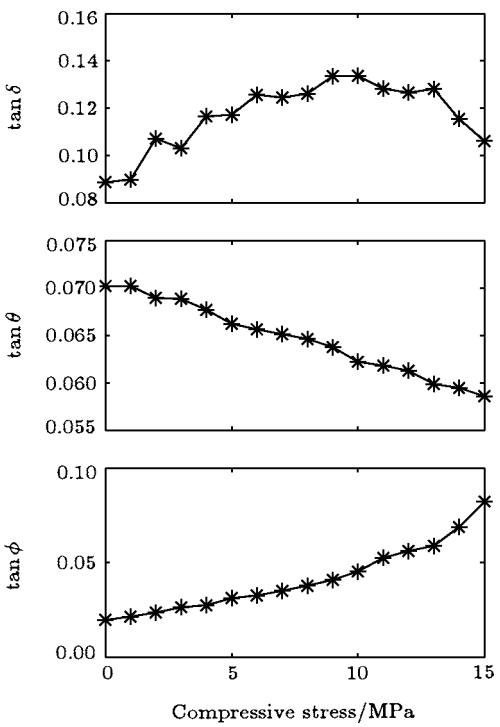


Fig.5. Dissipation factors vs. stress.

5. Conclusion

Dissipation factors are the representations of energy losses in piezoelectric materials, and are of importance in many applications where losses cannot be ignored. However, measurements of dissipation factors are always indirect and not easy. In this paper, we have proposed a method for obtaining the three dissipation factors of piezoelectric materials through equivalent circuit parameters. In an equivalent circuit with complex physical constants, electric components are connected to real and imaginary parts of complex physical constants, and defined as a new set of equivalent circuit parameters differing from the equivalent circuit parameters represented by real physical constants in the Mason's equivalent circuit. Considering that the dissipation factors are defined as ratios of imaginary to real parts of complex physical constants, we give the relationships between dissipation factors and equivalent circuit parameters. Through the well-developed instrumental techniques, equivalent circuit parameters can be detected easily. So, measurements of dissipation factors can be achieved.

In the experiments, PZT ceramic elements in thickness mode are discussed. However, the derivation can also be extended to other vibration modes, such as the longitudinal extension mode and shear vibration mode etc. In each vibration mode, the dissipation factor expressions in terms of the equivalent circuit parameters may take different components of physical constants. In any case, it provides a practical way for measuring dissipation factors. Compared with the results of the previous measurements implemented in many more complicated schemes, similar characteristics of dissipation factors have been observed.

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