Vol 15 No 6, June 2006
1009-1963/2006/15(06)/1231-08
Chinese Physics
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Analysis of stochastic bifurcation and chaos in stochastic Duffing–van der Pol system via Chebyshev polynomial approximation*

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(Received 23 June 2005; revised manuscript received 16 March 2006)

The Chebyshev polynomial approximation is applied to investigate the stochastic period-doubling bifurcation and chaos problems of a stochastic Duffing–van der Pol system with bounded random parameter of exponential probability density function subjected to a harmonic excitation. Firstly the stochastic system is reduced into its equivalent deterministic one, and then the responses of stochastic system can be obtained by numerical methods. Nonlinear dynamical behaviour related to stochastic period-doubling bifurcation and chaos in the stochastic system is explored. Numerical simulations show that similar to its counterpart in deterministic nonlinear system of stochastic period-doubling bifurcation and chaos may occur in the stochastic Duffing–van der Pol system even for weak intensity of random parameter. Simply increasing the intensity of the random parameter may result in the period-doubling bifurcation which is absent from the deterministic system.

Keywords: stochastic Duffing–van der Pol system, Chebyshev polynomial approximation, stochastic period-doubling bifurcation, stochastic chaos

PACC: 0547

1. Introduction

A stochastic system that we consider here means a system with random parameters, which are typically bounded in nature. Stochastic bifurcation that we consider here means bifurcation in a stochastic nonlinear system under harmonic excitations, which differs from another type of stochastic bifurcation resulting from a deterministic nonlinear system under harmonic excitations together with random excitations. There are several kinds of mathematical methods available for dealing with analysis of a stochastic system. The first one is the Monte Carlo method,[1−3] which is simple and universal, but usually involves a quite amount of computational effort. The second one is the stochastic perturbation method,[4,5] which involves the least computational effort, but usually is restricted to a system with random variables of small fluctuations only. The third one is the orthogonal polynomial approximation method for stochastic systems, which was first introduced by Spanos and Ghanem,[6] later by Li and Liao.[7] The third method does not require the small random perturbation assumption, thus providing more applicability. Recently, Fang *et al*[8] first applied Chebyshev polynomial approximation to the evolutionary random response problem of a linear stochastic system, and then Leng [9] further applied it to study bifurcation and chaos of a stochastic Duffing system. The authors of Ref.[10] studied the stochastic bifurcation behaviour in a stochastic Duffing–van der Pol system with bounded parameters and explored the feature of the stochastic period-doubling bifurcation. Some researchers also used other numerical methods to solve bifurcation problems of stochastic system.[11,12]

The Duffing–van der Pol (DP for short) system, capable of describing many practical nonlinear dynamical problems, is a typical nonlinear system hav-

*Project supported by the National Natural Science Foundation of China (Grants Nos 10472091 and 10332030).
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ing both the self-excitation property of the van der Pol system and the cubic nonlinear stiffness of the Duffing system in the same model. Holmes and Rand[13] studied the phase portraits and bifurcation of a DP system free from excitation. Kozlov *et al*[14] explored the complicated dynamical behaviour in a system of coupled DP oscillators. The principal parametric resonance of a DP oscillator subjected to combined harmonic and bounded random excitations was studied by the multiple scale method.[15] Global analysis of stochastic bifurcation in a DP oscillator by the generalized cell mapping method with diagraph was reported in Ref.[16] However, all the above-mentioned results are obtained for deterministic DP system or DP system under random excitation, and the stochastic DP system with bounded random parameters has not been examined, including the problems of stochastic bifurcation and stochastic chaos.

In this paper, the stochastic period-doubling bifurcation of a stochastic DP system under a harmonic excitation $F\cos\omega t$ is studied by the Chebyshev polynomial approximation method. The equivalent deterministic system for the stochastic DP system is derived first. Then, the responses of the equivalent system are obtained by numerical methods. Taking ω as the bifurcation parameters, we explore systematically nonlinear phenomena related to stochastic period-doubling (PD for short) bifurcation and stochastic chaos in the stochastic DP system.

This paper is organized as follows. In section 2, the second kind of Chebyshev polynomial is introduced as the base of orthogonal polynomial approximation. In section 3, transformation of the stochastic DP system into its equivalent deterministic one by Chebyshev polynomial approximation is shown. Section 4 is devoted to the investigation of various nonlinear dynamical phenomena related to stochastic PD bifurcation, the features of the stochastic PD bifurcation and the stochastic chaos in the stochastic DP system.

2. Chebyshev polynomials

Random parameters in some engineering structures are bounded in nature, so the probability density function (PDF for short) models for the bounded random variables is taken to be an arch-like PDF of the form

$$p(u)=\begin{cases}(2/\pi)\sqrt{1-u^2}, & |u|\le 1,\\ 0, & |u|>1.\end{cases} \tag{1}$$

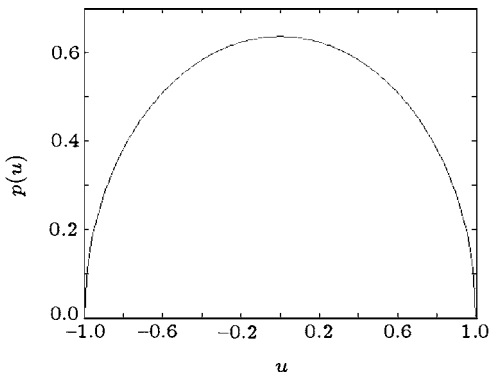


Fig.1. The arch-like PDF curve for random variable U.

Figure 1 shows the arch-like PDF curve for bounded random variable U. As the orthogonal polynomial basis for this kind of PDF, the second kind of Chebyshev polynomials is the only choice. The general expression for the second kind of Chebyshev polynomials can be expressed as follows:[17]

$$U_n(u)=\sum_{k=0}^{[n/2]}(-1)^k\frac{(n-k)!}{k!\,(n-2k)!}(2u)^{n-2k}. \tag{2}$$

Thus, we have

$$\begin{aligned}&U_0(u)=1,\\ &U_1(u)=2u,\\ &U_2(u)=4u^2-1,\\ &U_3(u)=8u^3-4u,\\ &U_4(u)=16u^4-12u^2+1,\\ &\cdots\cdots.\end{aligned} \tag{3}$$

The recurrent formula for Chebyshev polynomials of the second kind is

$$uU_n(u)=\frac{1}{2}\left(U_{n-1}(u)+U_{n+1}(u)\right). \tag{4}$$

The orthogonality of Chebyshev polynomials of the second kind can be expressed as

$$\int_{-1}^{1}\frac{2}{\pi}\sqrt{1-u^2}U_i(u)\,U_j(u)=\begin{cases}1, & (i=j),\\ 0, & (i\ne j).\end{cases} \tag{5}$$

Obviously, Eq.(5) represents a weighted orthogonal relationship. Since the weighting function is just the same as the arch-like PDF, $p(u)$ in Eq.(1), the left-hand side of Eq.(5) may be regarded as the expectation of the product of $U_i(u)$ and $U_j(u)$. Owing to the

orthogonality of Chebyshev polynomials, any measurable function $f(u)$ can be expanded into the following series form:

$$f(u)=\sum_{i=0}^{\infty}c_iU_i(u),$$

where $c_i=\int_{-1}^{1}p(u)f(u)U_i(u)\,\mathrm{d}u$. Similar results can be obtained for measurable functions of several mutually independent random variables. We call this expansion an orthogonal decomposition of random function $f(u)$, which is the theoretical base of orthogonal decomposition methods.

3. Chebyshev polynomial approximation for stochastic DP system

Consider a DP system with a bounded random parameter and subject it to harmonic excitation. The differential equation for its dynamical response may be expressed as follows:

$$\ddot{x}+\left(a+cx^2\right)\dot{x}+bx^3=f(t),\tag{6}$$

where a,c are constant, b is a random parameter, and $f(t)=F\cos(\omega t)$ is a given harmonic excitation. Suppose that b can be expressed as

$$b=\bar{b}+\delta u,\tag{7}$$

where $\bar{b}$ and $\delta/2$ are the mean value and the standard deviation of b respectively, δ is taken as the intensity of b and U is a random parameter defined on $[-1,1]$ with a given PDF, say an arch-like PDF. It follows from the orthogonal polynomial approximation that the response $x(t,u)$ of system (6) can be expressed by the following series:

$$x(t,u)=\sum_{i=0}^{N}x_i(t)\,U_i(u),\tag{8}$$

where $U_i(u)$ represents the ith Chebyshev polynomial, N represents the largest order of the polynomials we have taken, and $x_i(t)=\int_{-1}^{1}p(u)x(t,u)\,U_i(u)\,\mathrm{d}u$.

Substituting Eqs.(7) and (8) into Eq.(6), we have

$$\begin{aligned}&\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+a\frac{\mathrm{d}}{\mathrm{d}t}\right)\sum_{i=0}^{N}x_i(t)\,U_i(u)\\&+\frac{1}{3}c\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=0}^{N}x_i(t)\,U_i(u)\right)^3\\&+\bar{b}\left(\sum_{i=0}^{N}x_i(t)\,U_i(u)\right)^3\\&+\delta u\left(\sum_{i=0}^{N}x_i(t)\,U_i(u)\right)^3=f(t).\end{aligned}\tag{9}$$

The nonlinear term $\left(\sum_{i=0}^{N}x_i(x)\,U_i(u)\right)^3$ on the left side of Eq.(9) can be expanded into

$$\begin{aligned}\left(\sum_{i=0}^{N}x_i(t)\,U_i(u)\right)^3=&x_0^3(t)\,U_0^3(u)+\cdots+x_N^3(t)\,U_N^3(u)+3x_0^2(t)\,x_1(t)\,U_0^2(u)\,U_1(u)+\cdots\\&+3x_{N-1}(t)\,x_N^2(t)\,U_{N-1}(u)\,U_N^2(u)+6x_0(t)\,x_1(t)\,x_2(t)\,U_0(u)\,U_1(u)\,U_2(u)+\cdots\\&+6x_{N-2}(t)\,x_{N-1}(t)\,x_N(t)\,U_{N-2}(u)\,U_{N-1}(u)\,U_N(u).\end{aligned}\tag{10}$$

By the recurrent formulas, the triple product polynomial of Eq.(10) can be further reduced into a linear combination of related single polynomials. As for Chebyshev polynomials, $U_i(u)$, we have

$$\begin{aligned}&U_0^3(u)=U_0(u),\\&3U_0^2(u)\,U_1(u)=3U_1(u),\\&3U_1^2(u)\,U_2(u)=3U_4(u)+6U_2(u)+3U_0(u),\\&\cdots\cdots.\end{aligned}\tag{11}$$

By denoting the coefficient of $U_i(u)$ in the linear combination as $X_i(t)$, the right side of Eq.(10) can be finally expressed as

$$\begin{aligned}&\left(\sum_{i=0}^{N}x_i(t)\,U_i(u)\right)^3\\=&X_0(t)\,U_0(u)+\cdots+X_N(t)\,U_N(u)\\=&\sum_{i=0}^{3N}X_i(t)\,U_i(u).\end{aligned}\tag{12}$$

The expressions of $X_i(t),i=0,1,2,\cdots,N$, can be derived through computer algebraic system, such

as MAPLE.[18] By the recurrent formulas Eq.(4) of Chebyshev polynomials and by Eq.(12), the fourth term of Eq.(9) can be reduced to

$$\begin{aligned} & u\delta\left(\sum_{i=0}^{N} x_i(t)\,U_i(u)\right)^3 \\ =&\delta\left(u\sum_{i=0}^{3N} X_i(t)\,U_i(u)\right) \\ =&\frac{1}{2}\delta\sum_{i=0}^{3N} X_i(t)\,[U_{i-1}(u)+U_{i+1}(u)] \\ =&\frac{1}{2}\delta\sum_{i=0}^{3N}[X_{i-1}(t)+X_{i+1}(t)]U_i(u), \end{aligned} \tag{13}$$

where X_{-1} and $X_{N+1}$are supposed to be zero due to the approximation, Eq.(8). Substituting Eqs.(12), (13) into Eq.(9), we have

$$\begin{aligned} &\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+a\frac{\mathrm{d}}{\mathrm{d}t}\right)\sum_{i=0}^{N} x_i(t)U_i(u) \\ &+\frac{1}{3}c\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=0}^{3N} X_i(t)U_i(u) \\ &+\bar{b}\sum_{i=0}^{3N} X_i(t)U_i(u) \\ &+\frac{1}{2}\delta\sum_{i=0}^{3N}[X_{i+1}(t)+X_{i-1}(t)]U_i(u)=f(t). \end{aligned} \tag{14}$$

Multiplying both sides of Eq.(14) by $U_i(u)$in sequence and taking expectation, owing to the orthogonality of the Chebyshev polynomial, we finally have

$$\begin{aligned} &\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+a\frac{\mathrm{d}}{\mathrm{d}t}\right)x_0(t)+\frac{1}{3}c\frac{\mathrm{d}}{\mathrm{d}t}X_0(t)+\bar{b}X_0(t)+\frac{1}{2}\delta X_1(t)=f(t), \\ &\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+a\frac{\mathrm{d}}{\mathrm{d}t}\right)x_1(t)+\frac{1}{3}c\frac{\mathrm{d}}{\mathrm{d}t}X_1(t)+\bar{b}X_1(t)+\frac{1}{2}\delta[X_0(t)+X_2(t)]=0, \\ &\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+a\frac{\mathrm{d}}{\mathrm{d}t}\right)x_2(t)+\frac{1}{3}c\frac{\mathrm{d}}{\mathrm{d}t}X_2(t)+\bar{b}X_2(t)+\frac{1}{2}\delta[X_1(t)+X_3(t)]=0, \\ &\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+a\frac{\mathrm{d}}{\mathrm{d}t}\right)x_3(t)+\frac{1}{3}c\frac{\mathrm{d}}{\mathrm{d}t}X_3(t)+\bar{b}X_3(t)+\frac{1}{2}\delta[X_2(t)+X_4(t)]=0, \\ &\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}+a\frac{\mathrm{d}}{\mathrm{d}t}\right)x_4(t)+\frac{1}{3}c\frac{\mathrm{d}}{\mathrm{d}t}X_4(t)+\bar{b}X_4(t)+\frac{1}{2}\delta[X_3(t)+X_5(t)]=0. \end{aligned} \tag{15}$$

This is just the equivalent nonlinear deterministic system reduced by Chebyshev polynomial approximation for the stochastic DP system with a random parameter U of an arch-like PDF. When $N\to\infty$ in Eq.(8), $\sum_{i=0}^{N} x_i(t)\,U_i(u)$ is strictly equivalent to $x(t,u)$ of the stochastic system. Otherwise, if N is finite, Eq.(8) is just approximately valid with a minimal residual error. In the following numerical analysis, we take $N=4$ and obtain the numerical solution $x_i(t),(i=0,1,2,3,4)$ of Eq.(15) by effective numerical methods available. Then, the approximate random response of the original stochastic DP system can be expressed as

$$x(t,u)=\sum_{i=0}^{4} x_i(t)\,U_i(u). \tag{16}$$

If the random parameter $u=0$, namely $b=\bar{b}$, we obtain the so-called mean parameter system, which is significant in the following text. The sample response of the mean parameter system may be expressed as

$$\begin{aligned} x(t,0)=&\sum_{i=0}^{4} x_i(t)\,U_i(0) \\ =&x_0(t)-x_2(t)+x_4(t). \end{aligned} \tag{17}$$

Then, the ensemble mean response of the stochastic DP system can be obtained as

$$\begin{aligned} E[x(t,u)]=&\sum_{i=0}^{4} x_i(t)E[U_i(u)] \\ =&x_0(t). \end{aligned} \tag{18}$$

4. Stochastic period-doubling bifurcation and chaos in stochastic DP system

The original stochastic DP system may be rewritten as

$$\ddot{x}+\left(a+cx^2\right)\dot{x}+\left(\bar{b}+\delta u\right)x^3=f(t). \tag{19}$$

Its deterministic equivalent nonlinear system has been derived as Eq.(15). Now we apply numerical integration methods to Eq.(15) to explore nonlinear phenomena related to PD bifurcation and stochastic chaos in the original stochastic DP system. As $u = 0$, Eq.(19) reduces to the mean-parameter system, which is a deterministic one

$$\ddot{x} + \left(a + cx^2\right)\dot{x} + \bar{b}x^3 = f(t). \tag{20}$$

We will show the various types of responses corresponding to a series of specific initial conditions and excitations. Firstly, we find the random response $x(t,u)$ by Eq.(16); the sample response $x(t,0)$ by Eq.(17); and the ensemble mean response $E[x(t,u)]$ by Eq.(18). Then, the last one is further checked by deterministic response $x(t)$ of the mean parameter system (20) obtained directly by the Runge-Kutta method. Hence we offer the following three sets of response phase trajectories in the portraits: let SRM represent sample responses of the mean-parameter system $x(t,0)$; let DRM represent deterministic responses of the mean parameter system $x(t)$; and let EMR represent ensemble mean response of the stochastic DP system $E[x(t,u)]$. Let IC denote the initial conditions for the deterministic equivalent system Eq.(15):

$$\boldsymbol{x}(0) = [x_0(0), \cdots, x_4(0)]^{\mathrm{T}},$$
$$\boldsymbol{y}(0) = [\dot{x}_0(0), \cdots, \dot{x}_4(0)]^{\mathrm{T}},$$

and the initial conditions for the deterministic DP system Eq.(20) as $x_0 = x(0), y_0 = \dot{x}(0)$. In this paper, the parameter δ is taken as a small quantity, and we take the same initial conditions for the deterministic equivalent system Eq.(15) and deterministic DP system Eq.(20), namely

$$x_0 = x_0(0), y_0 = \dot{x}_0(0),$$

$$x_i(0) = \dot{x}_i(0) = 0, (i = 1, 2, 3, 4).$$

In this paper, the initial conditions are chosen as follows:

$$(\mathrm{IC}-1): \quad \begin{aligned} &\boldsymbol{x}(0) = [1.8, 0, 0, 0, 0]^{\mathrm{T}}, \\ &\boldsymbol{y}(0) = [2.5, 0, 0, 0, 0]^{\mathrm{T}}, \\ &x_0 = 1.8, y_0 = 2.5; \end{aligned}$$

$$(\mathrm{IC}-2): \quad \begin{aligned} &\boldsymbol{x}(0) = [3.0, 0, 0, 0, 0]^{\mathrm{T}}, \\ &\boldsymbol{y}(0) = [2.0, 0, 0, 0, 0]^{\mathrm{T}}, \\ &x_0 = 3.0, y_0 = 2.0; \end{aligned}$$

$$(\mathrm{IC}-3): \quad \begin{aligned} &\boldsymbol{x}(0) = [2.3, 0, 0, 0, 0]^{\mathrm{T}}, \\ &\boldsymbol{y}(0) = [1.84, 0, 0, 0, 0]^{\mathrm{T}}, \\ &x_0 = 2.3, y_0 = 1.84; \end{aligned}$$

$$(\mathrm{IC}-4): \quad \begin{aligned} &\boldsymbol{x}(0) = [1.3, 0, 0, 0, 0]^{\mathrm{T}}, \\ &\boldsymbol{y}(0) = [0.3, 0, 0, 0, 0]^{\mathrm{T}}, \\ &x_0 = 1.3, y_0 = 0.3; \end{aligned}$$

4.1. Stochastic period-doubling bifurcation phenomenon

It is proved in Ref.[19] that for a symmetrical nonlinear deterministic dynamical system, like the DP system, the PD bifurcation cannot happen to a response with self-symmetrical phase trajectory. In other words, PD bifurcation can only happen to a pair of responses resulting from symmetry-breaking bifurcation. This statement is most likely the same for the stochastic PD bifurcation.

In this case, system parameters are chosen as follows:

$$a = -0.1, \quad \bar{b} = 1.0, \quad c = 0.1, \quad \delta = 0.01, \quad F = 8.4;$$

while ω takes different values from 1.132 to 1.185. Numerical results for a few numbers of stochastic PD bifurcations are shown in Fig.2.

As we take $\omega = 1.185$, there are two mutually symmetric steady-state periodic trajectories with period $1T$, one of which is shown in Fig.2(a) corresponding to conditions (IC-1). Decreasing ω, for$\omega = 1.15$, there are two mutually symmetric steady-state periodic trajectories with period $2T$, one of which is shown in Fig.2(b) corresponding to conditions (IC-2). If $\omega = 1.132$, we find that there are two mutually symmetric steady-state periodic trajectories with period $4T$, one of which is shown in Fig.2(c) corresponding to conditions (IC-3).

From Fig.2, we can see that when ω decreases from 1.185 to 1.132, there are a set of periodic responses with period $1T$, $2T$ and $4T$, partly illustrating the stochastic PD bifurcation in the formal way in a stochastic system.

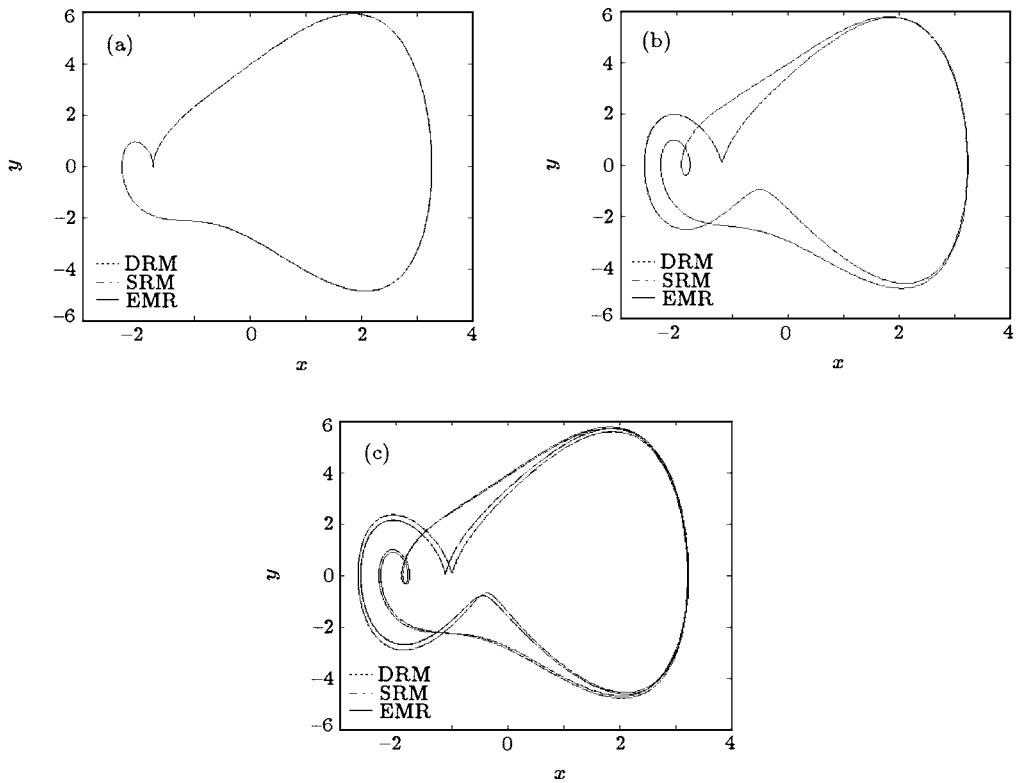


Fig.2. Phase portraits for illustration of PD bifurcation at $\omega = 1.185$ for (IC-1) (a), $\omega = 1.15$ for (IC-2) (b), and $\omega = 1.132$ for (IC-3) (c).

4.2. Features of stochastic bifurcation

In trying to explore the formal stochastic PD bifurcation from period $1T$ to $2T$, or $2T$ to $4T$, we find an interesting fact. When the random intensity of the random parameter in the stochastic Duffing–van der Pol system increases or decreases, the bifurcation behaviour can be influenced: namely, the bifurcation point might move. This case, we think, also exists in the transition of stochastic chaos in a stochastic nonlinear system. Our numerical simulations on the phenomena related to stochastic PD bifurcation have verified it

We do not change the other system parameters, only making $\delta = 0.04$. As we take $\omega = 1.185$ corresponding to initial condition (IC-4), there is a steady-state periodic trajectory with $1T$ for DRM which is shown in Fig.3(a), as well as a steady-state periodic trajectory with period $2T$ for EMR which is shown in Fig.3(b). Figure 3(c) is the local portrait of the pane in Fig.3(b). At another period-doubling point$\omega = 1.15$, corresponding to initial condition (IC-4), there is a steady-state periodic trajectory with period $2T$ for DRM, which is shown in Fig.3(d), as well as a steady-state periodic trajectory with period $4T$ for EMR, which is shown in Fig.3(e). From Fig.3, we know that with increasing random intensity δ, the bifurcation point of the period-doubling bifurcation can move in the stochastic system. If we further increase δ, the system can have a period-doubling bifurcation with a large period or chaotic motion.

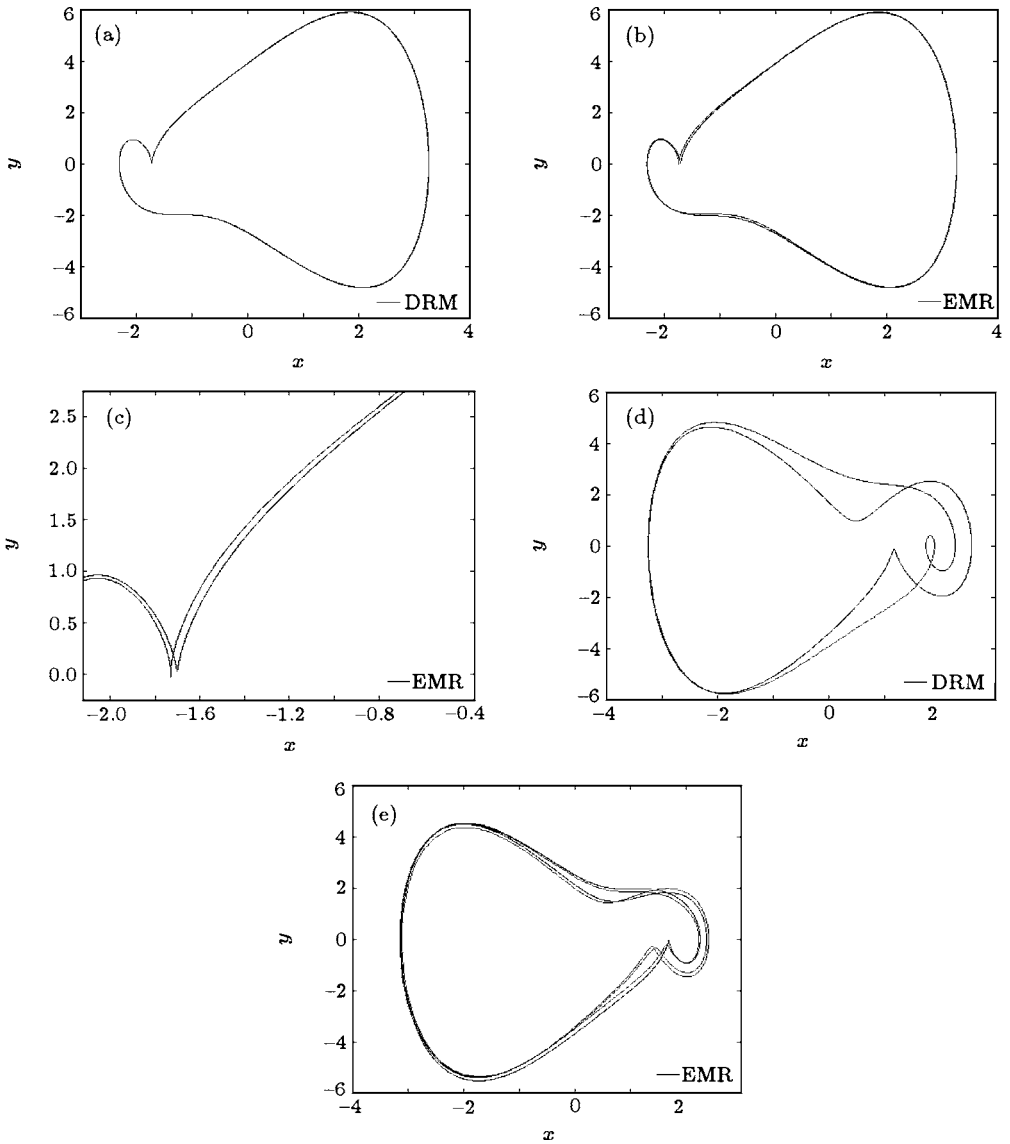


Fig.3. Phase portraits of (a) the DRM with $1T$ and (b) the EMR with period $2T$, for $\omega = 1.185$; (c) the local portraits of Fig.3(b). Phase portraits of (d) the DRM with $2T$ and (e) the EMR with period $4T$, for $\omega = 1.15$.

4.3. Stochastic chaos

It is well known that a series of PD bifurcations often pave the way to chaos in a deterministic nonlinear system. The similar way happens to stochastic chaos in a stochastic nonlinear dynamical system.

In this case, we take the system parameters same as those in section 4, but ω as $\omega = 1.08$. Figure 4 shows the Poincaré maps of DP system corresponding to initial condition (IC-1). The strange absorbers of DRM, SRM and EMR are shown in Figs.4(a), 4(b) and 4(c) respectively. By the algorithm listed in Ref.[20] the largest Lyapunov exponent of the deterministic system and the deterministic system equivalent to the stochastic Duffing–van der Pol system are 0.12756 and 0.2101 respectively. The chaos is so sensitive to various conditions that the strange absorber of the stochastic DP system keeps mainly the shape of the deterministic system after we increase the random factor.

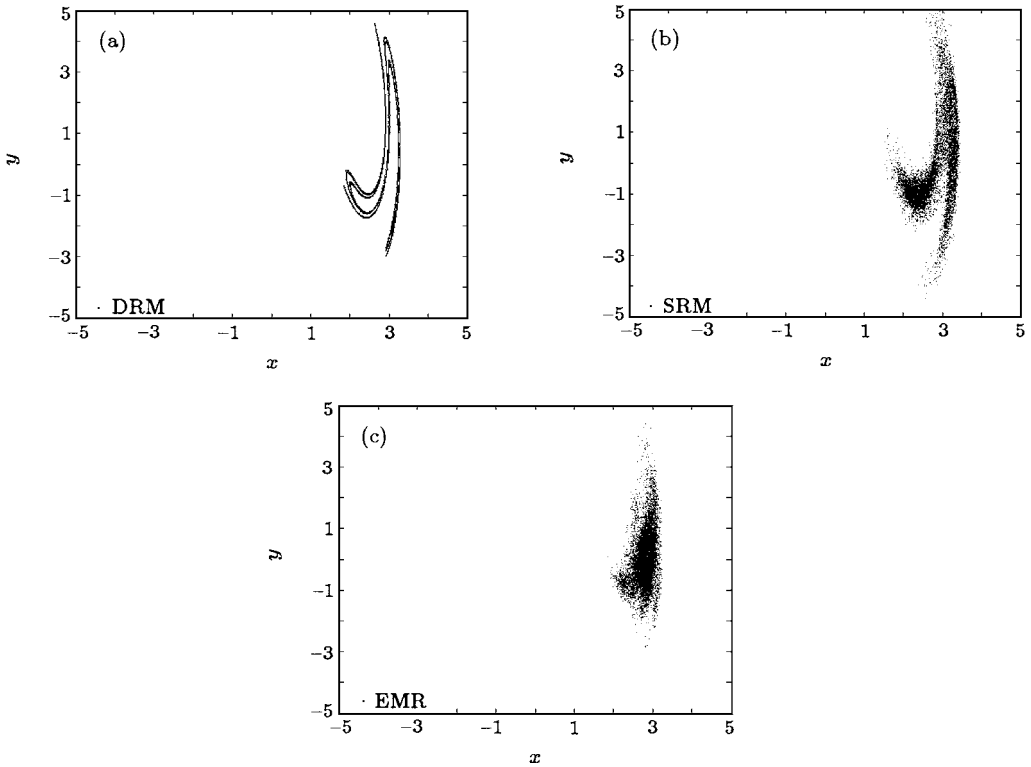


Fig.4. Poincaré maps of the chaotic state for $\omega = 1.08$ (a) DRM (b) SRM and (c) EMR.

5. Conclusions

The orthogonal polynomial approximation provides a new way to reduce a stochastic nonlinear system with bounded random parameter into an equivalent deterministic nonlinear system. Thus any numerical methods for a deterministic system can be applied to study the dynamical behaviour of a stochastic nonlinear system. In this paper, primary numerical results show that the equivalent deterministic nonlinear system preserves the nonlinear phenomena, such as period-doubling bifurcation and chaos of the Duffing–van der Pol system while the harmonic excitation amplitude ω is taken as bifurcation parameter. At the same time, we find that with increasing random intensity of the random variable, the bifurcation point of the period-doubling bifurcation can be moved. From the above, we can see that the proposed Chebyshev polynomial approximation could be a new approach to explore the abundant phenomena in stochastic DP systems.

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