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Probabilistically implementing nonlocal operations between two distant qutrits*

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We propose a method to probabilistically implement a nonlocal operation, $\exp[i\xi U_A U_B]$, between two distant qutrits A and B, where $\xi \in [0, 2\pi]$ and U_A, U_B are local unitary and Hermitian operations for qutrits A and B respectively. The consumptions of resource for one performance of the method are a single non-maximally entangled qutrit state and 1-trit classical communication. For a given ξ, the successful probability of the method depends on the forms of both entanglement resource and Bob's partial-measurement basis. We systematically discuss the optimal successful probabilities and their corresponding conditions for three cases: adjustable entanglement resource, adjustable partial-measurement basis, adjustable entanglement resource and partial-measurement basis. It is straightforward to generalize the method for producing nonlocal unitary operations between any two N-level systems.

Keywords: nonlocal operation, qutrit, optimal successful probability
PACC: 0367, 0365

Entanglement is a very mysterious and important phenomenon in quantum physics that has been studied extensively and deeply both in theory and in experiment. It plays a pivotal role in many aspects of quantum information and quantum computation, such as teleportation, dense coding, quantum cryptography and distributed quantum computation.[1] Recently, an interesting application of entanglement has been put forward that involves the implementation of nonlocal quantum gates or interactions between spatially separated systems, without actually moving the physical systems together.[2−13] The nonlocal unitary operations are essentially important to realize the distributed quantum computation,[14] because all quantum unitary operations between distributed systems can be realized by combining this kind of nonlocal gates with local operations. The key problem is how to implement the desired nonlocal gates with the optimal successful probability[7,8,15−18] using the given limited resources including entangled qubits and classical bits, which is still worthwhile to be studied. Recently an experimentally feasible scheme to realize the nonlocal gates has been proposed in cavity quantum electrodynamics (C-QED).[19]

In addition, quantum entanglement in higher-dimensional Hilbert spaces has potential applications in quantum information and quantum computation.[20−23] Since particles in higher-dimensional systems, i.e. qudits, can carry more information than particles in two-dimensional systems, i.e. qubits, they can increase information flux in quantum communication, as well as security in quantum cryptography by exploiting the higher-dimensional systems.[24,25] Further, higher-dimensional systems have advantages in fault-tolerant quantum computation and quantum error-correction.[26] Therefore, extending nonlocal operations to higher-dimensional systems is important.

For quantum computation and quantum information processing of higher-dimensional systems, qutrit system holds a very important position. This is because, on the one hand, qutrit is the simplest and most typical delegate of higher-dimensional systems. Many new characteristics in quantum compu-

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tation and quantum information processing of higher-dimensional systems may be observed by using a qutrit system. On the other hand, it is more important that qutrit-based quantum computation can optimize the Hilbert-space dimensionality and is expected to be more powerful than other qudit quantum computation.[27] Reznik *et al*[6] presented a method to deterministically implement a kind of nonlocal operation for arbitrary discrete dimensional systems by employing a state-operator tool. However, it requires consumption of too many entanglement resources and classical communications. Producing a nonlocal operation between two distant qutrits requires consuming a pair of maximally entangled qutrit states and 2-trit classical communication. Further, it only works well for maximally entangled resources. In this paper, we show an approach to the probabilistic implementation of a nonlocal operation between two spatially separated qutrits A and B, which only needs a single non-maximally entangled state and 1-trit classical communication. This nonlocal operation has the following general form:

$$U_{AB}(\xi) = \exp[\mathrm{i}\xi U_A U_B]\,, \qquad \xi \in [0, 2\pi] \tag{1}$$

with U_A and U_B being local unitary and Hermitian operations on qutrits A and B respectively. For a given ξ, the successful probability depends on the forms of both entanglement resource and the partial-basis measurement performed by Bob. We will show the optimal successful probabilities and their corresponding conditions for three cases: i) only entanglement resource is adjustable; ii) only the partial-measurement basis is adjustable; iii) both entanglement resource and the partial-measurement basis are adjustable.

Let us consider two qutrit systems A and B that are remotely separated in space, the partners Alice and Bob previously shared an accessorial entangled qutrit state

$$|\Psi_{ab}\rangle = \lambda_0 |0_a 0_b\rangle + \lambda_1 |1_a 1_b\rangle + \lambda_2 |2_a 2_b\rangle\,. \tag{2}$$

Here, the accessorial qutrits a and b belong to Alice and Bob respectively. For simplicity, we assume that the coefficients λ_i are real and normalized: $\lambda_0^2 + \lambda_1^2 + \lambda_2^2 = 1$. We can easily obtain the entanglement of this state expressed as

$$E(|\Psi_{ab}\rangle) = -\sum_{i=0}^{2} \lambda_i^2 \log(\lambda_i^2)\,. \tag{3}$$

As usual, we use $\log(x)$ to denote logarithms to base 2.

In order to obtain the expected nonlocal operations on qutrits A and B, Alice and Bob firstly perform the following local unitary operation respectively:

$$\begin{aligned} U_{aA} =& |0_a\rangle\langle 0_a| \otimes I_A + |1_a\rangle\langle 1_a| \otimes U_A \\ &+ |2_a\rangle\langle 2_a| \otimes U_A^2\,, \\ U_{bB} =& |0_b\rangle\langle 0_b| \otimes I_B + |1_b\rangle\langle 1_b| \otimes U_B \\ &+ |2_b\rangle\langle 2_b| \otimes U_B^2\,. \end{aligned} \tag{4}$$

Here U_{aA} (U_{bB}) denotes that the unitary operation is performed between accessorial qutrit a (b) and target particle A (B). We assume that U_A and U_B are both unitary and Hermitian local operations for target particles A and B, i.e. they satisfy $U_A = U_A^+ = U_A^{-1}$, $U_B = U_B^+ = U_B^{-1}$. After performing these local unitary operations, they acquire a stator[6] in the following form:

$$\begin{aligned} S =& \lambda_0 |0_a 0_b\rangle \otimes I_A I_B + \lambda_1 |1_a 1_b\rangle \otimes U_A U_B \\ &+ \lambda_2 |2_a 2_b\rangle \otimes U_A^2 U_B^2\,. \end{aligned} \tag{5}$$

Introducing a new set of basis for the accessorial qutrit a

$$\begin{aligned} |0'_a\rangle &= \frac{1}{\sqrt{3}}[|0_a\rangle - \mathrm{i}|1_a\rangle + |2_a\rangle]\,, \\ |1'_a\rangle &= \frac{1}{\sqrt{3}}[|0_a\rangle - \mathrm{i}\mathrm{e}^{(-2\pi\mathrm{i}/3)}|1_a\rangle + \mathrm{e}^{(-4\pi\mathrm{i}/3)}|2_a\rangle]\,, \\ |2'_a\rangle &= \frac{1}{\sqrt{3}}[|0_a\rangle - \mathrm{i}\mathrm{e}^{(-4\pi\mathrm{i}/3)}|1_a\rangle + \mathrm{e}^{(-2\pi\mathrm{i}/3)}|2_a\rangle]\,, \end{aligned} \tag{6}$$

then the stator S can be rewritten as follows:

$$S = \frac{1}{\sqrt{3}}(S'_b|0'_a\rangle + S''_b|1'_a\rangle + S'''_b|2'_a\rangle)\,, \tag{7}$$

where

$$\begin{aligned} S'_b =& \lambda_0|0_b\rangle \otimes I_A I_B + \mathrm{i}\lambda_1|1_b\rangle \otimes U_A U_B \\ &+ \lambda_2|2_b\rangle \otimes U_A^2 U_B^2\,, \end{aligned} \tag{8}$$

$$\begin{aligned} S''_b =& \lambda_0|0_b\rangle \otimes I_A I_B + \mathrm{i}\mathrm{e}^{(2\pi\mathrm{i}/3)}\lambda_1|1_b\rangle \otimes U_A U_B \\ &+ \mathrm{e}^{(4\pi\mathrm{i}/3)}\lambda_2|2_b\rangle \otimes U_A^2 U_B^2\,, \end{aligned} \tag{9}$$

$$\begin{aligned} S'''_b =& \lambda_0|0_b\rangle \otimes I_A I_B + \mathrm{i}\mathrm{e}^{(4\pi\mathrm{i}/3)}\lambda_1|1_b\rangle \otimes U_A U_B \\ &+ \mathrm{e}^{(2\pi\mathrm{i}/3)}\lambda_2|2_b\rangle \otimes U_A^2 U_B^2\,. \end{aligned} \tag{10}$$

Apparently, if Alice now performs a measurement on the stator S of Eq.(7) with respect to the accessorial qutrit a in the basis of Eq.(6), then the post-measurement stator would be S'_b, S''_b or S'''_b, each taking place with a probability 1/3. Afterward, Alice

transmits his measurement outcome (1-trit classical information) to Bob, and Bob will do nothing or perform the transformations

$$\widehat{C}_1 = |0_b\rangle\langle 0_b| + \mathrm{e}^{-2\pi \mathrm{i}/3}|1_b\rangle\langle 1_b| + \mathrm{e}^{-4\pi \mathrm{i}/3}|2_b\rangle\langle 2_b|,$$
$$\widehat{C}_2 = |0_b\rangle\langle 0_b| + \mathrm{e}^{-4\pi \mathrm{i}/3}|1_b\rangle\langle 1_b| + \mathrm{e}^{-2\pi \mathrm{i}/3}|2_b\rangle\langle 2_b|, \quad (11)$$

on his accessorial qutrits b, conditional on his received message $|0'_a\rangle$, $|1'_a\rangle$ or $|2'_a\rangle$. As

$$\widehat{C}_1 S''_b = \widehat{C}_2 S'''_b = S'_b,$$

thus Bob will always come back to the stator S'_b no matter what information he receives.

Now, Bob performs a measurement on the stator S'_b in the following partial basis:

$$|\Phi\rangle_b = \alpha|0_b\rangle + \beta|1_b\rangle + \gamma|2_b\rangle. \quad (12)$$

For simplicity, we assume that the coefficients are real and satisfy the normalization: $\alpha^2+\beta^2+\gamma^2 = 1$. With the probability

$$P = (\lambda_0\alpha + \lambda_2\gamma)^2 + (\lambda_1\beta)^2, \quad (13)$$

we will obtain a nonlocal unitary operation

$$U_{AB} = [(\lambda_0\alpha + \lambda_2\gamma) + \mathrm{i}\lambda_1\beta U_A U_B]/\sqrt{P}. \quad (14)$$

In the process of deriving these equations, we have used the unitary and Hermitian properties of U_A and U_B i.e. $U_A^2 = U_B^2 = 1$. The unitary operator U_{AB} can further be rewritten as

$$U_{AB}(\xi) = \exp[\mathrm{i}\xi U_A U_B] \quad (15)$$

with

$$\xi = \arctan\Big[\frac{\lambda_1\beta}{\lambda_0\alpha + \lambda_2\gamma}\Big]. \quad (16)$$

From the above deduction, we see clearly that the consumptions of resource for probabilistically producing $U_{AB}(\xi)$ are a single (usually non-maximally) entangled qutrit state and 1-trit classical communication from Alice to Bob. Of course, they need also to perform some local unitary operations and measurements. Compared with the method in Ref.[6] that needs to consume a pairs of maximally entangled qutrit states and 2-trit classical communication for deterministically producing a nonlocal operation, our method can reduce half of consumptions of resource. In addition, taking into account the effects of quantum decoherence or some other factors, it is usually difficult to create maximally entangled states. Therefore, our method that uses a non-maximally entangled resource to implement nonlocal operations on qutrits is practical.

It is worthwhile to point out that the method to produce nonlocal operations on qutrit systems is, in a sense, a generalization of the method in Ref.[15] for qubit systems. Indeed, if we regard qutrit operators U_A and U_B as the generalization of qubit operators σ_{n_A} and σ_{n_B}, then our method is in principle very similar to theirs. Maybe, the only visible difference is that, instead of the rotation before σ_z-measurement (see Eq.(14) in Ref.[15]) we execute directly a partial-basis measurement.

Note that the nonlocal unitary operation $U_{AB}(\xi)$ and the corresponding successful probability P depend on the forms of not only entangled resource but also the partial-measurement basis, i.e. depend on the parameters λ_0, λ_1, λ_2 and α, β, γ. In practical applications, ξ is usually a given constant, but the parameters of entangled resource and the partial-measurement basis may be partially or overall adjustable. In what follows, we discuss the problem of how to optimize (i.e. maximize) the successful probability P according to the adjustability of parameters λ_0, λ_1, λ_2 or/and α, β, γ, for a given ξ. We now illuminate this problem in three distinguishable cases.

i) Only the parameters of entanglement resource, λ_0, λ_1, λ_2, are adjustable. In this case, parameters α, β, γ and ξ are all given constants. The problem of optimizing the successful probability P is equivalent to that of finding the maximum of P, under the restricted conditions of $\lambda_0^2 + \lambda_1^2 + \lambda_2^2 = 1$ and $\lambda_1\beta/(\lambda_0\alpha + \lambda_2\gamma) = \tan\xi$. This problem can be easily solved using the method of Lagrange multipliers. By constructing an auxiliary function

$$\begin{aligned} F(\lambda_0,\lambda_1,\lambda_2) =& [(\alpha\lambda_0 + \gamma\lambda_2)^2 + (\beta\lambda_1)^2] \\ &+ \eta_1(\lambda_0^2 + \lambda_1^2 + \lambda_2^2 - 1) \\ &+ \eta_2[\beta\lambda_1 - (\alpha\lambda_0 + \gamma\lambda_2)\tan(\xi)] \end{aligned} \quad (17)$$

with η_1 and η_2 being Lagrange multipliers, and letting its Partial derivatives equal zero, we have

$$\frac{\partial F}{\partial \lambda_0} = \frac{\partial F}{\partial \lambda_1} = \frac{\partial F}{\partial \lambda_2} = 0. \quad (18)$$

Combining this set of equations with restricted conditions of

$$\lambda_0^2 + \lambda_1^2 + \lambda_2^2 = 1\,, \quad \lambda_1\beta/(\lambda_0\alpha + \lambda_2\gamma) = \tan\xi,$$

we can easily obtain the values of λ_i, which, corresponding to the optimal successful probability, are

$$\lambda_0 = \frac{\alpha\beta}{\sqrt{[(\alpha^2+\gamma^2)\tan^2\xi+\beta^2](\alpha^2+\gamma^2)}}, \quad (19)$$

$$\lambda_1 = \tan\xi\sqrt{\frac{\alpha^2+\gamma^2}{(\alpha^2+\gamma^2)\tan^2\xi+\beta^2}}, \quad (20)$$

$$\lambda_2 = \frac{\beta\gamma}{\sqrt{[(\alpha^2+\gamma^2)\tan^2\xi+\beta^2](\alpha^2+\gamma^2)}}. \quad (21)$$

Then the optimal successful probability can be written as

$$P_1^{\rm opt} = \frac{\beta^2(\alpha^2+\gamma^2)(1+\tan^2\xi)}{(\alpha^2+\gamma^2)\tan^2\xi+\beta^2}. \quad (22)$$

Especially, if we set $\alpha = \beta = \gamma = 1/\sqrt{3}$, i.e. Bob's partial-measurement basis is fixed in a uniform superposition state, then the optimal successful probability becomes $2(1+\tan^2\xi)/(3+6\tan^2\xi)$. The variational relation for this special optimal successful probability with ξ is plotted in Fig.1 (solid line). It is clear that the curve exhibits regular periodicity, with its maximal value 2/3 occurring for $\xi = 0,\ \pi,\ 2\pi$ and minimal value 1/3 occurring for $\xi = \pi/2,\ 3\pi/2$.

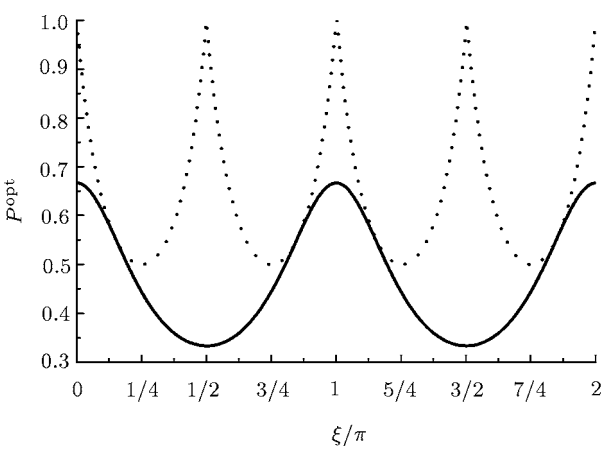


Fig.1. The optimal successful probability is plotted as a function of ξ. The solid line represents the case of adjustable entanglement resource, which is plotted according to Eq.(22) with $\alpha = \beta = \gamma = 1/\sqrt{3}$. The dotted line represents the case where both entanglement resource and partial measurement basis are adjustable, which is plotted according to Eq.(27).

That is to say, if Bob performs his partial measurement in a uniform superposition basis (i.e. $\alpha = \beta = \gamma = 1/\sqrt{3}$), by designing the parameters of entanglement resource to be satisfied by Eqs.(19)–(21), then the successful probability for producing the nonlocal operation (15) will not be smaller than 1/3 for any given $\xi \in [0, 2\pi]$, and the maximal successful probability will be close to 2/3. Note that when $\xi = 0,\ \pi$ or 2π, the nonlocal operation (15) becomes trivial. Thus here the maximal successful probability can not reach 2/3 exactly. Also note that when $\xi = \pi/2$ or $3\pi/2$, the nonlocal operation degenerates to local operations U_A and U_B.

ii) Only the parameters of partial-measurement basis, α, β, γ, are adjustable. Instead of the adjustment of entanglement resource, now Bob's partial-measurement basis becomes adjustable. This case is maybe more consistent with practicality, because in most practical circumstances, it may be difficult to alter randomly the entanglement resource. Employing the method of Lagrange multipliers, and noting that the restricted conditions for this case become $\alpha^2+\beta^2+\gamma^2 = 1$ and $\lambda_1\beta/(\lambda_0\alpha+\lambda_2\gamma) = \tan\xi$, we can in a similar way deduce the optimal successful probability to be

$$P_2^{\rm opt} = \frac{\lambda_1^2(\lambda_0^2+\lambda_2^2)\left(1+\tan^2\xi\right)}{(\lambda_0^2+\lambda_2^2)\tan^2\xi+\lambda_2^2}, \quad (23)$$

and the corresponding values of adjustable parameters to be

$$\alpha = \frac{\lambda_0\lambda_1}{\sqrt{\left(\lambda_0^2+\lambda_2^2\right)^2\tan^2\xi+\lambda_1^2\left(\lambda_0^2+\lambda_2^2\right)}}, \quad (24)$$

$$\beta = \frac{\sqrt{\lambda_0^2+\lambda_2^2}\tan\xi}{\sqrt{(\lambda_0^2+\lambda_2^2)\tan^2\xi+\lambda_1^2}}, \quad (25)$$

$$\gamma = \frac{\lambda_2\lambda_1}{\sqrt{\left(\lambda_0^2+\lambda_2^2\right)^2\tan^2\xi+\lambda_1^2\left(\lambda_0^2+\lambda_2^2\right)}}. \quad (26)$$

That is to say, in the case of a given entanglement resource, we can adjust the parameters of Bob's partial-measurement basis to be satisfied by Eqs.(24)–(26) so that the successful probability for producing the nonlocal operation (15) reaches its optimal value expressed by Eq.(23). In Fig.2, we show the variation of $P_2^{\rm opt}$ with ξ for three different sets of entanglement-resource parameters: $\lambda_0 = \lambda_1 = \lambda_2 = 1/\sqrt{3}$ (solid line), $\lambda_0 = \lambda_2 = 1/\sqrt{6}$ and $\lambda_1 = \sqrt{2/3}$ (dotted line), $\lambda_0 = \lambda_1 = 1/\sqrt{6}$ and $\lambda_2 = \sqrt{2/3}$ (dashed line). It is clearly seen from the figure that the curves each take on some regular periodicity. Further, for a given ξ, which means the production of a given nonlocal operation $U_{AB}(\xi)$, when the amount of entanglement of the entangled resource is maximal (i.e. when $\lambda_0 = \lambda_1 = \lambda_2 = 1/\sqrt{3}$), the successful probability is

not always maximal.

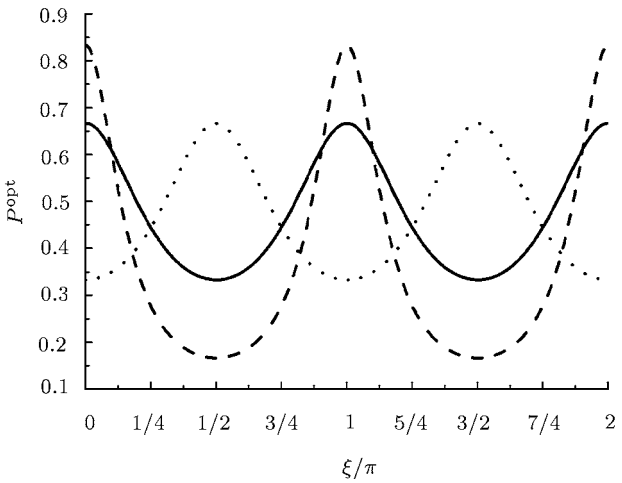


Fig.2. In the case where only Bob's partial measurement basis is adjustable, the optimal successful probability expressed by Eq.(23) is plotted as a function of ξ for three different sets of entanglement-resource parameters: $\lambda_0 = \lambda_1 = \lambda_2 = 1/\sqrt{3}$ (solid line), $\lambda_0 = \lambda_2 = 1/\sqrt{6}$ and $\lambda_1 = \sqrt{2/3}$ (dotted line), $\lambda_0 = \lambda_1 = 1/\sqrt{6}$ and $\lambda_2 = \sqrt{2/3}$ (dashed line).

iii) The parameters of both entanglement resource and partial-measurement basis are adjustable. It will be seen that in this case the optimal successful probability is greatest, i.e. greater than either of the probabilities for the previous two cases. Taking into account the restricted conditions, $\lambda_0^2 + \lambda_1^2 + \lambda_2^2 = 1$, $\alpha^2 + \beta^2 + \gamma^2 = 1$ and $\lambda_1\beta/(\lambda_0\alpha + \lambda_2\gamma) = \tan\xi$, then the method of Lagrange multipliers gives the optimal successful probability and the corresponding parameter values as

$$P_3^{\rm opt} = \frac{1 + |\tan\xi|^2}{(1 + |\tan\xi|)^2}, \tag{27}$$

$$\lambda_1 = \pm\sqrt{\frac{|\tan\xi|}{1 + |\tan\xi|}}, \tag{28}$$

$$\beta = \pm\sqrt{\frac{|\tan\xi|}{1 + |\tan\xi|}}, \tag{29}$$

$$\lambda_0^2 + \lambda_2^2 = \frac{1}{1 + |\tan\xi|}, \tag{30}$$

$$\frac{\alpha}{\lambda_0} = \frac{\gamma}{\lambda_2} = 1. \tag{31}$$

The choice of '$\pm$' for λ_1 and β should obey rules: they have the same signs for ξ belonging to quadrants 1 and 3, and opposite signs for ξ belonging to quadrants 2 and 4. Note, in this case, that the parameters λ_0 and λ_2 (and correspondingly α and γ) are not completely determined. It is sufficed as long as Eqs.(30) and (31) are fulfilled. In practical applications, we can choose those values of λ_0 and λ_2 that minimize the amount of entanglement of entangled resource. That's to say, in a given successful probability, we should try using the smallest entanglement consumption to produce a given nonlocal operation. In Fig.1 (dotted line), we show the optimal successful probability $P_3^{\rm opt}$ as a function of parameter ξ. Obviously, it also possesses regular periodicity. The minimum of successful probability is 1/2 which occurs at $\xi = \pi/4$, $3\pi/4$, $5\pi/4$ or $7\pi/4$, and the maximum is close to unit which occurs as ξ approaches 0, $\pi/2$, π, $3\pi/2$ or 2π. In addition, it is also clear that, for any given ξ, the successful probability $P_3^{\rm opt}$ is always larger than the successful probabilities $P_1^{\rm opt}$ (solid line in Fig. 1) and $P_2^{\rm opt}$ (see Fig.2). That is to say, when both the entangled resource and the partial measurement basis are adjustable, the successful probability of producing the nonlocal operation (15) reaches its maximum, in other words, the method works in its optimal state.

The nonlocal operator $U_{AB}(\xi)$ represents the broad nonlocal operations or interactions in the sense that the local unitary operations U_A and U_B are alterable. For example, if we regard the qutrits as the spin-1 particle, with eigenstates of L_z denoted by $|0\rangle$, $|1\rangle$, $|2\rangle$ and the corresponding eigenvalues -1, 0, 1 respectively, then a straightforward choice for U_A and U_B may be written as

$$U_A = \exp(\mathrm{i}\pi L_{n_A}), \qquad U_B = \exp(\mathrm{i}\pi L_{n_B}), \tag{32}$$

where L_{n_A} and L_{n_B} are the n ($n = x, y, z$) components of angular momentum $\boldsymbol{L}$ for qutrits A and B. Correspondingly, the nonlocal operation in this case can be simplified as

$$U_{AB}(\xi) = \mathrm{e}^{\mathrm{i}\xi}\mathrm{e}^{-\mathrm{i}2\xi L_{n_A}^2}\mathrm{e}^{-\mathrm{i}2\xi L_{n_B}^2}\mathrm{e}^{\mathrm{i}4\xi L_{n_A}^2 L_{n_B}^2}. \tag{33}$$

Except some local operations $\exp(-\mathrm{i}2\xi L_{n_A}^2)$ and $\exp(-\mathrm{i}2\xi L_{n_B}^2)$, this operation is equivalent to the nonlocal interaction $\exp(\mathrm{i}4\xi L_{n_A}^2 L_{n_B}^2)$.

This nonlocal operation $U_{AB}(\xi)$ is interesting. Specially, if we set $L_{n_A} = L_{x_A}$, $L_{n_B} = L_{x_B}$ and let the initial state of the target qutrits be

$$|\Psi_{AB}\rangle = |0_A 0_B\rangle,$$

then we have

$$U_{AB}(\xi)|\Psi_{AB}\rangle = \cos\xi|0_A 0_B\rangle + \mathrm{i}\sin\xi|2_A 2_B\rangle. \tag{34}$$

For $\xi = \pi/4$, it can be regarded as a maximally entangled state with respect to effective levels $|0\rangle$ and $|2\rangle$ for particles A and B.

Finally, we point out that our method to construct nonlocal unitary operations can be generalized straightforwardly to any N-level system. For simplicity, we only outline the main clues as follows. The entangled resource shared by Alice and Bob now is $|\Psi_{ab}\rangle = \sum_{j=0}^{N-1} \lambda_j |j_a j_b\rangle$ with $\sum_{j=0}^{N-1} \lambda_j^2 = 1$. After Alice and Bob perform local unitary operations $U_{aA} = \sum_{j=0}^{N-1} |j\rangle_{aa}\langle j| \otimes U_A^j$ and $U_{bB} = \sum_{j=0}^{N-1} |j\rangle_{bb}\langle j| \otimes U_B^j$, we have the generalized counterpart of (5) as follows:

$$S = \sum_{j=0}^{N-1} \lambda_j |j_a j_b\rangle \otimes U_A^j U_B^j. \tag{35}$$

The measurement basis (6) is now generalized as follows:

$$|m\rangle_a' = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} (-\mathrm{i})^{j \bmod 2} \mathrm{e}^{-2\pi \mathrm{i} m j/N} |j\rangle_a \tag{36}$$

with $m = 0, 1, \cdots, N-1$. After this measurement, Alice has obtained a set of collapsed stators

$$S_b(m) = \sum_{j=0}^{N-1} (i)^{j \bmod 2} \mathrm{e}^{2\pi \mathrm{i} m j/N} \lambda_j |j\rangle_b \otimes U_A^j U_B^j, \tag{37}$$

each taking place with the probability $1/N$. Then Alice broadcasts her measurements $|0\rangle_a'$, $|1\rangle_a'$, $\cdots$, $|N-1\rangle_a'$ to Bob through 1-Nit classical communication. If Bob receives the information $|m\rangle_a'$, then he will perform a correcting transform $\widehat{C}(m) = \sum_{j=0}^{N-1} \mathrm{e}^{-2\pi \mathrm{i} m j/N} |j\rangle_{bb}\langle j|$. Thus he can always obtain a stator

$$S_b = \sum_{j=0}^{N-1} (i)^{j \bmod 2} \lambda_j |j\rangle_b \otimes U_A^j U_B^j, \tag{38}$$

regardless of what information being received. Finally, Bob performs a local partial measurement with respect to basis

$$|\Phi\rangle_b = \sum_{j=0}^{N-1} \alpha_j |j\rangle_b$$

with $\sum_{j=0}^{N-1} \alpha_j^2 = 1$. With the probability

$$P = \Big(\sum_{j\ (\mathrm{even})} \alpha_j \lambda_j \Big)^2 + \Big(\sum_{j\ (\mathrm{odd})} \alpha_j \lambda_j \Big)^2, \tag{39}$$

he obtains a nonlocal unitary gate for two remotely separated N-level systems A and B

$$U_{AB} = \Big[\sum_{j\ (\mathrm{even})} \alpha_j \lambda_j + \mathrm{i} \sum_{j\ (\mathrm{odd})} \alpha_j \lambda_j U_A U_B \Big] \Big/ \sqrt{P}. \tag{40}$$

Here, we have used the unitary and Hermitian properties of U_A and U_B, i.e. $U_A^2 = U_B^2 = 1$. In principle, we can also discuss the optimality of the successful probability (39) with respect to entanglement resource and Bob's partial-measurement basis $|\Phi\rangle_b$ in a similar way. Owing to the complexity of derivation, we omit it here.

In conclusion, we have proposed a method to probabilistically implement a nonlocal operation between two distant qutrits. The consumptions of resource for one performance of our method are a single non-maximally entangled qutrit state and 1-trit classical communication. The successful probability of the method depends on the forms of both entanglement resource and Bob's partial-measurement basis. We have systematically discussed the optimal successful probabilities and their corresponding conditions in three cases, i.e. adjustable entanglement resource, adjustable partial-measurement basis, adjustable entanglement resource and partial-measurement basis. For producing a given nonlocal operation, the successful probability in the last case is maximal that has its maximum close to unit and minimum never below 1/2. This kind of successful probability is already rather high. Taking into account the effects of quantum decoherence or some other factors on quantum entanglement channels, our method of using non-maximally entangled resource to implement nonlocal operations on qutrits is practical. We have also shown the possibility to generalize our method for producing nonlocal gates between any two N-level systems.

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