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Thermodynamical quantities around a RNAdS black hole*

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The entropy density, energy density, pressure and equation of state around the RNAdS black hole are calculated in the WKB approximation on the Teukolsky-type master equation. The appearance of spin-dependent terms is demonstrated. The existence of these terms shows that the black hole radiation is not exactly thermal radiation and the black hole entropy is not strictly proportional to the area of the event horizon.

Keywords: thermodynamical quantities, equation of state, RNAdS black hole, WKB approximation, Boulware state

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1. Introduction

It is well known in flat spacetime that, using the quantization procedure referred to as Minkowski time and Minkowski vacuum, the entropy density σ, the energy density ρ, and the pressure P of a perfect relativistic gas are given in terms of the temperature T as follows:

$$\sigma = \frac{2\pi^2}{45}\omega^* T^3\,, \quad \rho = \frac{\pi^2}{30}\omega^* T^4\,, \quad P = \frac{\pi^2}{90}\omega^* T^4\,, \quad (1)$$

where ω^* is the number of bosonic degrees of freedom plus 7/8 times the number of fermionic degrees of freedom. The second formula in Eq.(1) is the Stefan–Boltzmann law. Obviously, Eq.(1) includes the equation of state

$$\rho - 3P = 0\,. \quad (2)$$

The characteristics of Eq.(1) are not dependent on the spin, but the fields are dependent on the number of degrees of freedom.

In curved spacetime, one generally assumes that the thermodynamical quantities take the same forms as in flat spacetime if T is replaced by the local Tolman temperature $T(x^a) = T_\infty/\sqrt{g_{00}}$, where T_∞ is the temperature measured by the observer at the spatial infinity. The physical reason can be traced back to the equivalence principle.[1,2] Recently, Hawking[3] has argued that quantum gravity is unitary and information is preserved in black hole formation and evaporation. This implies that the black hole radiation is not exactly thermal radiation and the black hole entropy is not strictly proportional to the area of the event horizon. In fact, in 2004, using the quantization procedure referred to as Boulware vacuum state[4] and Schwarzschild Killing time, the thermodynamical quantities around the Schwarzschild black hole were studied and it was found that near the event horizon there existed the subleading order terms with spin dependence beyond the expected Minkowskian high-temperature contribution.[5] Obviously, the result is interesting and useful for a deep understanding of the properties of black holes.

The purpose of the present paper is to extend this study to the Reissner–Nordström–anti-de Sitter (RNAdS) spacetime. The spacetime structure differs from that of the Schwarzschild and Reissner–Nordström black holes, since it is an asymptotical anti-de Sitter space. Anti-de Sitter space is a spacetime that has the maximal symmetry and a negative constant curvature, supported by a negative cosmological constant. As shown by Hawking and Page,[6] the

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presence of a negative cosmological constant makes it possible for black holes to reach thermal equilibrium with a heat bath. Since we are considering spin fields in a thermal state, it seems most appropriate to consider black holes for which this configuration is stable.

The remainder of this paper is organized as follows. In Section 2 we summarize the main properties of the RNAdS spacetime, including its Petrov classification. In Section 3 we review the quantization of the massless scalar ($s = 0$), neutrino ($s = 1/2$), electromagnetic ($s = 1$), massless Rarita–Schwinger ($s = 3/2$) and gravitational ($s = 2$) fields with respect to the Killing time t, and derive the Teukolsky-type master equation for these fields. In Section 4 we evaluate the entropy density, energy density, pressure and equation of state around the RNAdS black hole in the WKB approximation on the Teukolsky-type master equation. Finally we conclude with a discussion in Section 5.

2. RNAdS spacetime

The Reissner-Nordström black hole solution of Einstein's equation in free space with a negative cosmological constant $\Lambda = -3/\chi^2$ is given by (see e.g. Ref.[7])

$$\mathrm{d}s^2 = B\mathrm{d}t^2 - B^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2), \quad (3)$$

with

$$B = 1 - \frac{r_+}{r} - \frac{r_+^3}{\chi^2 r} - \frac{Q^2}{r_+ r} + \frac{Q^2}{r^2} + \frac{r^2}{\chi^2}. \quad (4)$$

The asymptotic form of this spacetime is anti-de Sitter. The mass of the black hole is given by

$$M = \frac{1}{2}\Big(r_+ + \frac{r_+^3}{\chi^2} + \frac{Q^2}{r_+}\Big). \quad (5)$$

The Hawking inverse temperature is given by the expression

$$\beta_H = 4\pi r_+\big(1 - \frac{Q^2}{r_+^2} + \frac{3r_+^2}{\chi^2}\Big)^{-1}. \quad (6)$$

In general, r_+ and the charge Q are independent, but in the extremal case they are relatedly

$$1 - \frac{Q^2}{r_+^2} + \frac{3r_+^2}{\chi^2} = 0. \quad (7)$$

The horizons are determined by $B = 0$. For a nonextremal RNAdS black hole, one has two positive real roots, $r_- < r_+$, which may be interpreted as the inner Cauchy horizon and the outer event horizon, respectively. The other roots, r_{+i} and r_{-i}, are a pair of complex conjugates and therefore unphysical, which satisfy

$$r_{+i} + r_{-i} = -(r_+ + r_-), \quad (8)$$

$$r_{+i}r_{-i} = \chi^2 + r_+^2 + r_-^2 + r_+ r_-. \quad (9)$$

For the extremal case where Eq.(7) is satisfied, the inner and outer horizons coincide.

In the spacetime expressed by Eq.(3), the null tetrad vectors are established as follows:

$$l^\mu = (B^{-1}, 1, 0, 0),$$
$$n^\mu = \frac{1}{2}(1, -B, 0, 0),$$
$$m^\mu = \frac{1}{\sqrt{2}r}(0, 0, 1, \mathrm{i}\csc\theta), \quad (10)$$

which satisfy $l_\mu n^\mu = -m_\mu \overline{m}^\mu = 1$, with all other inner products being zero.

Using the Newman–Penrose[8] formula, we can obtain from Eq.(3) and the expressions (10) the spin coefficients

$$\kappa = \pi = \varepsilon = \lambda = \sigma = \nu = \tau = 0,$$
$$\rho = -\frac{1}{r}, \qquad \mu = \frac{B}{2r},$$
$$\gamma = \frac{B'}{4}, \qquad \alpha = -\frac{\cot\theta}{2\sqrt{2}r} = -\beta, \quad (11)$$

and the tetrad components of the Weyl tensor

$$\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0,$$
$$\Psi_2 = -\frac{1}{2r^3}\Big(r_+ + \frac{r_+^3}{\chi^2} + \frac{Q^2}{r_+} - \frac{2Q^2}{r}\Big), \quad (12)$$

where the prime denotes the derivative with respect to r. Equations (11) and (12) tell us that the RNAdS metric is of Petrov type D.

3. Teukolsky-type master equation

In this section, we review the quantization of the massless scalar, neutrino, electromagnetic, massless Rarita–Schwinger, and gravitational fields with respect to the Killing time t and derive a master equation for these fields. The quantization of the spin fields has been done in many places before (see, for example, Refs.[9] and [10]), here we just sketch the procedure. For simplicity, we first consider a massless spin-0 field.

In curved spacetime, the massless spin-0 field is described by a scalar wavefunction Ψ with an action

$$I = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi. \quad (13)$$

We use the following mode decomposition for the quantum field Ψ:

$$\Psi = \sum_q (a_q \Phi_q + a_q^\dagger \Phi_q^*)\,, \tag{14}$$

where q represents the set of quantum numbers, a_q and $a_q^\dagger$ are called the annihilation and creation operators, respectively; they obey the standard commutation relations

$$[a_q, a_{q'}^\dagger] = \delta_{qq'}\,, \quad [a_q, a_{q'}] = [a_q^\dagger, a_{q'}^\dagger] = 0\,. \tag{15}$$

On the RNAdS spacetime background, the mode functions Φ_q read

$$\Phi_{Elm} = R_{lE}(r) Y_l^m(\theta,\varphi) \mathrm{e}^{-\mathrm{i}Et}\,. \tag{16}$$

Here $q = (Elm)$, $E \in [0,\infty)$ and l, m are integers satisfying the inequalities $l \geq 0$, $-l \leq m \leq l$. $Y_l^m(\theta,\varphi)$ is a spherical harmonic and $R_{lE}(r)$ satisfies the following equation:

$$\Big\{\frac{\mathrm{d}}{\mathrm{d}\,r}\Big[(r^2 B)\frac{\mathrm{d}}{\mathrm{d}\,r}\Big] + \frac{r^2E^2}{B} - l(l+1)\Big\} R_{lE}(r) = 0\,. \tag{17}$$

The normal ordered Hamiltonian of quantized scalar field is given by

$$:H: = \sum_q E a_q^\dagger a_q\,. \tag{18}$$

In the above discussion we have quantized the scalar field with respect to the Killing time t. Hence, the corresponding ground state is called the Boulware state $|B\rangle$, which is determined by

$$a_q |B\rangle = 0\,. \tag{19}$$

The Hilbert space of all quantum states can be constructed as a symmetric Fock space in the Boulware state, and the complete basis is defined as

$$|\{N_q\}\rangle = \prod_q \frac{1}{\sqrt{N_q!}} (a_q^\dagger)^{N_q} |B\rangle\,, \tag{20}$$

where N_q is an integer, and $N_q \geq 0$. The partition function Z is given by a trace in Fock space[9]

$$Z = \mathrm{Tr}\,\mathrm{e}^{-\beta_H :H:} = \prod_q (1 - \mathrm{e}^{-\beta_H E})^{-1}\,. \tag{21}$$

Now let us consider the gravitational, massless Rarita–Schwinger, electromagnetic and neutrino fields. In fact, the quantization procedure for electromagnetic and gravitational perturbations will probably differ very little from those for scalar waves except for the additional problem of gauge. For the quantization of massless Rarita–Schwinger and neutrino fields, it should be noted that the annihilation and creation operators obey the standard anticommutation relations, and neutrinos are purely 'left-handed'. Thus, in a type-D metric, mode functions of these fields satisfy the linearized equations[11,12]

$$\begin{aligned}\{&[D - (2s-1)\varepsilon + \overline{\varepsilon} - 2s\rho - \overline{\rho}](\Delta - 2s\gamma + \mu)\\ &- [\delta + \overline{\pi} - \overline{\alpha} - (2s-1)\beta - 2s\tau]\\ &\times (\overline{\delta} + \pi - 2s\alpha) - (2s-1)(s-1)\,\Psi_2\}\Phi_q = 0\,,\end{aligned} \tag{22}$$

for spin states $p = s = 1/2, 1, 3/2, 2$, and

$$\begin{aligned}\{&[\Delta + (2s-1)\gamma - \overline{\gamma} + 2s\mu + \overline{\mu}](D + 2s\varepsilon - \rho)\\ &- [\overline{\delta} - \overline{\tau} + \overline{\beta} + (2s-1)\alpha + 2s\pi]\\ &\times (\delta - \tau + 2s\beta) - (2s-1)(s-1)\,\Psi_2\}\Phi_q = 0\,,\end{aligned} \tag{23}$$

for spin states $p = -s = -1/2, -1, -3/2, -2$, where D, Δ, and δ are the directional derivatives given by

$$D = l^\mu \partial_\mu\,, \quad \Delta = n^\mu \partial_\mu\,, \quad \delta = m^\mu \partial_\mu\,. \tag{24}$$

Choosing normal modes to be positive frequency with respect to the Killing vector ∂_t of the metric (3), mode functions Φ_q in the RNAdS spacetime are given by

$$\Phi_q = \Phi_{Elmp} = r^{p-s}\, {}_pR_{lE}(r)\, {}_pY_l^m(\theta,\varphi)\, \mathrm{e}^{-\mathrm{i}Et}\,, \tag{25}$$

where $q = (Elmp)$. Using the spin coefficients, the components of the Weyl tensor and the directional derivatives written down in Eqs.(11), (12), and (24), we find that the radial and angular functions obey the equations

$$\begin{aligned}\Big\{&(r^2B)^{-p}\frac{\mathrm{d}}{\mathrm{d}\,r}\Big[(r^2B)^{p+1}\frac{\mathrm{d}}{\mathrm{d}\,r}\Big]\\ &+ \frac{r^4E^2 + \mathrm{i}Ep(2r^3B - r^4B')}{r^2B}\\ &+ \frac{1}{12}[s(2s-1)(s-1) + 6(s+p)]\\ &\times [(r^2B)'' - 2] - \lambda^2\Big\}\, {}_pR_{lE}(r) = 0\,,\end{aligned} \tag{26}$$

$$\begin{aligned}\Big[&\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\Big(\sin\frac{\partial}{\partial\theta}\Big) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{2\mathrm{i}p\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi}\\ &- p^2\cot^2\theta + p + \lambda^2\Big]\, {}_pY_l^m(\theta,\varphi) = 0\,.\end{aligned} \tag{27}$$

Equation (27) shows that ${}_pY_l^m(\theta,\varphi)$ is a spin-weighted spherical harmonic,[13] and the separation constant λ satisfies

$$\lambda = \sqrt{(l-p)(l+p+1)}\,. \tag{28}$$

Obviously, if $s = 0$, then $\Phi_{Elm0} = \Phi_{Elm}$. Therefore Eqs.(26) and (27) can also be shown to describe the behaviour of a massless scalar field on the RNAdS background. In the Schwarzschild limit, Eq.(26) and Teukolsky equation[14] coincide. Thus Eq.(26) can be named as the Teukolsky-type master equation. In a similar way to that in the case of scalar field, by using the single-particle spectrum, one can obtain the free energy F of the system in the following form:

$$-\beta_H F = \ln Z = \pm \sum_q \ln(1 \pm \mathrm{e}^{-\beta_H E}) . \tag{29}$$

Here the plus sign in Eq.(29) corresponds to the Fermi case, while the minus sign corresponds to the Bose case.

4. Calculation of thermodynamical quantities

In this section, we investigate the entropy density, energy density, pressure and equation of state around the RNAdS black hole by using the brick-wall model.[15] Although some people[16] have doubted of the validity and consistency of the brick-wall model, Mukohyama and Israel[9] showed that some objections to the brick-wall model could be overcome when the ground state was correctly identified. Meanwhile, they have shown that the local description of the statistical mechanics is equivalent to that of the quantized field on the curved background, which is defined globally, and whose ground state is the Boulware state.

For explicit calculation of the thermodynamical quantities of spin fields, we adopt the WKB approximation. Thus, the radial mode function $r^{p-s}{}_pR_{lE}(r)$ can be written as[17]

$$r^{p-s}{}_pR_{lE}(r) = \exp[\mathrm{i}S(r,p,l,E)] . \tag{30}$$

Then Eq.(26) yields the r-dependent wave number as

$$k^2(r,p,l,E) = [\partial_r S(r,p,l,E)]^2 = \frac{E^2}{B^2} + \frac{1}{r^2 B}\Big\{\frac{1}{12}[s(2s-1)(s-1)+6(s+p)][(r^2B)''-2] + \frac{1}{r}(s-p)(p+1)(r^2B)' + (s-p)(s-p-1)B - (l-p)(l+p+1)\Big\} . \tag{31}$$

According to Ref.[15], the energy spectrum of Eq.(26) is given by the semiclassical quantization condition

$$\int_V \mathrm{d}\, r k(r,p,l,E) = \pi n . \tag{32}$$

Here the quantum number $n > 0$ is integers, $V = \{(r,\theta,\varphi) \mid r_+ + \varepsilon \le r \le L\}$, where ε is a small positive constant. The total number of modes with energy less than or equal to E is given by

$$g(E) = \sum_p \sum_l (2l+1)n = \frac{1}{\pi}\sum_p \int_V \frac{\mathrm{d}\, r}{rB} \int_s^{l_{\max}} \mathrm{d}\, l(2l+1)[E^2r^2 - B(l-p)(l+p+1) + B(s-p) - B\eta(r,p)]^{1/2} = \frac{2}{3\pi}\sum_p \int_V \frac{\mathrm{d}\, r}{rB^2}[E^2r^2 - B\eta(r,p)]^{3/2} , \tag{33}$$

where

$$\eta(r,p) = s(s-p)\Big(\frac{r_+}{r} + \frac{r_+^3}{\chi^2 r} + \frac{Q^2}{r_+ r}\Big) - (s-p)(s-p-1)\frac{Q^2}{r^2} - \frac{r^2}{\chi^2}[s(2s-1)(s-1) + (s-p)(2p+3) + 6(s+p)]; \tag{34}$$

the upper limit $l_{\max}$ of integration with respect to the variable l is due to the fact that $k^2(r,p,l,E)$ has to be positive.

Substituting Eq.(33) into Eq.(29), we obtain the free energy,

$$F = \mp\frac{1}{\beta_H}\int \mathrm{d}\, E \frac{\mathrm{d}\, g(E)}{\mathrm{d}\, E}\ln(1 \pm \mathrm{e}^{-\beta_H E}) = -\frac{2}{3\pi}\sum_p \int_V \frac{\mathrm{d}\, r}{rB^2}\int [E^2r^2 - B\eta(r,p)]^{3/2}\frac{\mathrm{d}\, E}{\mathrm{e}^{\beta_H E} \pm 1} . \tag{35}$$

In Eq.(35), the requirement that the argument of the square root is positive bounds E as follows:

$$E^2 > \max\Big[0,\ \frac{B}{r^2}\eta(r,p)\Big]\ . \tag{36}$$

We can explicitly perform the E integral to find

$$F \approx -\frac{2\pi^3\omega^*}{45\beta_H^4}\int_V \frac{r^2}{B^2}\mathrm{d}\,r + \frac{\pi\gamma^*}{6\beta_H^2}\sum_p\int_V \frac{\eta(r,p)}{B}\mathrm{d}\,r\ . \tag{37}$$

Here γ^* is 1 for bosons and 1/2 for fermions. Hence the total entropy S and energy U are calculated as

$$\begin{aligned} S &= \beta_H^2\frac{\partial F}{\partial\beta_H} \\ &= \frac{8\pi^3\omega^*}{45\beta_H^3}\int_V \frac{r^2}{B^2}\mathrm{d}\,r - \frac{\pi\gamma^*}{3\beta_H}\sum_p\int_V \frac{\eta(r,p)}{B}\mathrm{d}\,r\,; \end{aligned} \tag{38}$$

$$\begin{aligned} U &= \frac{\partial}{\partial\beta_H}(\beta_H F) \\ &= \frac{2\pi^3\omega^*}{15\beta_H^4}\int_V \frac{r^2}{B^2}\mathrm{d}\,r - \frac{\pi\gamma^*}{6\beta_H^2}\sum_p\int_V \frac{\eta(r,p)}{B}\mathrm{d}\,r\ . \end{aligned} \tag{39}$$

On the other hand, the total entropy is given by the integral

$$S = \int_V \sigma(r)\ \frac{4\pi r^2\mathrm{d}\,r}{\sqrt{B}}\,, \tag{40}$$

where we have taken account of the proper volume element as given by the metric (3). The factor $1/\sqrt{B}$ does not, however, appear in the integral for the total energy of the thermal excitations:[1]

$$U = \int_V \rho(r)4\pi r^2\mathrm{d}\,r\,. \tag{41}$$

In comparison with Eqs.(38) and (39), the entropy density $\sigma(r)$ and the energy density $\rho(r)$ in Eqs.(40) and (41) are given by

$$\sigma(r) = \frac{2\pi^2}{45}\omega^* T^3(r) - \frac{\gamma^*}{12r^2}T(r)\sum_p\eta(r,p)\,; \tag{42}$$

$$\rho(r) = \frac{\pi^2}{30}\omega^* T^4(r) - \frac{\gamma^*}{24r^2}T^2(r)\sum_p\eta(r,p)\,. \tag{43}$$

Here the local temperature $T(r)$ is defined by the Tolman's law

$$T(r) = \frac{1}{\beta_H\sqrt{B}}\,. \tag{44}$$

Using the relation derived by Unruh and Wald[18]

$$P(r) = \sigma(r)T(r) - \rho(r)\,, \tag{45}$$

the pressure $P(r)$ is given by

$$P(r) = \frac{\pi^2}{90}\omega^* T^4(r) - \frac{\gamma^*}{24r^2}T^2(r)\sum_p\eta(r,p)\,. \tag{46}$$

The equation of state is

$$\rho - 3P = \frac{\gamma^*}{12r^2}T^2(r)\sum_p\eta(r,p)\,, \tag{47}$$

which is not zero.

5. Conclusions and discussion

we have investigated the entropy density, energy density, pressure and equation of state around the RNAdS black hole in the WKB approximation on the Teukolsky-type master equation. They are given by Eqs.(42), (43), (46) and (47), respectively. Similar to the Schwarzschild case,[5] any of these equations is the sum of two terms. The first term has completely the same form as that in flat spacetime, while the second term vanishes when $s = 0$ and therefore this term arises from the spin of the fields. The presence of this term leads to $\rho - 3P \neq 0$: namely, the equation of state does not satisfy the usual relation (2). The usual results that the entropy density, energy density and pressure take the same forms as in flat spacetime hold only for the leading term in powers of the temperature. In particular, at sufficiently low temperatures, for example, in near-extremal black hole cases, the spin-dependent terms are important. It is common knowledge that black-body radiation may be regarded as a perfect gas consisting of relativistic particles and the energy density is proportional to the fourth power of the temperature. Therefore, here the existence of the spin-dependent terms shows that the black hole radiation is not exactly thermal radiation and the black hole entropy is not strictly proportional to the area of the event horizon.

On the other hand, when cosmological constant $\Lambda = -3/\chi^2 = 0$, where the spacetime becomes an asymptotical flat one, the spin-dependent terms decrease as $1/r^3$ or more rapidly for large r, and hence the terms can be neglected more often than not. However, for an asymptotical anti-de Sitter spacetime ($\Lambda < 0$), at large r the first term falls off faster than the second term in any of Eqs.(42), (43) and (46), and therefore the spin-dependent terms cannot be neglected. Obviously, the deviation arises from the difference of the spacetime structures. It should be noted that when $Q = 0$ and $\Lambda = 0$, the results reduce to those in the case discussed in Ref.[5], and when $M = 0$, $Q = 0$, and $\Lambda = 0$, we recover the flat spacetime case.

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