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Dynamic behaviours of mix-game model and its application*

Gou Cheng-Ling(苟成玲)†

Department of Physics, Beijing University of Aeronautics and Astronautics, Beijing 100083, *China*

Department of Physics, University of Oxford, Clarendon Laboratory, Parks Road, Oxford, OX1 3PU, UK

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In this paper a minority game (MG) is modified by adding into it some agents who play a majority game. Such a game is referred to as a mix-game. The highlight of this model is that the two groups of agents in the mix-game have different bounded abilities to deal with historical information and to count their own performance. Through simulations, it is found that the local volatilities change a lot by adding some agents who play the majority game into the MG, and the change of local volatilities greatly depends on different combinations of historical memories of the two groups. Furthermore, the analyses of the underlying mechanisms for this finding are made. The applications of mix-game mode are also given as an example.

Keywords: minority game, majority game, mix-game
PACC: 0550

1. Introduction

Since the study of complex systems started at the Santa Fe Institute in 1984, [1,2] more and more physicists, economists, mathematicians and sociologists have been involved in this subject. Several methods have been established to study complex systems.[3,4] Among them, an agent-based model is a powerful approach to study of complex systems. A minority game (MG) is an important agent-based financial market model proposed by Challet and Zhang.[5] Johnson *et al*[6,7] extended MG by allowing a variable number of active traders at each time-step—they referred to their modified game as the Grand Canonical Minority Game (GCMG). The GCMG, and the MG itself, can reproduce the stylized facts of financial markets, such as volatility clustering and fat-tail distributions.

However, there are some weaknesses in the MG and GCMG. First, the diversity of agents is limited, since agents all have the same historical memory and time-horizon. Second, in real markets, some agents are trend-followers, i.e. 'noise traders',[8−15] who effectively play a majority game; while the others are 'fundamentalist', who effectively play a minority game. Some researches on mixed Minority/Majority Games have been done analytically and simulationally.[16−20] In their models, the time-horizon was infinite and all agents had the same ability to deal with historical information and could switch between the majority side and minority side. They studied the stationary macroscopic properties and the dynamical features of the systems with different information structures, i.e. agents received the information that was either exogenous ('random') or endogenous ('real'). They found (a) a significant loss of informational efficiency as compared with a pure minority game (in particular, an efficient, unpredictable phase existed only if the fraction of agents in the majority group $< 1/2$), and (b) a catastrophic increase in global fluctuation if the fraction of agents in majority group $> 1/2$ on the condition that agents received random information.[17] They also found that a small amount of herding tendency could change the collective behaviour dramatically if agents received endogenous information (real history).[19] Zhong et al studies a similar model in which they introduced contrarians into MG who deliberately preferred to hold an opinion that was contrary to the prevailing idea of the commons or normal agents. Their results of numerical simulations revealed that the average success rate among the

*Project supported by China Scholarship Council (Grant No 200307A15).
†E-mail: gouchengling@hotmail.com

agents depended non-monotonically on the fraction of contrarians.[21]

In order to create an agent-based model that more closely mimics a real financial market in this paper, the MG model is modified by dividing agents into two groups: each group has different historical memories and time-horizons; one group plays the minority game and the other plays the majority game. For this reason, we refer to this system as a 'mix-game' model. The difference between the mix-game and the mixed Minority/Majority Games studied by Marsili and Martino *et al* is that the two groups of agents in the mix-game had different bounded abilities to deal with historical information and to count their own performance. This feature of the mix-game represents that agents have bounded rationality. [9]

In this paper, the dynamic features of mix-game are investigated under different combinations of historical memories of the two groups and agents receiving real historical information. The study focuses on the interplay between the strategy spaces of these two groups. Reference [22] reported that almost all agents played with historical memory of 6 or less in a typical minority game with evolution. This means that memory of 6 or less is more important and worth investigating. Therefore, we focus on studying the effect on the MG by adding some agents playing majority game with historical memory of 6 or less. Section 2 introduces the mix-game model and the simulation condition. Section 3 reports the simulation results and discussion. Section 4 gives an example of application of mix-game model. Section 5 presents the conclusions of this study.

2. The mix-game model and simulation condition

In the mix-game, there are two groups of agents; Group 1 plays a majority game, and the Group 2 plays a minority game. N is the total number of the agents and N_1 is number of agents in Group 1. The system resource is $R = 0.5 * N$. All agents compete in the system for the limited resource R. T_1 and T_2 are the time-horizons of the two groups of agents, and m_1 and m_2 denote the historical memories of the two groups of agents, respectively.

Only the global information available to the agents is a common bit-string 'memory' of the m_1 or m_2 for most recent competition outcomes. A strategy consists of a response, i.e. -1 or 1, to each possible bit string; hence there are $2^{2^{m_1}}$ or $2^{2^{m_2}}$ possible strategies for Group 1 or Group 2, respectively, which form full strategy spaces (FSS). At the beginning of the game, each agent is assigned to s strategies and keeps them unchanged during the game. After each turn, agents assign one (virtual) point to a strategy that would have predicted the correct outcome. For agents in Group 1, they will reward their strategies with one point if they are in the majority side; for agents in Group 2, they will reward their strategies with one point if they are in the minority side. Agents collect the virtual points for their strategies over the time-horizon T_1 or T_2, and they use their strategy that has the highest virtual point in each turn. If there are two strategies that have the highest virtual point, agents use coin toss to decide which strategy to be used. Excess demand $D[(t)^-]$ is equal to the number of buy-order that agents choose minus the number of sell-order that agents choose.

$$\begin{aligned} D[(t)^-] &= n_{\text{buy}-\text{orders}}[t-1] - n_{\text{sell}-\text{orders}}[t-1] \\ &= \sum_{i=1}^{N_1} a_i(t-1) + \sum_{j-1}^{N_2} a_j(t-1)\,, \end{aligned} \tag{1}$$

where $a_i(t-1)$ is the action—buy (1) or sell (-1)—that the agent i in Group 1 will take and $a_j(t-1)$ is the action—buy (1) or sell (-1)—that the agent j in Group 2 will take.

According to a widely accepted assumption that excess demand exerts a force on the price of the assets and the change of price is proportional to the excess demand in a financial market, [23,24] the time series of prices of the assets $P[(t)]$can be calculated based on the time series of excess demand as follows:

$$P[(t)] = D[t^-]/\lambda + P[t-1]. \tag{2}$$

Volatility of the time series of prices is represented by the variance of the increases in price. The local volatility Vol is calculated at each time-step by calculating the variance of the increases in price within a small window whose width is d time-step. At time-step t, Vol$[t]$ is calculated between time-step $(t-d)$ and time-step t. This small window moves along the time series of prices as t increases. For simplicity, one can choose $\lambda = 1$. Then the increase in price is just the excess demand. Therefore, we obtain the following formula:

$$\text{Vol}[t] = \frac{1}{d}\sum_{t-d}^{t} D[t^-]^2 - \left(\frac{1}{d}\sum_{t-d}^{t} D[t^-]\right)^2. \tag{3}$$

In simulations, the distribution of initial strategies of agents is randomly uniform in FSS and remains unchanged during the game. Each agent has two strategies. The simulation running time-steps are 3000 in number. The window length of local volatility is $d = 5$.

3. Simulation results and discussion

3.1. The effect of mix-game on local volatilities

To see the effect of mix-game on local volatilities, simulations are performed with different parameter configurations of historical memories, the number of total agents and fraction of agents in Group 1. In order to see the interplay between agent strategy spaces of the two groups, we choose three categories of configurations of historical memories: (a) $m_1 < m_2 = 6$; (b) $m_1 = m_2 \leq 6$; (c) $m_1 = 6 > m_2$. The number of total agents (N) varies from 101 to 401. Time horizons (T_1 and T_2) are relatively stable. We will look at the influence of time horizons (T_1 and T_2) on volatilities of mix-game in the future.

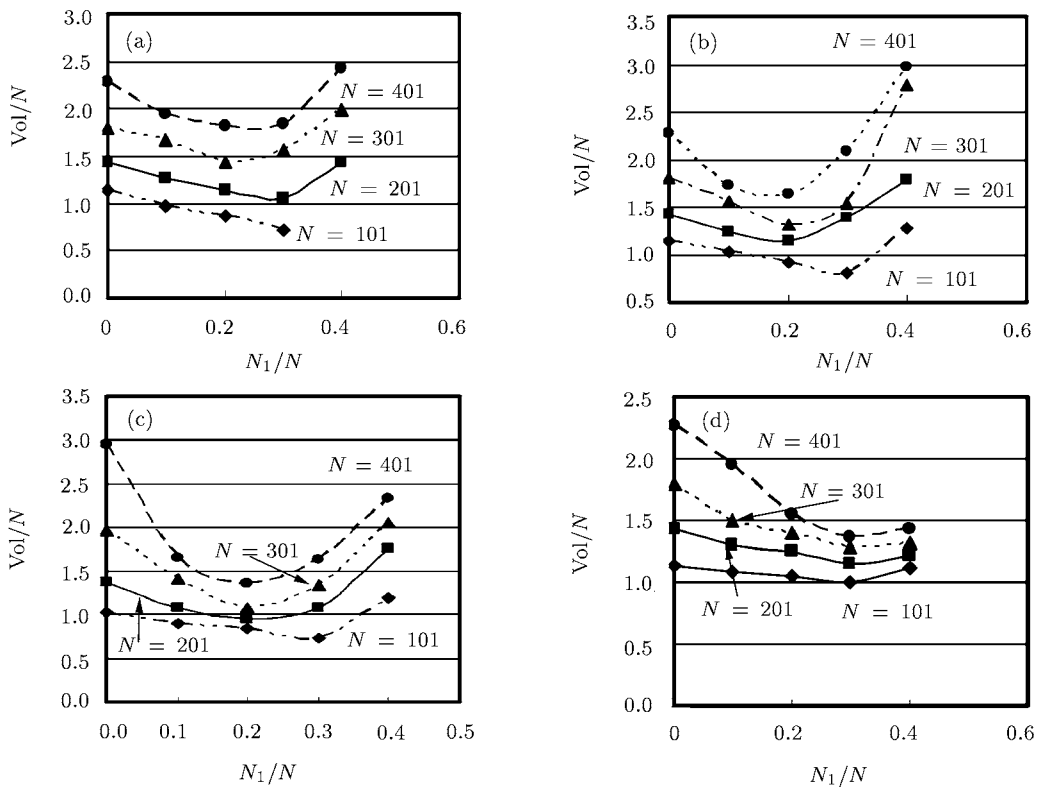


Fig.1. The Vol/N representing the mean of local volatilities per agent which is equal to the mean of local volatilities divided by the number of total agents (N) changes with N_1/N under the condition of $m_1 < m_2$, $T_1 < T_2$ and N varying from 101 to 401. (a), (b), (c) and (d) show the simulation results with parameters of (a) $m_1 = 1$, $m_2 = 6$, $T_1 = 12$ and $T_2 = 60$; (b) $m_1 = 3$, $m_2 = 6$, $T_1 = 12$ and $T_2 = 60$; (c) $m_1 = 3$, $m_2 = 6$, $T_1 = 50$ and $T_2 = 100$; (d) $m_1 = 5$, $m_2 = 6$, $T_1 = 12$ and $T_2 = 60$, respectively.

Figure 1 shows that the means of local volatilities per agent change as N_1/N increases from 0 to 0.4 under the condition of $m_1 < m_2 = 6$, $T_1 < T_2$, N from 101 to 401. We can find that the means of local volatilities per agent have the lowest points at about $N_1 1/N = 0.2$ or 0.3. We can also notice that the means of local volatilities per agent increase as N increases from 101 to 401. Comparing Fig.1(b) with Fig.1(c), we can find that the means of local volatilities per agent are influenced by T_1 and T_2. Vol/N in Fig.1(d) increases less at $N_1/N = 0.4$ than in Figs.1(a), (b) and (c). Reference [21] indicates similar results when contrarians are introduced into the MG, but the difference is that the lowest point of volatility

appears when the fraction of contrarians is 0.4.

From Fig.2 we can find that the means of local volatilities per agent decrease and approach 1 with N_1/N increasing from 0 to 0.4 under the condition of $m_1 = m_2 \leq 6$, $T_1 = T_2 = 12$ and $N = 101, 201, 301, 401$. This means that the efficiency of a system can be improved by adding some agents with the same parameters (i.e. $m_1 = m_2$, $T_1 = T_2$.) who play the majority game.

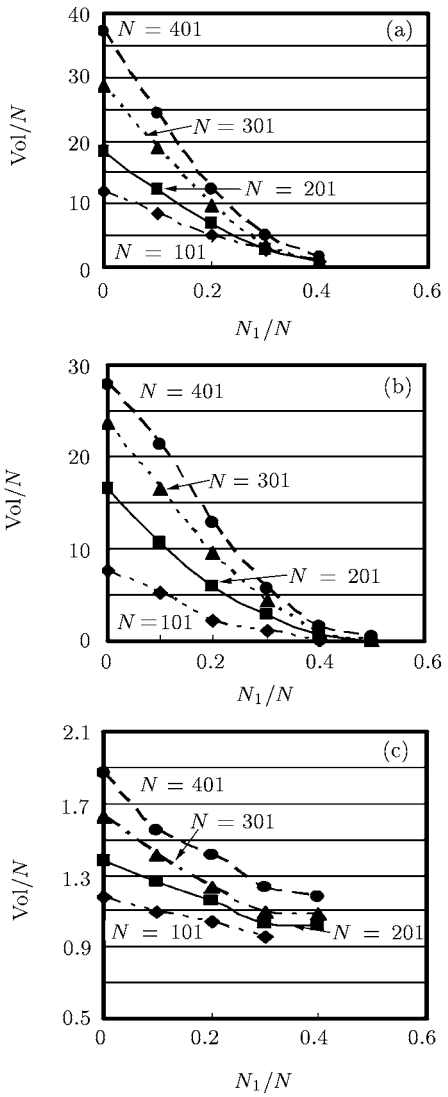


Fig.2. Vol/N representing the mean of local volatilities per agent which is equal to the mean of local volatilities divided by the number of total agents (N) changes with N_1/N under the condition of $m_1 = m_2 \leq 6$, $T_1 = T_2 = 12$ and N varying from 101 to 401.

Reference [17] reported similar results under the condition of 'random information' with $N_1/N < 0.5$. And Ref. [25] observed the similar phenomena in distributed computing systems. Comparing Figs.2(a) and (b) with Fig.2(c), we can find that the means of local volatilities per agent in Figs.2(a) and (b) are much larger than those in Fig.2(c). This means that smaller m_1 and m_2 make the means of local volatilities per agent increase dramatically.

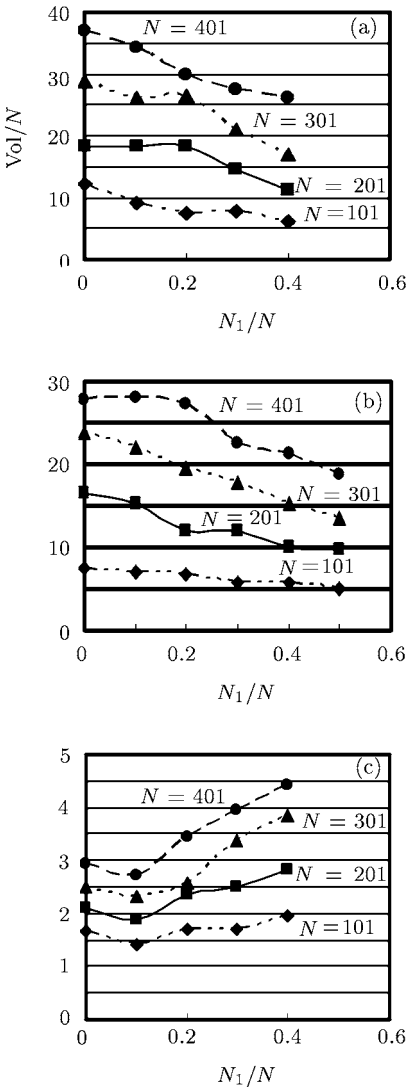


Fig.3. Vol/N representing the mean of local volatilities per agent which is equal to the mean of local volatilities divided by the number of total agents (N) changes with N_1/N under the condition of $m_1 = 6 > m_2$, $T_1 = 60$, $T_2 = 12$ and N varying from 101 to 401.

Figure 3 shows the simulation results with $m_1 = 6 > m_2$ and $T_1 > T_2$, where the means of local volatilities per agent decrease some times, but increase some times as N_1/N increases from 0 to 0.4. For cases of $m_1 = 6$, $m_2 = 1$ and $m_1 = 6$, $m_2 = 3$, the means of local volatilities per agent decrease as N_1/N increases from 0 to 0.4; for the case of $m_1 = 6$, $m_2 = 5$, the means of local volatilities per agent have the lowest points at $N_1/N = 0.1$. However, we can see that the means of local volatilities per agent increase with

N increasing from 101 to 401. Comparing Figs.3(a) and (b) with Figs.2(a) and Fig.2(b), we can find that Vol/N in Figs.3(a) and (b) decrease much more slowly than in Figs.2(a) and (b) as N_1/N increases from 0 to 0.4. Comparing Fig.3(c) with Fig.1(d) and Fig.2(c), we notice that Vol/N in Fig.3(c) behaves quite differently from that in Fig.1(d) or Fig.2(c).

Through simulations, we find that local volatilities drop to zeros in most cases when N_1/N is larger than or equal to 0.5 under the simulation condition of $m_1 = m_2$ or $m_1 < m_2$ with some initial strategy distributions. This phenomenon also can be influenced by T_1. But this phenomenon has not been found under the simulation condition of $m_1 > m_2$.

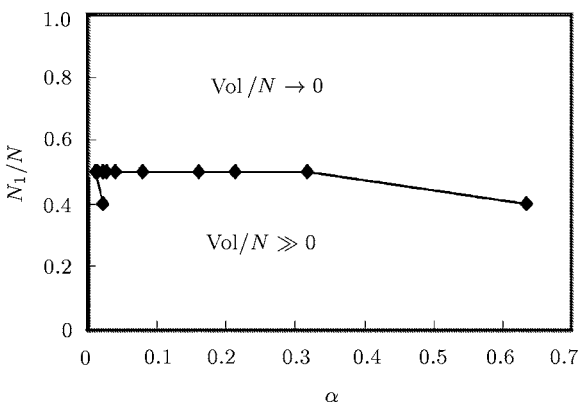


Fig.4. The two-phase phenomenon of local volatilities under a simulation condition of $m_1 = m_2$ or $m_1 < m_2$, where $\alpha = 2^{m_1}/N$.

Figure 4 shows the two-phase phenomenon of local volatilities under the simulation condition of $m_1 = m_2$ or $m_1 < m2$, where $\alpha = 2^{m_1}/N$. The reason why local volatilities drop to zero is that all agents quickly adapt themselves to using one strategy and stick to it, i.e. all agents are 'frozen' in these simulation conditions. Local volatilities dropping to zero can be found frequently in a pure majority game, and the smaller the historical memory is, the more easily the agents become 'frozen'. Therefore, we can say that characteristics of minority game dominates in the regime of $N_1/N < 0.5$ and characteristics of majority game dominates in the regime of $N_1/N \geq 0.5$ except for two points as shown in Fig.4 where the phases change at $N_1/N = 0.4$. However, Refs. [17] and [21] did not report such a phenomenon. A possible explanation for this is that agents received 'random information' for Ref. [17] and agents rewarded their strategies according to minority rule for Ref. [21] so that agents failed to adapt themselves to the keeping of using one strategy.

3.2. Analysis and discussion

In order to understand the simulation results, a theoretical analysis can be made as follows. First, we consider the case of $m_1 = m_2$. Because these two groups have the same historical memories, i.e. their strategy spaces are the same, and Group 1 plays the majority game and Group 2 plays the minority game, their strategies can be fully anti-correlated if agents in two groups uniformly distribute in the strategy spaces. Therefore, the dynamic process of the mix-game with $m_1 = m_2$ behaves like the dynamic process of pure minority game with "the effective number of agents" if $x = N_1/N < 0.5$. Reference [26] also shows the dynamics behaviour of the mix-game with $m_1 = m_2$ is similar to that of the pure MG with the same parameters. According to Crowd–Anticrowd theory,[6] the volatility of MG is proportional to square of the number of agents, i.e. $\mathrm{Vol}(x = 0) \propto N^2$. So in the mix-game with $m_1 = m_2$, the volatility is proportional to square of 'the effective number of agents', i.e. $\mathrm{Vol}(x) \propto (N^{\mathrm{eff.}})^2$. Because the strategies of these two groups are fully anti-correlated, 'the effective number of agents' is equal to the number of agents in Group 2 minus the number of agents in Group 1, i.e. $N^{\mathrm{eff.}} = N_2 - N_1$. Since $N_2 = (1-x)N$ and $N_1 = xN$, we have $N^{\mathrm{eff.}} = (1-2x)N$. Therefore, we obtain

$$\mathrm{Vol}(x) \propto (N^{\mathrm{eff.}})^2 = (1-2x)^2 N^2. \tag{4}$$

By defining

$$y = \frac{\mathrm{Vol}(x)}{Vol(x=0)}, \tag{5}$$

we obtain

$$y = (1-2x)^2 . \tag{6}$$

Figure 5 shows the relation between y and x of formula (6). Comparing Fig.5 with Fig.2, we can find that Fig.2(a) and Fig.2(b) qualitatively match Fig.5, but the volatility in Fig.2(c) decreases slowly. This implies that the strategies of Group 1 are indeed fully anti-correlated with those of Group 2 in the mix-game when historical memories are $m_1 = m_2 = 1$ or 3, and the strategies of Group 1 is partly anti-correlated with those of Group 2 when historical memory is $m_1 = m_2 = 6$. The reason for this is that the strategy space of historical memory $m = 6$ is much larger than that of $m = 1$ or 3 so that agents less uniformly distribute in the strategy space of $m = 6$ than in strategy space of $m = 1$ or 3.

Second, we look at the case of $m_1 = 6 > m_2$. Because the strategy space of Group 1 is much larger than that of Group 2, the distribution of agents in the strategy space 1 is not uniform so that the strategies held by Group 1 have little chance to anti-correlate with those held by Group 2. Therefore, we can assume that these two groups behave independently. The volatility can be calculated as follows:

$$\mathrm{Vol}(x) = Vol_{m1=6}(x) + \mathrm{Vol}_{m2}(x). \tag{7}$$

So we have

$$y = \frac{\mathrm{Vol}(x)}{\mathrm{Vol}(x=0)} = \frac{\mathrm{Vol}_{m1=6}(x) + \mathrm{Vol}_{m2}(x)}{\mathrm{Vol}_{m2}(x=0)}. \tag{8}$$

Through simulation, we obtain $\mathrm{Vol}_{m1=6}(x)$, $\mathrm{Vol}_{m2}(x=0)$, and $\mathrm{Vol}_{m2}(x)$. Figure 6 shows the relation between y and x of the formula (8) with $N = 401$, $m_1 = 6$, $m_2 = 1$, 3, 5 and $T_1 = T_2 = 12$. Comparing Fig.6 with Fig.3, we can find volatilities in Fig.6 qualitatively match those in Fig.3. This shows that the argument that these two groups behave independently is reasonable.

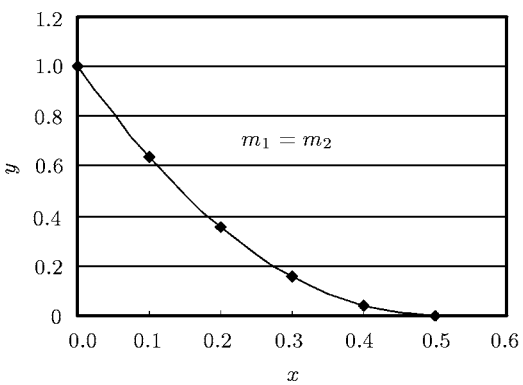


Fig.5. Relation between y and x of the formula (6).

Finally, we discuss the case of $m_1 < m_2 = 6$. Because the strategy space of Group 1 is much smaller than that of Group 2, the distribution of agents in the strategy space 1 is relatively uniform so that the strategies held by Group 1 have chance to partly anti-correlate with those held by Group 2. We refer to this as 'anti-correlated effect'. On the other hand, through simulations we find that the volatilities of the pure majority game are several times larger than those of the pure minority game with the same parameters. We refer to this as "majority game effect". Therefore, the dynamic processes of the mix-game with $m_1 < m_2 = 6$ are influenced by the interplay between the anti-correlated effect and the majority game effect. This is why the volatilities have the minimums at $x_{\min}$. If $x < x_{\min}$, the anti-correlated effect dominates in the mix-game. If $x > x_{\min}$, the majority game effect dominates in the mix-game. The mix-game in this case is more interesting than in the previous two cases because the mix-game with $m_1 < m_2 = 6$ has potential applications in modelling financial markets. We will discuss this point in the following section.

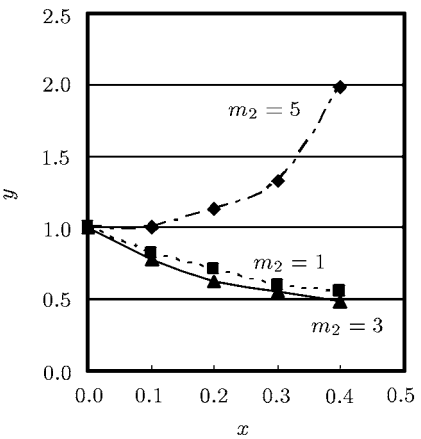


Fig.6. Relation between y and x of the formula (8).

4. Application of mix-game model

4.1. The effect of mix-game on price time series

In order to see whether the mix-game is a potential model for financial markets, one needs to know the effect of mix-game on time series of prices. Therefore, we examine the time series of prices of MG and mix-game with the same parameters as those in the above section. We find that agents' playing majority game almost does not change the stylized features of price time series of MG except for $m_1 = 1$ or $2 < m_2 = 6$ if $N_1/N < 0.5$. A naive explanation for this is that the minority game dominates in mix-game when $N_1/N < 0.5$. Figure 7 shows the examples of the time series of prices that are performed under the condition of $N = 201$, $N_1 = 72$, $T_1 = T_2 = 12$ and $m_1 = 1$, 2, $3 < m_2 = 6$ in the mix-game.

Comparing with the price time series of the MG with $m = 6$, $T = 12$ and $N = 201$, one can find that the features of price time series in the mix-game almost do not change if some agents with $6 \geq m_1 \geq 3$ play the majority game and the other agents with $m_2 = 6$ play minority game. Figure 7(c) is an example of this phenomenon. However, the features of price time series change dramatically if some agents with $m_1 = 1$ or 2 play the majority game under the condition of $m_2 = 6$ as shown in Fig.7(b) and Fig.7(d).

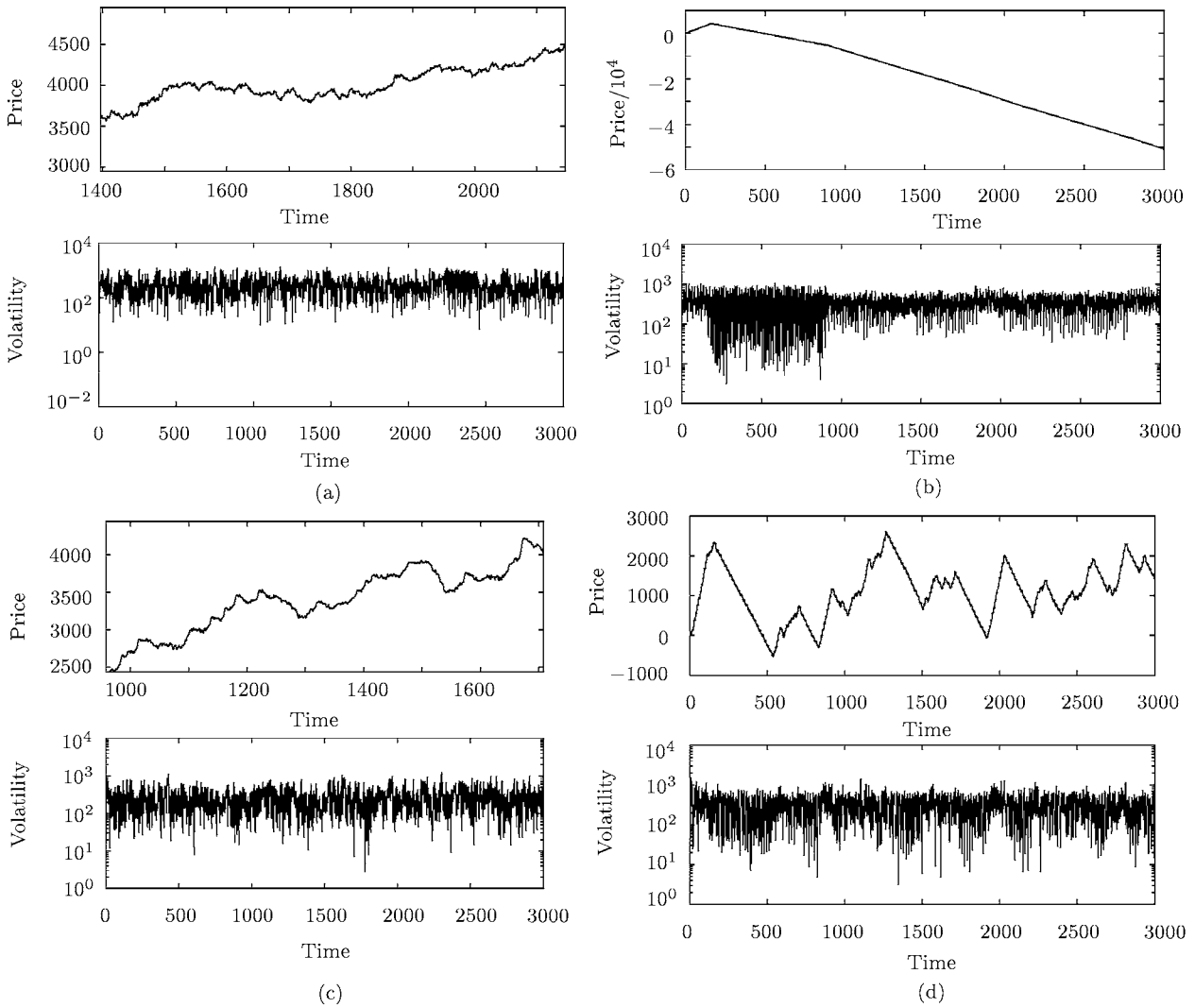


Fig.7. Time series of prices and local volatilities of MG with $m = 6$, $T = 12$ and $N = 201$, and mix-game with $T_1 = T_2 = 12$, $N = 201$, $N_1 = 72$ and $m_1 = 1, 2, 3 < m_2 = 6$ in (b), (d) and (c), respectively; the upper half figures in (a), (b), (c) and (d) are for price time series and the lower half figures are for time series of local volatilities; the x-axes represent time steps.

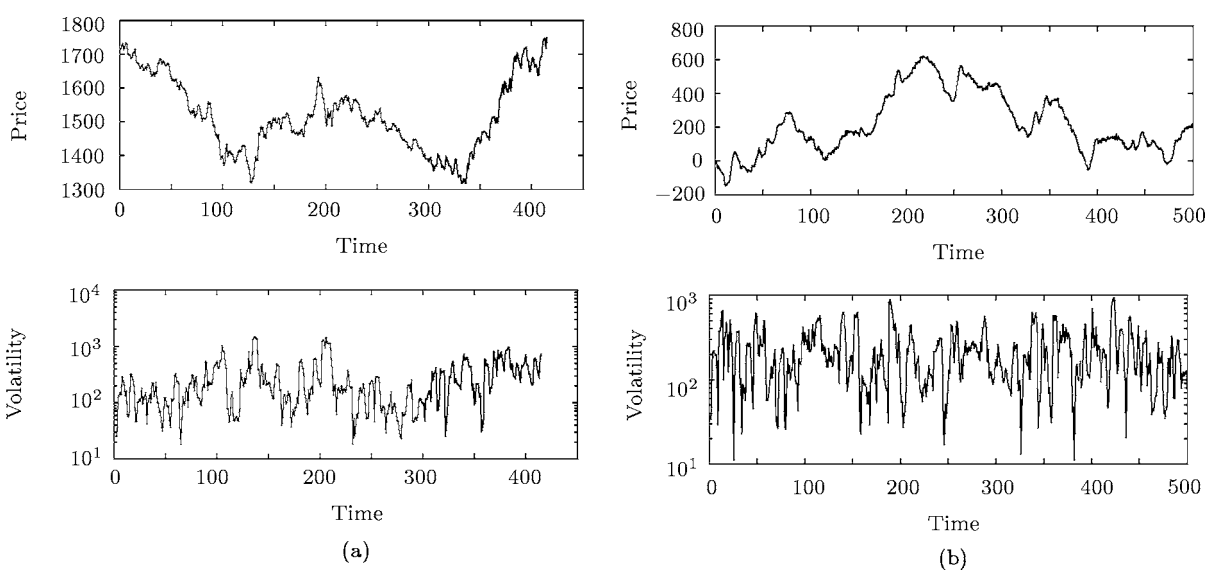


Fig.8. Time series and local volatilities of Shanghai index daily data in (a) and mix-game with parameters of $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, $N = 201$, $N_1 = 40$, and $s = 2$ in (b).

The price time series show strong trends, which do not appear in real price time series of financial markets. Andersen *et al* already noticed a similar phenomenon for the \$-game in Ref. [8]. In Ref. [26], we find that the agents' average winnings in the mix-game under the condition of $m_1 = 1$ or 2, $m_2 = 6$, $T_1 = T_2 = 12$, $N = 201$ and $N_1 = 72$ are much bigger than those in the MG under the condition of $m = 6$, $N = 201$ and $T = 12$ as shown in Table 1. This result implies that agents in the mix-game who are not only in the majority group but also in minority group can take the advantage of the trends of price time series to improve their performances.

Table 1. Average winnings of both groups in mix-game and MG where $N = 201$, $N_1 = 72$ and $T_1 = T_2 = T = 12$.

	mix-game		MG
	$m_1 = 1$, $m_2 = 6$	$m_1 = 2$, $m_2 = 6$	$m = 6$
Group 1	0.693	0.612	0.466
Group 2	0.511	0.493	

4.2. The rules and the example for simulating financial markets by mix-game

From the previous sections, one can find out that the mix-game with some parameters reproduces the stylized features of financial time series, but some fail to do so. Therefore, one needs to choose parameters of m_1, m_2, T_1, T_2, N and N_1 by using the mix-game to model financial markets. The following aspects need to be considered.

1) Firstly, make sure that the time series of mix-game can reproduce the stylized facts of price time series of financial markets by choosing proper parameters such that m_1 and m_2 must be larger than 2, and N_1/N is smaller than 0.5;

2) Secondly, pay attention to the fluctuation of local volatilities, and ensure that the median of local volatilities of mix-game is similar to that of the target time series by selecting N_1 and N since Vol/N increases with the increase of N;

3) Thirdly, make sure that the log-log plots of absolute returns look similar.

Since the median value of local volatilities of Shanghai Index daily data from 2nd January 1992 to 19th March 2004 is 222, the combination of parameters of the mix-game has similar median values of local volatilities according to Fig.1(b). This combination is $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, $N = 201$, $N_1 = 40$, and $s = 2$.

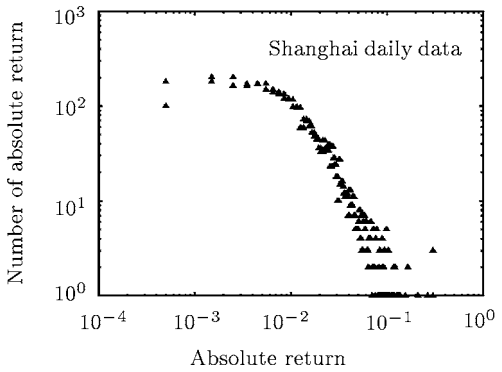


Fig.9. The log–log plot of Shanghai index daily absolute returns which is non-Gaussian.[27].

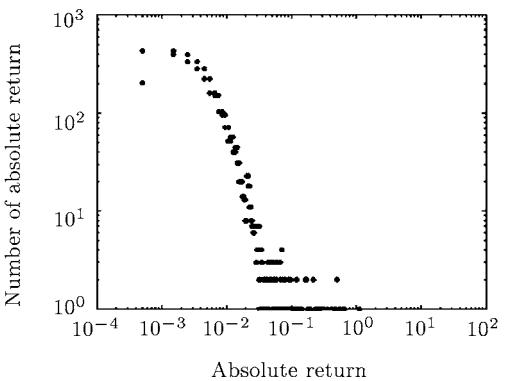


Fig.10. The log-log plot of mix-game absolute returns with parameters of $m_1 = 3$, $m_2 = 6$, $T_1 = 12$, $T_2 = 60$, $N = 201$, $N_1 = 40$, and $s = 2$.

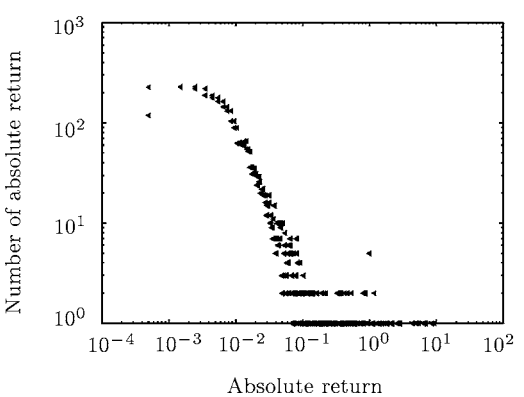


Fig.11. The log–log plot of MG absolute returns with parameters of $m = 6$, $T = 60$, $N = 201$, and $s = 2$.

Figures 8, 9 and 10 show that the mix-game with $m_1 = 3$, $T_1 = 12$, $m_2 = 6$, $T_2 = 60$, $N_1 = 40$, $N = 201$ and $s = 2$ can reproduce similar stylized facts as of the Shanghai stock market. As shown in Fig.11, the MG with parameters of $m = 6$, $T = 60$, $N = 201$, and $s = 2$ has a much fatter tail in the log–log plot of its absolute returns than with Shanghai index daily data.

So for simulating the Shanghai Index, the mix-game behaves better than the MG.

4. Conclusion

In the mix-game, different configurations of agent historical memories greatly influence the local volatilities. Means of local volatilities decrease monotonously with N_1/N increasing from 0 to 0.4 if $m_1 = m_2 = 1$, 3 or 6; means of local volatilities decrease first and then increase with N_1/N increasing from 0 to 0.4 if $m_1 < m_2 = 6$; means of local volatilities sometimes increase and sometimes decrease if $m_1 = 6 > m_2$ with N_1/N increasing from 0 to 0.4. It is found that local volatilities drop to zero in most cases if $N_1/N \geq 0.5$ under the simulation condition of $m_1 = m_2$ or $m_1 < m_2$ with some initial strategy distributions.

The underlying mechanisms for the above findings are as follows: if these two groups have the same historical memories, their strategies are anti-correlated; if the historical memory of Group 1 is 6 and larger than that of Group 2, their strategies are uncorrelated; if the historical memory of Group 2 is 6 and larger than that of Group 1, their strategies are partly anti-correlated.

Besides, historical memories, m for the MG, m_1 and m_2 for the mix-game, influence the dynamics of price time series. Finite time horizons, T for the MG, T_1 and T_2 for the mix-game, also have an influence on the dynamics of price time series as studied by Hart and Johnson[28] and Chalet.[29] Therefore, further study will focus on the combined effects caused by changing time horizons and historical memories

The mix-game can be a potentially good model for financial markets with specific parameters. For Shanghai Index, there is one suitable configuration of parameters: $m_1 = 3$, $T_1 = 12$, $m_2 = 6$, $T_2 = 60$, $N_1 = 40$, $N = 201$, and $s = 2$. For any other financial markets, parameters need to be adjusted correspondingly.

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